UCLA Computer Science

STAR AI RESEARCH LAB UCLA

From Probabilistic Circuits to Probabilistic Programs and Back

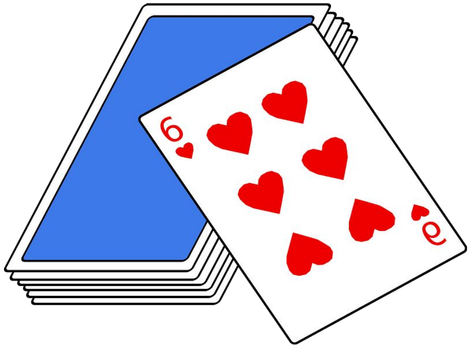
Guy Van den Broeck

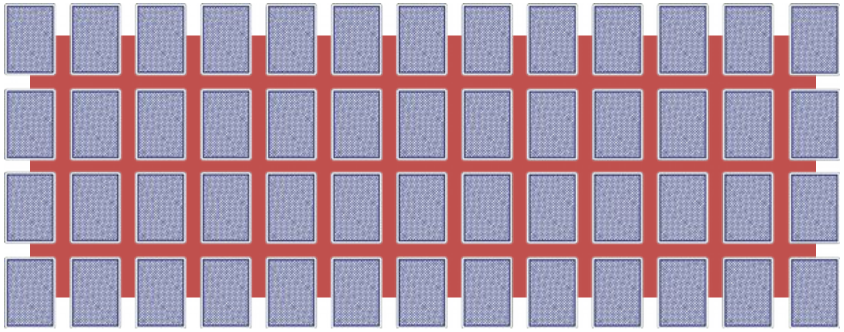
RWTH Aachen - Oct 6, 2020

Let me be provocative

Probabilistic graphical models is how we do probabilistic AI!

Graphical models of variable-level (in)dependence are a broken abstraction.





[VdB KRR15]

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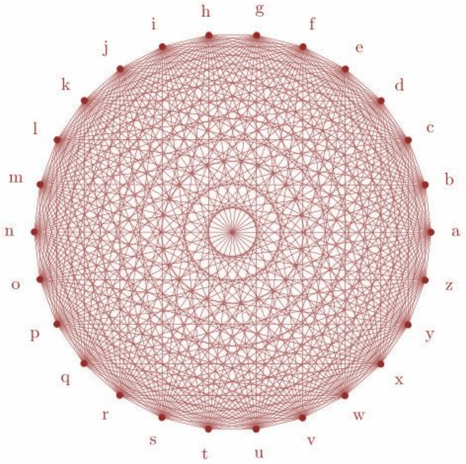
3.14 Smokes(x) $\wedge$ Friends(x,y) $\Rightarrow$ Smokes(y)



[VdB KRR15]

Let me be provocative

Probabilistic graphical models is how we do probabilistic AI!

Graphical models of variable-level (in)dependence are a broken abstraction.

Bean Machine

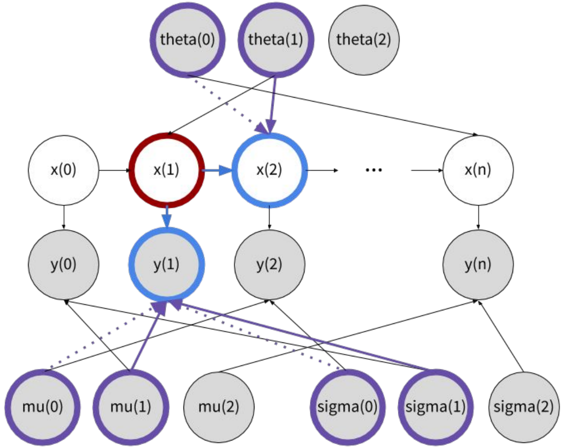
$$\mu_k \sim \text{Normal}(\alpha, \beta)$$

$$\sigma_k \sim \text{Gamma}(\nu, \rho)$$

$$\theta_k \sim \text{Dirichlet}(\kappa)$$

$$x_i \sim \begin{cases} \text{Categorical}(init) & \text{if } i = 0 \\ \text{Categorical}(\theta_{x_{i-1}}) & \text{if } i > 0 \end{cases}$$

$$y_i \sim \text{Normal}(\mu_{x_i}, \sigma_{x_i})$$



[Tehrani et al. PGM20]

Let me be provocative

We may have gotten stuck in a local optimum

The choice of representing a distribution primarily by its variable-level (in)dependencies is a little arbitrary…

What if we made some different choices?

Computational Abstractions

Let us think of probability distributions as objects that are computed.

Abstraction = Structure of Computation

‘closer to the metal’

Two examples:

1. Probabilistic Circuits
2. Probabilistic Programs

Probabilistic Circuits

Fully factorized

NaiveBayes AndOrGraphs PDGs

Trees PSDDs CNets LTMs SPNs NADEs

Thin Junction Trees ACs MADEs MAFs VAEs

DPPs FVSBNs TACs IAFs NAFs RAEs

Mixtures BNs NICE FGs GANs

RealNVP MNs

The *Alphabet Soup* of probabilistic models

Fully factorized

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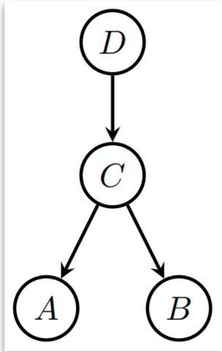
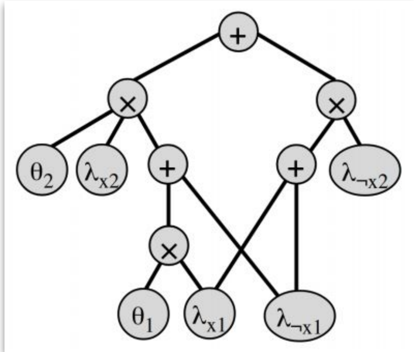
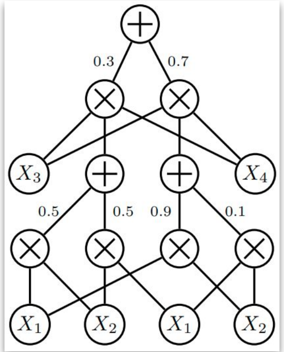
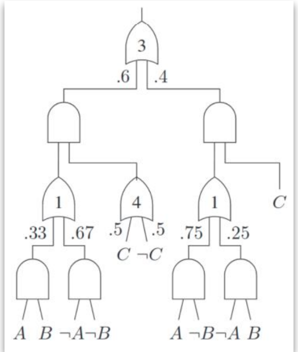
Intractable and *tractable* models

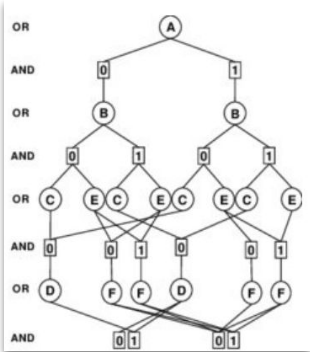
Tractable Probabilistic Models

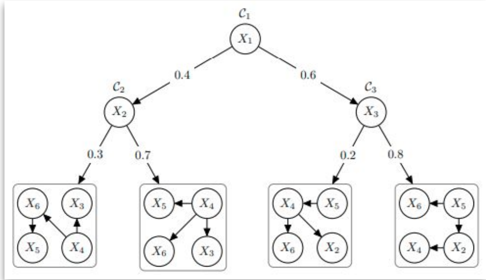


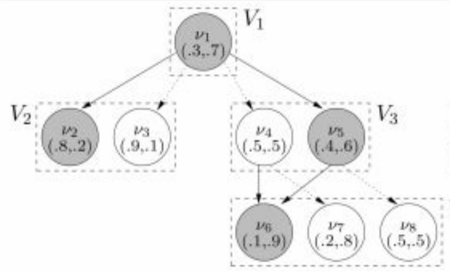


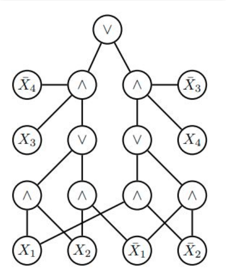


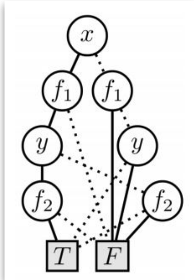


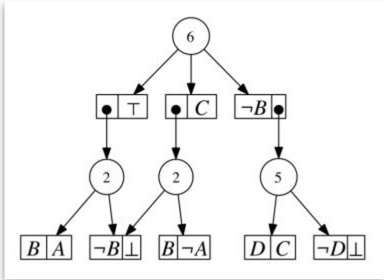












"*Every talk needs a joke and a literature overview slide, not necessarily distinct*"

- after Ron Graham

Fully factorized

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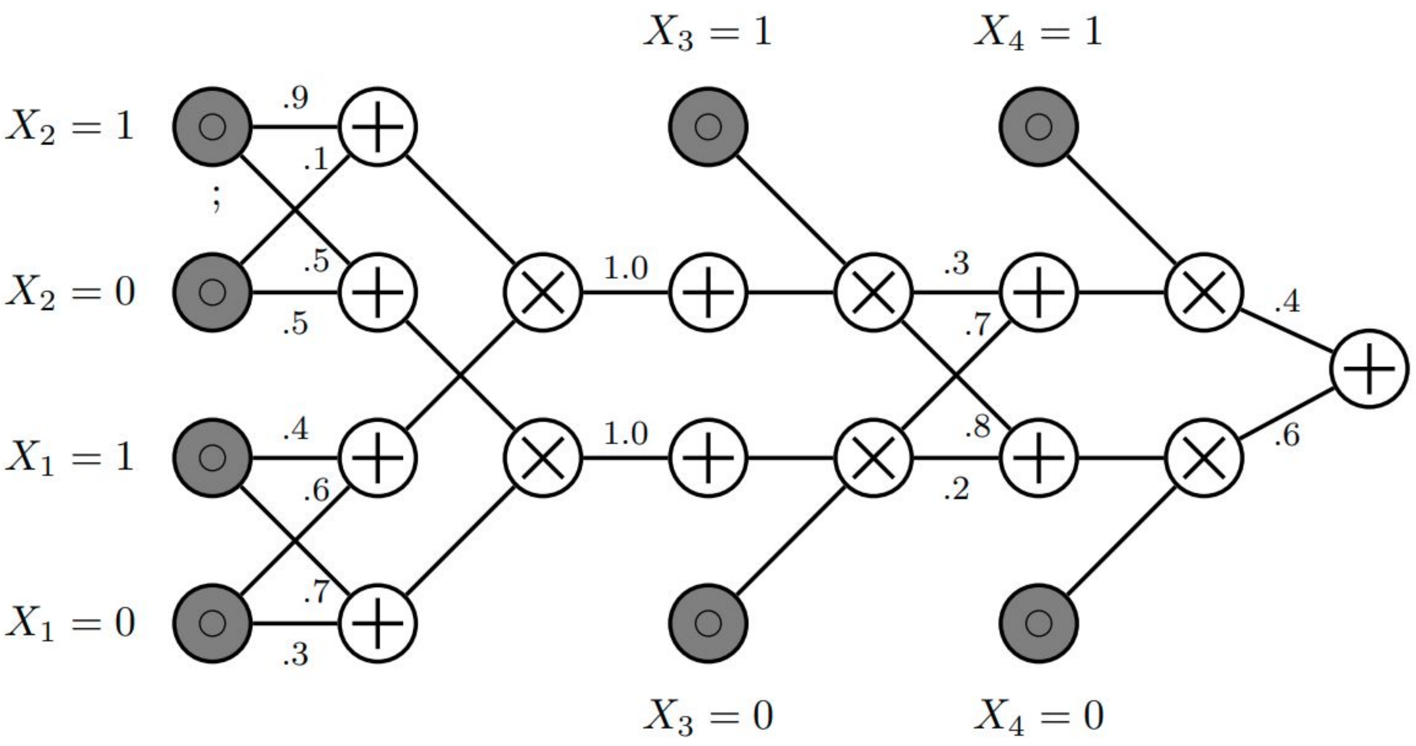
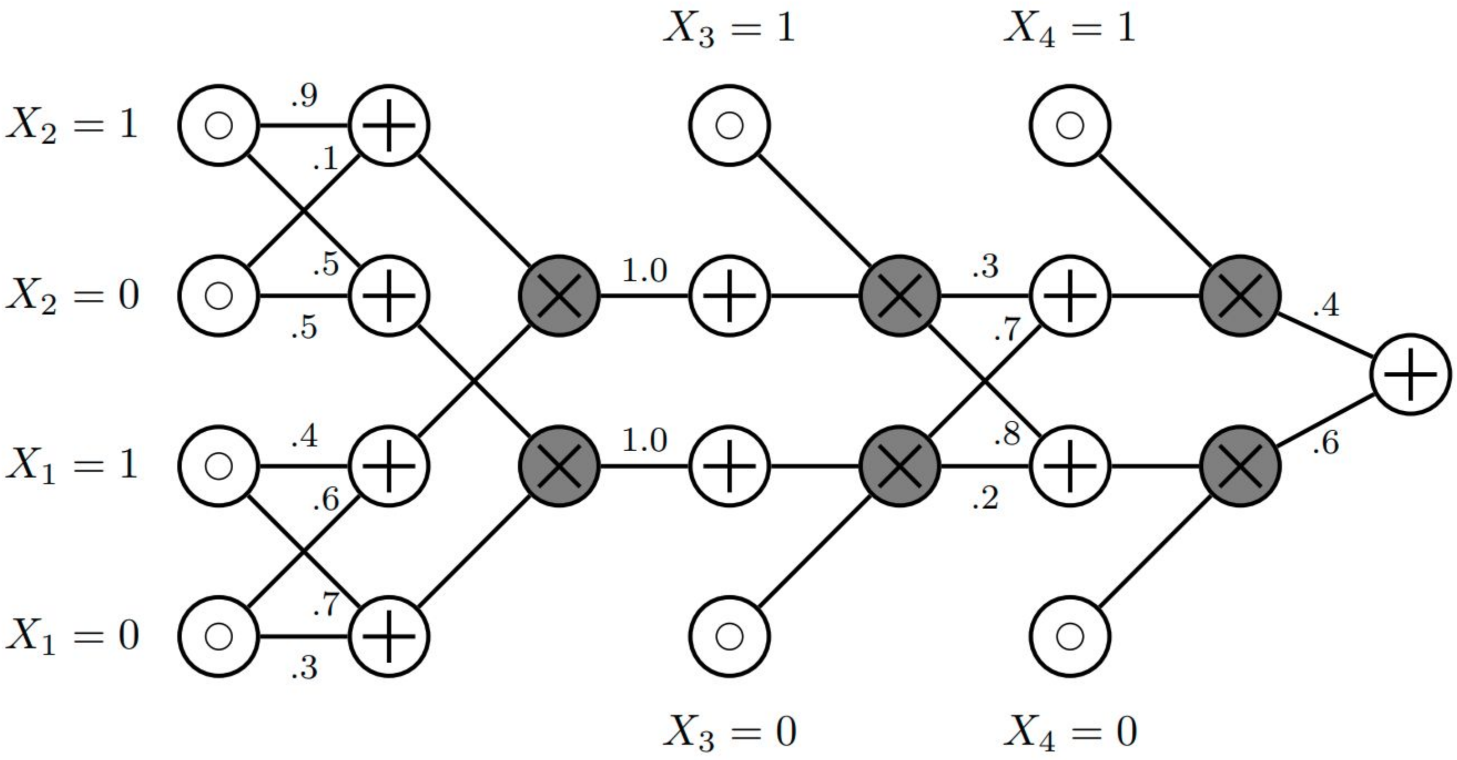
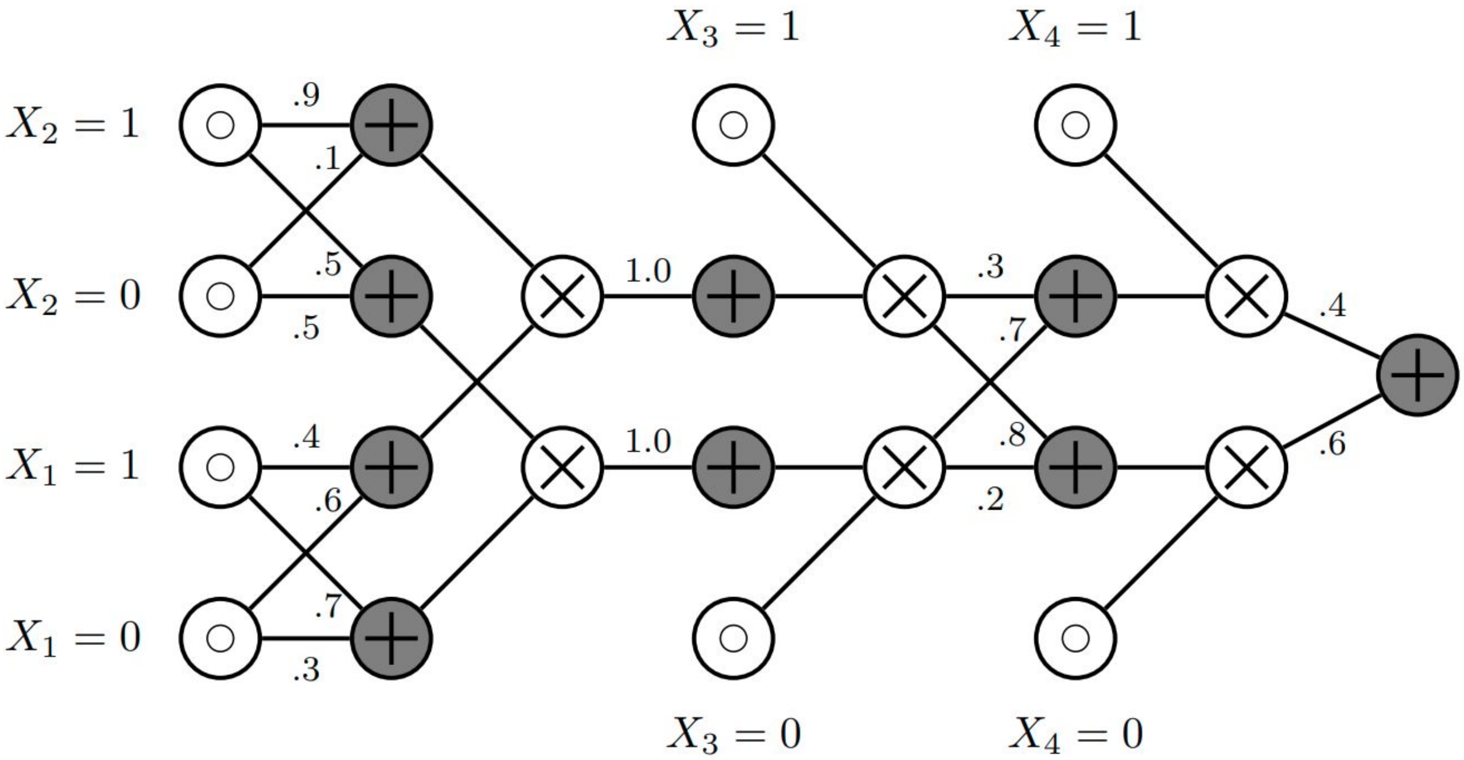
a *unifying framework* for tractable models



Input nodes c are tractable (simple) distributions,
e.g., univariate gaussian or indicator $p_c(X=1) = [X=1]$



Product nodes are factorizations $\prod_{c \in \mathsf{in}(n)} \mathrm{p}_c(\mathbf{x})$



Sum nodes are mixture models $\sum_{c \in \mathsf{in}(n)} \theta_{n,c} \, \mathrm{p}_c(\mathbf{x})$

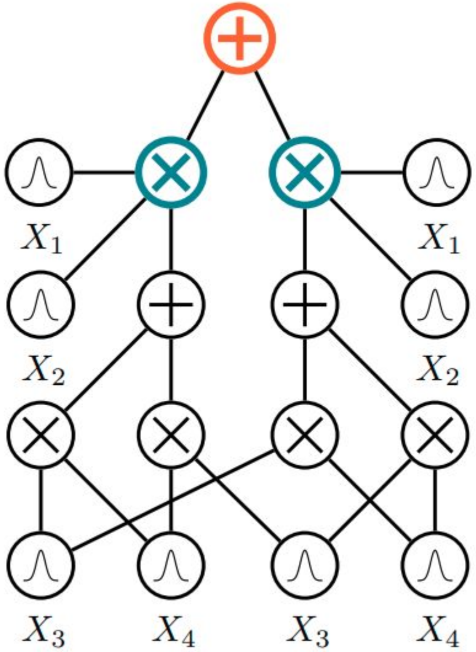
Smoothness + decomposability = tractable MAR

If $p(\mathbf{x}) = \sum_i w_i p_i(\mathbf{x})$, (***smoothness***):

$$\int p(\mathbf{x}) d\mathbf{x} = \int \sum_i w_i p_i(\mathbf{x}) d\mathbf{x} =$$

$$= \sum_i w_i \int p_i(\mathbf{x}) d\mathbf{x}$$

$\Rightarrow$ *integrals are "pushed down" to children*



[Darwiche & Marquis JAIR 2001, Poon & Domingos UAI11]

Smoothness + **decomposability** = **tractable MAR**

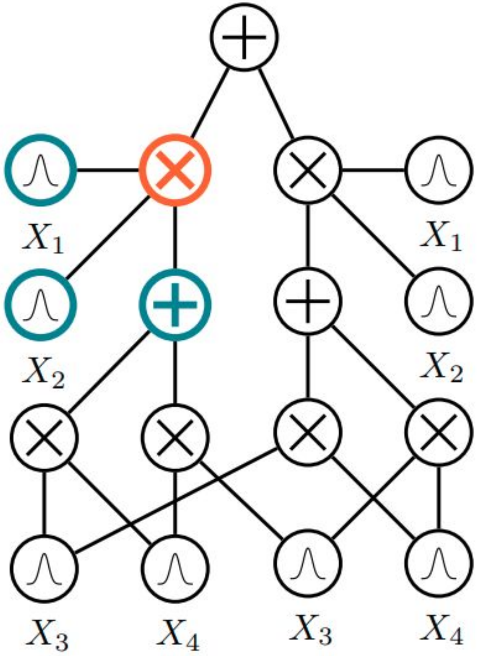
If $p(\mathbf{x}, \mathbf{y}, \mathbf{z}) = p(\mathbf{x})p(\mathbf{y})p(\mathbf{z})$, (***decomposability***):

$$\int\int\int p(\mathbf{x}, \mathbf{y}, \mathbf{z})d\mathbf{x}d\mathbf{y}d\mathbf{z} =$$

$$= \int\int\int p(\mathbf{x})p(\mathbf{y})p(\mathbf{z})d\mathbf{x}d\mathbf{y}d\mathbf{z} =$$

$$= \int p(\mathbf{x})d\mathbf{x}\int p(\mathbf{y})d\mathbf{y}\int p(\mathbf{z})d\mathbf{z}$$

$\Rightarrow$ *integrals decompose into easier ones*



Smoothness + decomposability = tractable MAR

Forward pass evaluation for MAR

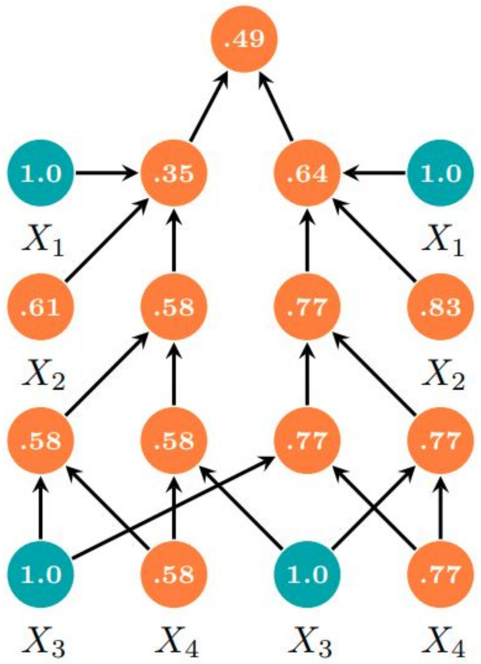
$\Rightarrow$ *linear in circuit size!*

E.g. to compute $p(x_2, x_4)$:

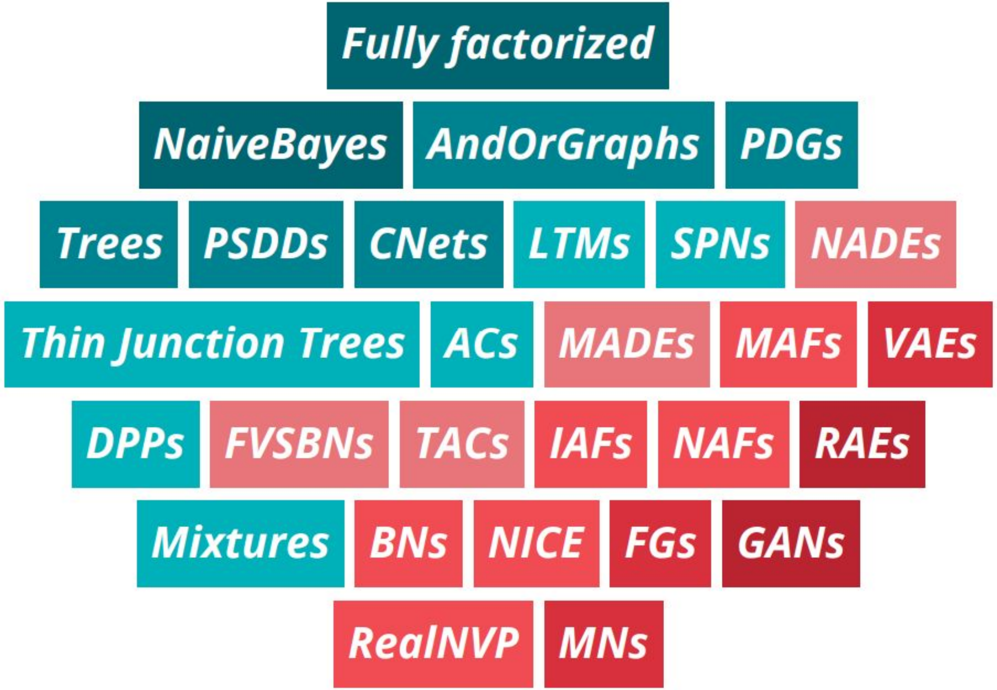
- leafs over X_1 and X_3 output $Z_i = \int p(x_i) dx_i$

  $\Rightarrow$ *for normalized leaf distributions:* 1.0

- leafs over X_2 and X_4 output ***EVI***
- feedforward evaluation (bottom-up)



$\mathcal{P}$ \ $\mathcal{Q}$:	MAR marginal queries	CON conditional queries	MOM moments (mean,...)	MAP maximum a posteriori	MMAP marginal MAP	ENT entropy	DIV divergences (KLD,...)	EXP expected predictions
smoothness **SMO**	✓	✓	✓	✗	✗	✓	✓	✓
decomposability **DEC**	✓	✓	✓	✗	✗	✗	✗	✗
consistency **CON**	✗	✗	✗	✓	✓	✗	✗	✗
determinism **DET**	✗	✗	✗	✓	✗	✗	✗	✗
marginal determinism **MAR-DET**	✗	✗	✗	✗	✓	✓	✓	✗
structured decomposability **STR-DEC**	✗	✗	✗	✗	✗	✓	✗	✗
paired str. decomposability **P-STR-DEC**	✗	✗	✗	✗	✗	✗	✓	✓



tractability is a spectrum

	smooth	*dec.*	det.	str.dec.
Arithmetic Circuits (ACs) *[Darwiche 2003]*	✔	✔	✔	✘
Sum-Product Networks (SPNs) *[Poon et al. 2011]*	✔	✔	✘	✘
Cutset Networks (CNets) *[Rahman et al. 2014]*	✔	✔	✔	✘
Probabilistic Decision Graphs *[Jaeger 2004]*	✔	✔	✔	✔
(Affine) ADDs *[Hoey et al. 1999; Sanner et al. 2005]*	✔	✔	✔	✔
AndOrGraphs *[Dechter et al. 2007]*	✔	✔	✔	✔
PSDDs *[Kisa et al. 2014a]*	✔	✔	✔	✔

Fully factorized

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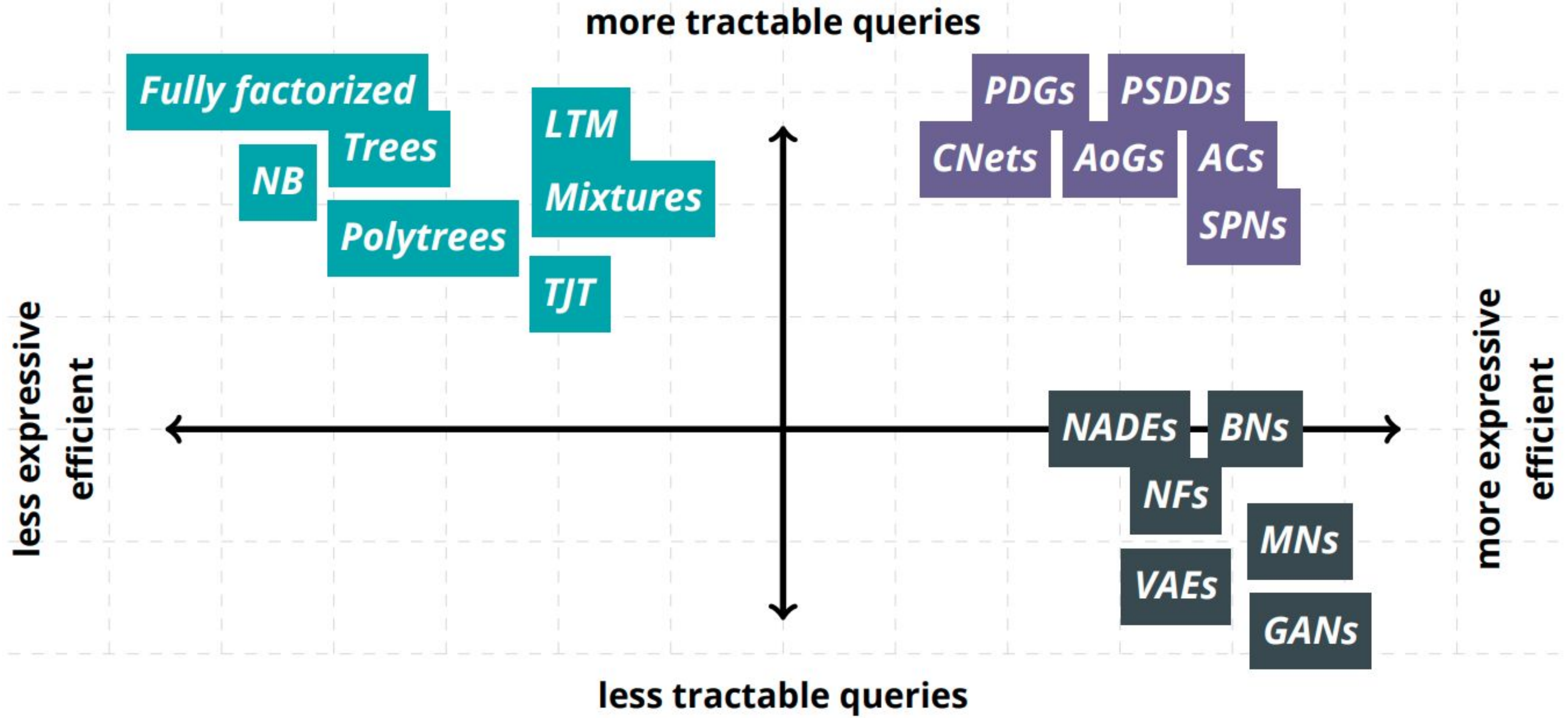
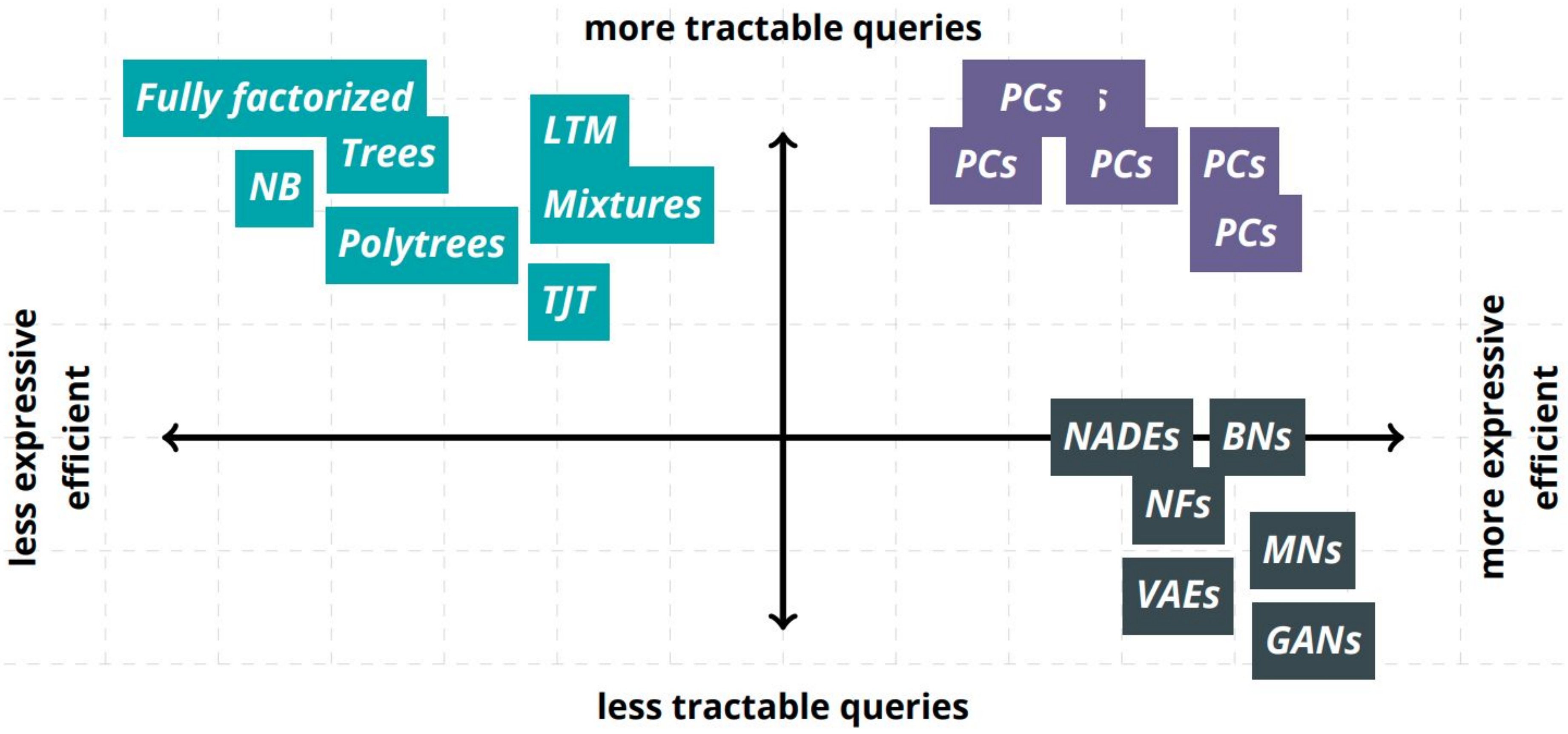
Expressive models without *compromises*

How expressive are probabilistic circuits?

density estimation benchmarks

dataset	best circuit	BN	MADE	VAE	dataset	best circuit	BN	MADE	VAE
nltcs	**-5.99**	-6.02	-6.04	**-5.99**	***dna***	**-79.88**	-80.65	-82.77	-94.56
msnbc	**-6.04**	**-6.04**	-6.06	-6.09	***kosarek***	**-10.52**	-10.83	-	-10.64
kdd	-2.12	-2.19	**-2.07**	-2.12	***msweb***	-9.62	-9.70	**-9.59**	-9.73
plants	**-11.84**	-12.65	-12.32	-12.34	***book***	-33.82	-36.41	-33.95	**-33.19**
audio	-39.39	-40.50	-38.95	**-38.67**	***movie***	-50.34	-54.37	-48.7	**-47.43**
jester	-51.29	**-51.07**	-52.23	-51.54	***webkb***	-149.20	-157.43	-149.59	**-146.9**
netflix	-55.71	-57.02	-55.16	**-54.73**	***cr52***	-81.87	-87.56	-82.80	**-81.33**
accidents	-26.89	**-26.32**	-26.42	-29.11	***c20ng***	-151.02	-158.95	-153.18	**-146.9**
retail	**-10.72**	-10.87	-10.81	-10.83	***bbc***	**-229.21**	-257.86	-242.40	-240.94
pumbs*	-22.15	**-21.72**	-22.3	-25.16	***ad***	-14.00	-18.35	**-13.65**	-18.81


more tractable queries
Fully factorized
Trees
NB
LTM
Mixtures
Polytrees
TJT
PDGs
PSDDs
CNets
AoGs
ACs
SPNs
less expressive
efficient
more expressive
efficient
NADEs
BNs
NFs
MNs
VAEs
GANs
less tractable queries


more tractable queries
Fully factorized
Trees
NB
Polytrees
LTM
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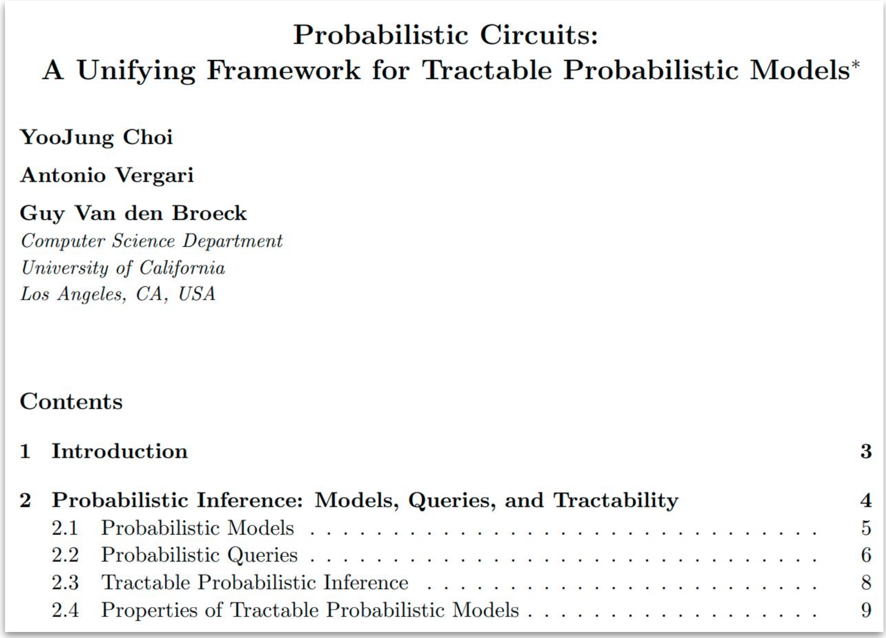
TJT
PCs
PCs
PCs
PCs
PCs
less expressive
efficient
more expressive
efficient
NADEs
BNs
NFs
MNs
VAEs
GANs
less tractable queries

Want to learn more?

Tutorial (3h)

https://youtu.be/2RAG5-L9R70

Overview Paper (80p)



Probabilistic Circuits:
A Unifying Framework for Tractable Probabilistic Models*

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Antonio Vergari
Guy Van den Broeck
Computer Science Department
University of California
Los Angeles, CA, USA

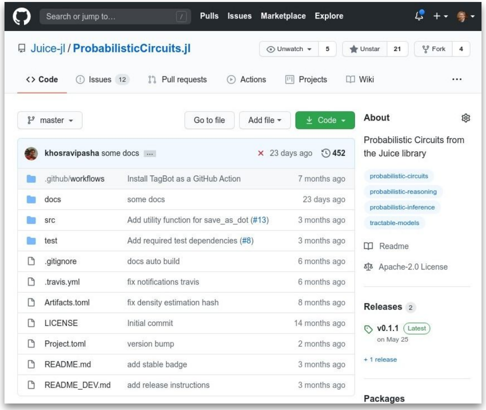
Contents

http://starai.cs.ucla.edu/papers/ProbCirc20.pdf

Training PCs in Julia with Juice.jl

Training maximum likelihood parameters of probabilistic circuits

```
julia> using ProbabilisticCircuits;
julia> data, structure = load(...);
julia> num_examples(data)
17412
julia> num_edges(structure)
270448
julia> @btime estimate_parameters(structure , data);
  63 ms
```



Custom SIMD and CUDA kernels to parallelize over layers and training examples.

[https://github.com/Juice-jl/]

Probabilistic circuits seem awfully general.

Are all tractable probabilistic models probabilistic circuits?

Determinantal Point Processes (DPPs)

DPPs are models where probabilities are specified by (sub)determinants

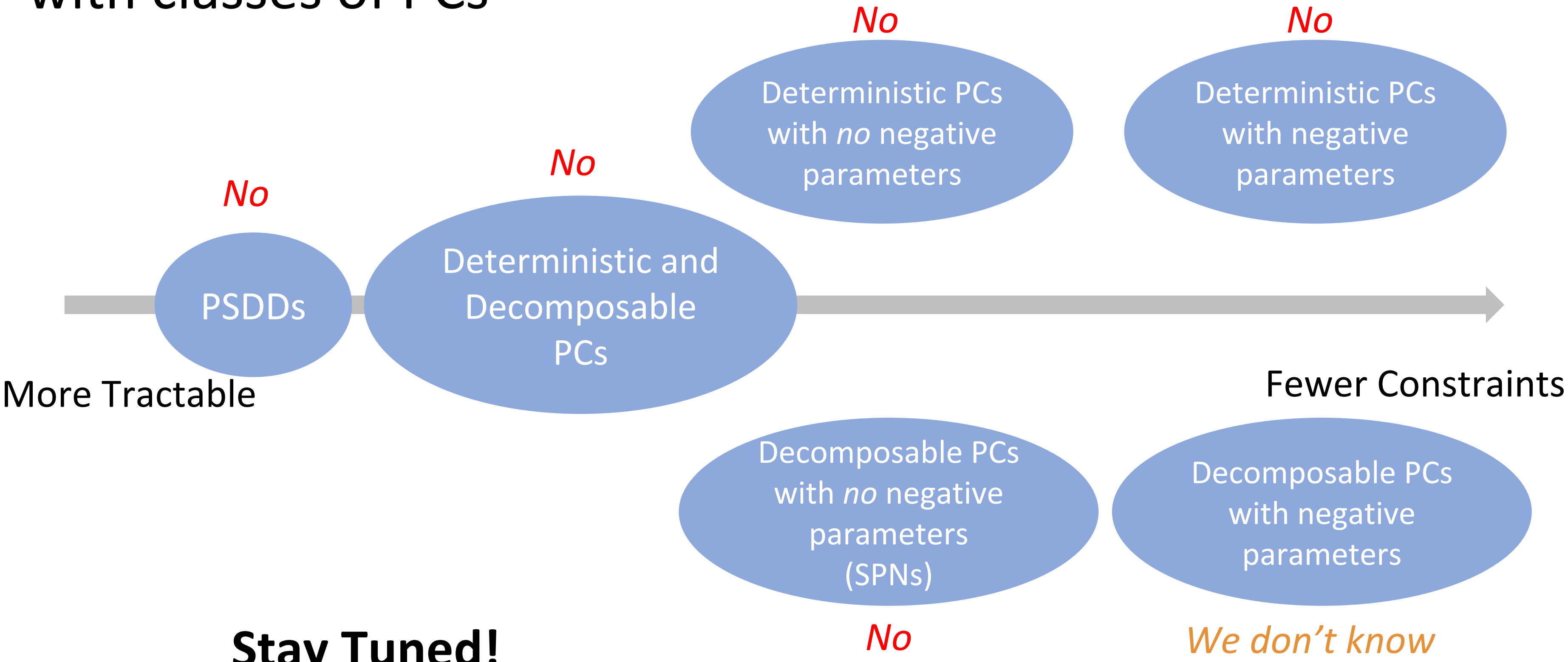
$$L = \begin{bmatrix} 1 & 0.9 & 0.8 & 0 \\ 0.9 & 0.97 & 0.96 & 0 \\ 0.8 & 0.96 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Pr_L(X_1 = 1, X_2 = 0, X_3 = 1, X_4 = 0) = \frac{1}{\det(L + I)} \det(L_{\{1,2\}})$$

Computing marginal probabilities is *tractable.*

[Zhang et al. UAI20]

We cannot tractably represent DPPs with classes of PCs



Stay Tuned!

[Zhang et al. UAI20; Martens & Medabalimi Arxiv15]

The AI Dilemma

Pure Logic | **Pure Learning**

The AI Dilemma

Pure Logic **Pure Learning**

- Slow thinking: deliberative, cognitive, model-based, extrapolation
- Amazing achievements until this day
- "*Pure logic is brittle*"
  noise, uncertainty, incomplete knowledge, …

The AI Dilemma

Pure Logic **Pure Learning**

- Fast thinking: instinctive, perceptive, model-free, interpolation
- Amazing achievements recently
- "*Pure learning is brittle*"

  bias, algorithmic fairness, interpretability, explainability, adversarial attacks, unknown unknowns, calibration, verification, missing features, missing labels, data efficiency, shift in distribution, general robustness and safety

  fails to incorporate a sensible model of the world

Pure Logic **Probabilistic World Models** **Pure Learning**

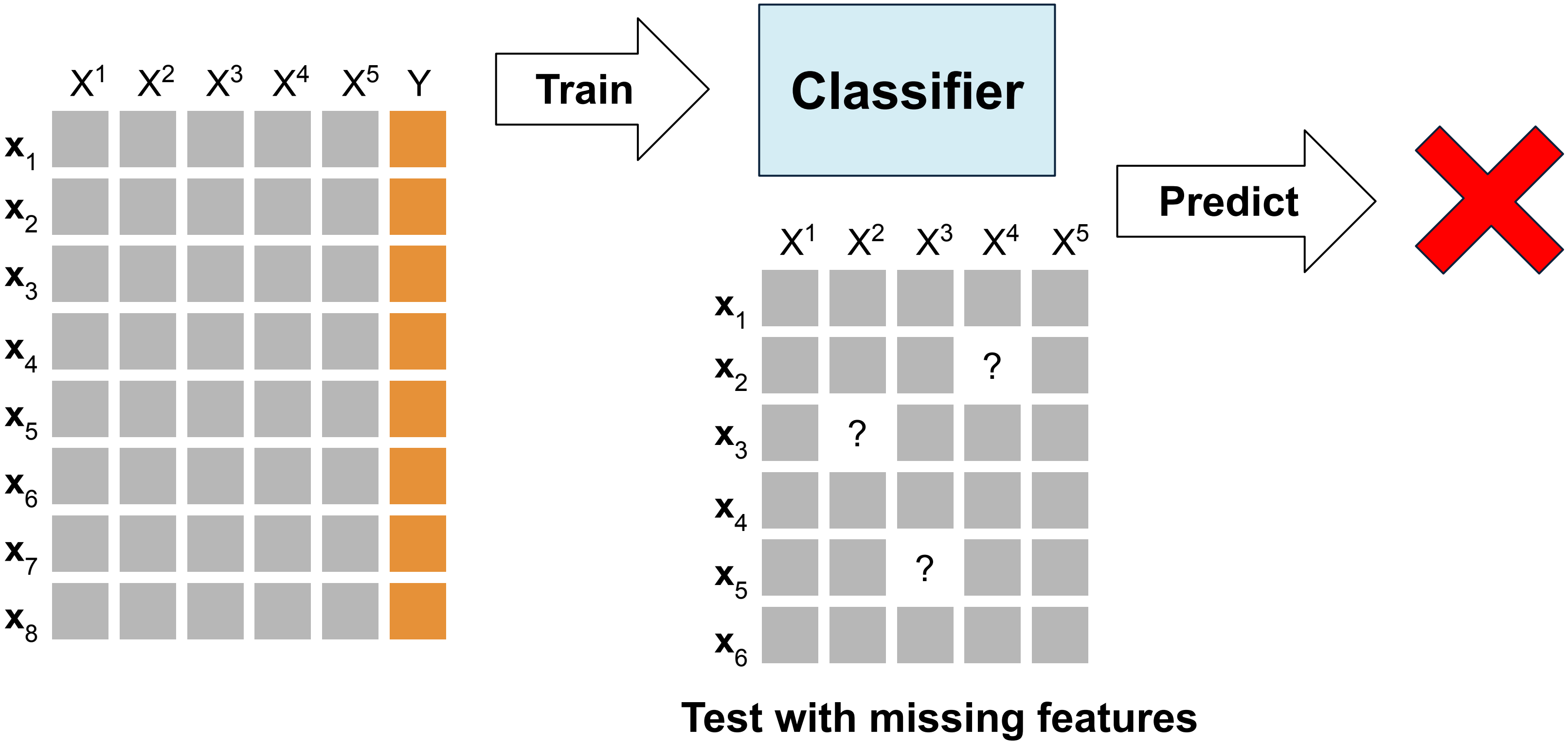
A New Synthesis of Learning and Reasoning

- "*Pure learning is brittle*"

  bias, **algorithmic fairness**, interpretability, **explainability**, adversarial attacks, unknown unknowns, calibration, verification, **missing features**, missing labels, data efficiency, shift in distribution, general robustness and safety

  fails to incorporate a sensible model of the world

Prediction with Missing Features



Expected Predictions

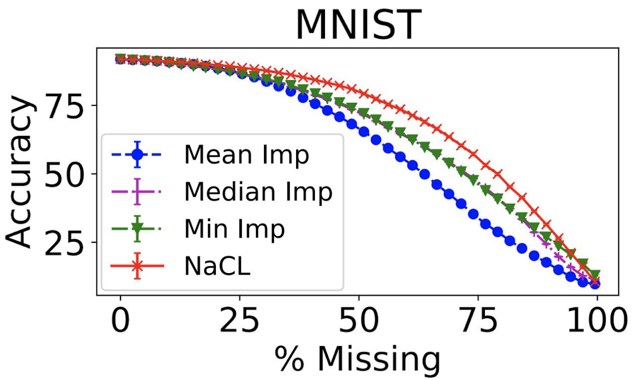
Consider **all possible complete inputs** and **reason** about the *expected* behavior of the classifier

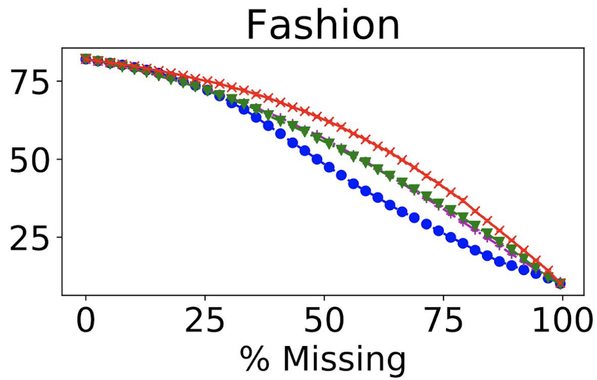
$$\mathbb{E}_{\mathbf{x}^m \sim p(\mathbf{x}^m|\mathbf{x}^o)}\left[f\left(\mathbf{x}^m\mathbf{x}^o\right)\right]$$

x^{o} = observed features
x^{m} = missing features

Experiment:

- *f(x)* = logistic regres.
- *p(x)* = naive Bayes





[Khosravi et al. IJCAI19, NeurIPS20, Artemiss20]

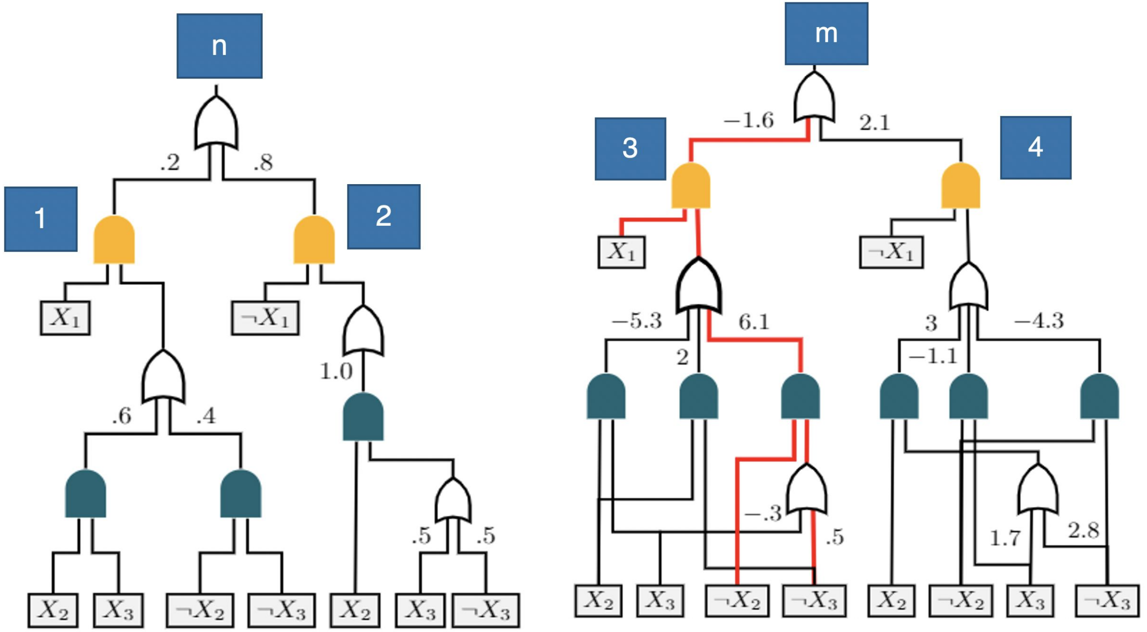
What about complex feature distributions?

- feature distribution is a compatible probabilistic circuits
- classifier is a regression circuit

Recursion that "breaks down" the computation.

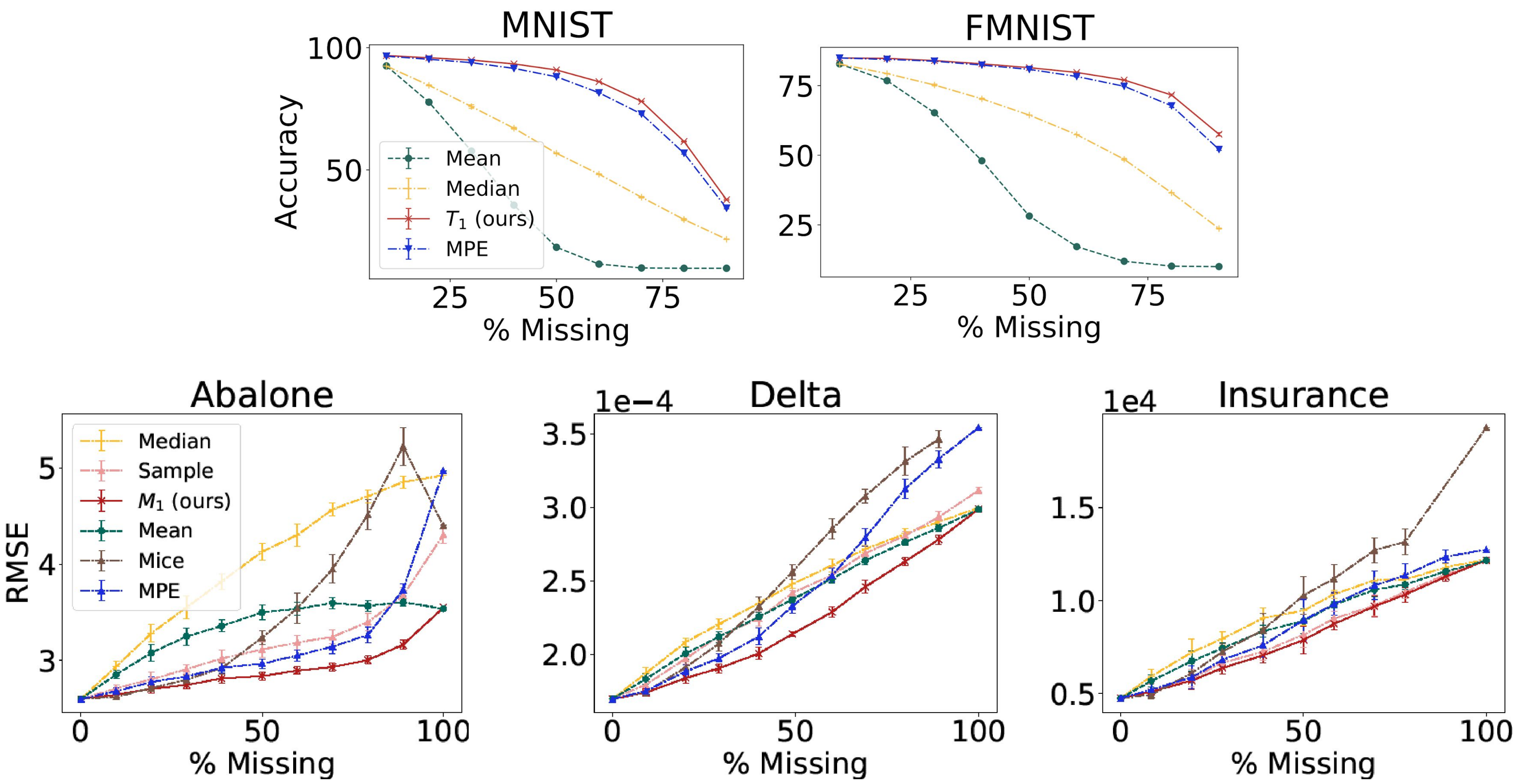
Expectation of function m w.r.t. dist. n ?

Solve subproblems: (1,3), (1,4), (2,3), (2,4)



[Khosravi et al. IJCAI19, NeurIPS20, Artemiss20]

Experiments with Probabilistic Circuits



[Khosravi et al. IJCAI19, NeurIPS20, Artemiss20]

ADV inference in Julia with Juice.jl

```
using ProbabilisticCircuits
pc = load_prob_circuit(zoo_psdd_file("insurance.psdd"));
rc = load_logistic_circuit(zoo_lc_file("insurance.circuit"), 1);
```

q_9: *Is the predictive model biased by gender?*

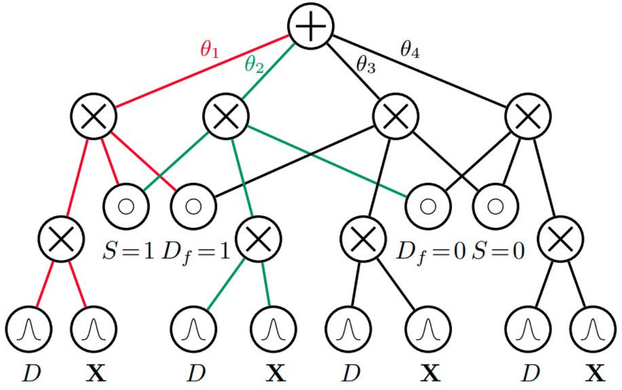
```
groups = make_observations([["male"], ["female"]])
exps, _ = Expectation(pc, rc, groups);
println("Female  : \$ $(exps[2])");
println("Male    : \$ $(exps[1])");
println("Diff    : \$ $(exps[2] - exps[1])");
Female  : $ 14170.125469335406
Male    : $ 13196.548926381849
Diff    : $ 973.5765429535568
```

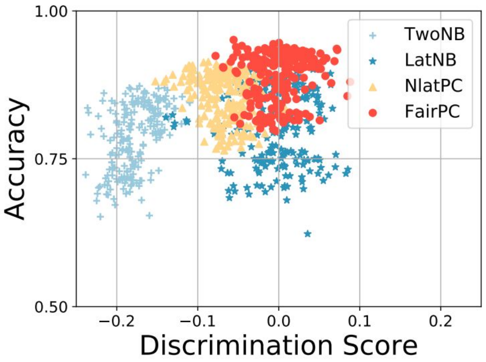
Model-Based Algorithmic Fairness: FairPC

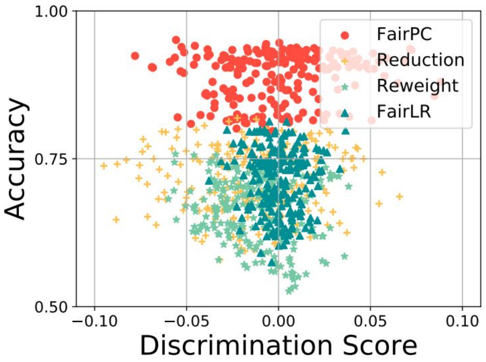
Learn classifier given
- features S and X
- training labels D

Group fairness by demographic parity:
Fair decision Df should be independent of the sensitive attribute S

Discover the latent fair decision Df by learning a PC.







[Choi et al. Arxiv20]

Probabilistic Sufficient Explanations

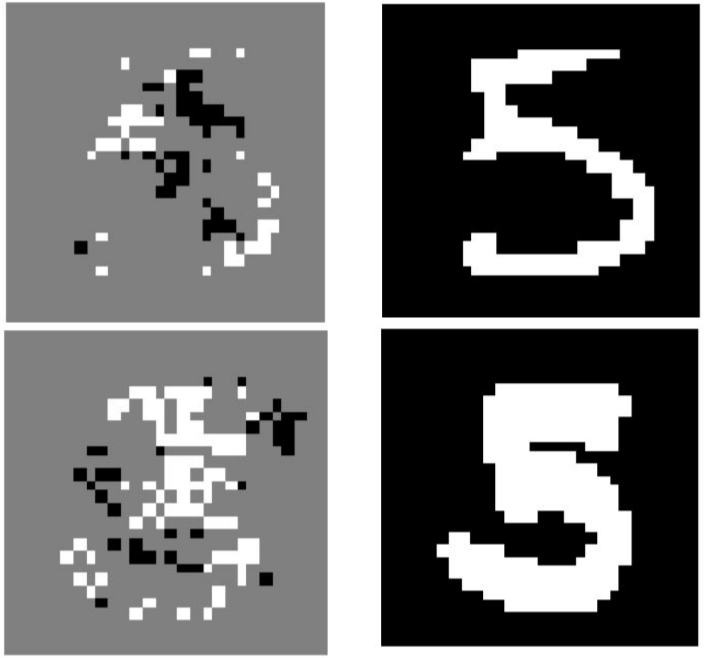
<u>Goal</u>: explain an instance of classification (a specific prediction)

Explanation is a subset of features, s.t.

1. The explanation is "probabilistically sufficient"

   Under the feature distribution, given the explanation, the classifier is likely to make the observed prediction.

2. It is minimal and "simple"



[Khosravi et al. IJCAI19, Wang et al. XXAI20]

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A New Synthesis of Learning and Reasoning

“*Pure learning is brittle*”

bias, **algorithmic fairness**, interpretability, **explainability**, adversarial attacks, unknown unknowns, calibration, verification, **missing features**, missing labels, data efficiency, shift in distribution, general robustness and safety

We need to incorporate a sensible probabilistic model of the world

Probabilistic Programs

What are probabilistic programs?

```
let x = flip 0.5 in
let y = flip 0.7 in
let z = x || y in
let w = if z then
      my_func(x,y)
else
      ...
in
observe(z);
```

means “flip a coin, and output true with probability ½”

Standard (functional) programming constructs: let, if, ...

means “reject this execution if z is not true”

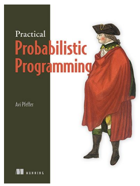
Why Probabilistic Programming?

PPLs are proliferating

HackPPL

Edward

Figaro



Pyro

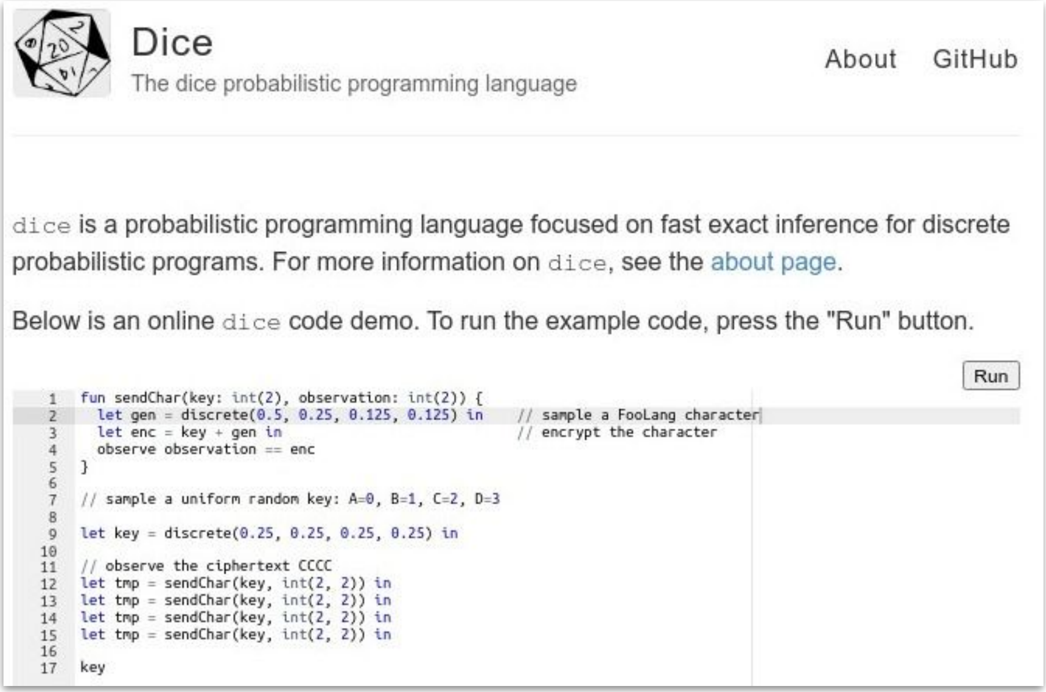
Stan

Venture, Church, IBAL, WebPPL, Infer.NET, Tensorflow Probability, ProbLog, PRISM, LPADs, CPLogic, CLP(BN), ICL, PHA, Primula, Storm, Gen, PRISM, PSI, Bean Machine, etc. … *and many many more*

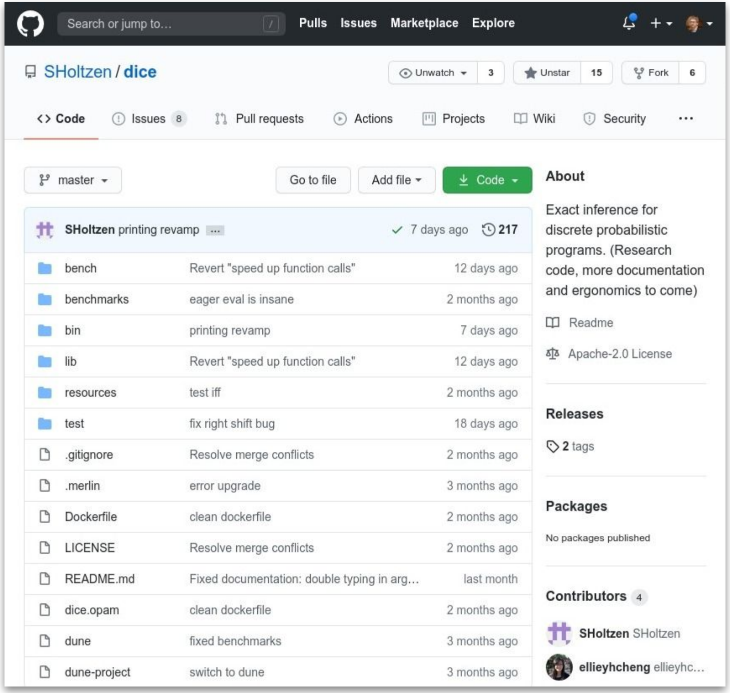
Programming languages are humanity's biggest knowledge representation achievement!

Dice probabilistic programming language

http://dicelang.cs.ucla.edu/



https://github.com/SHoltzen/dice



[Holtzen et al. OOPSLA20]

Possible world = Execution path

```
let x = flip 0.4 in
let y = flip 0.7 in
let z = x || y in
let x = if z then
      x
else
      1
in (x,y)
```

Execution A	Execution B	Execution C	Execution D
x=1	x=1	x=0	x=0
x=1, y=1	x=1, y=0	x=0, y=1	x=0, y=0
x=1, y=1, z=1	x=1, y=0, z=1	x=0, y=1, z=1	x=0, y=0, z=0
x=1, y=1, z=1	x=1, y=0, z=1	x=0, y=1, z=1	
			x=1, y=0, z=0
(1, 1)	(1,0)	(0,1)	(1,0)
P = 0.4*0.7	**P = 0.4*0.3**	**P = 0.6*0.7**	**P = 0.6*0.3**

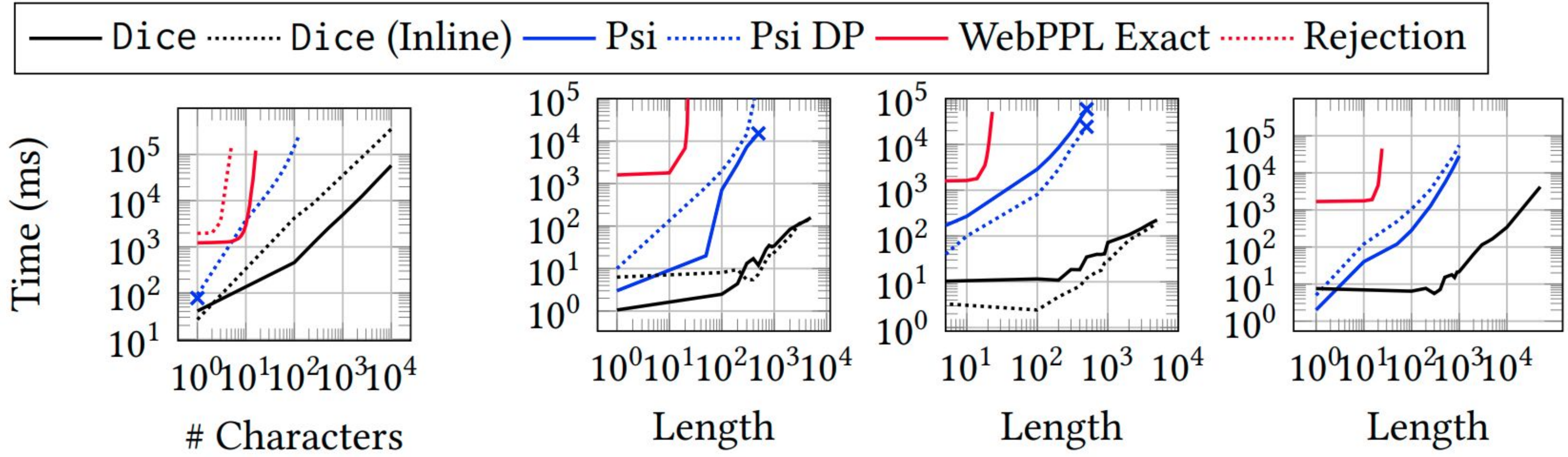
Why should I care?

Better abstraction than probabilistic graphical models:

- Beyond variable-level dependencies (contextual)
- modularity through functions
  reuse (cf. relational graphical models)
- intuitive language for local structure; arithmetic
- data structures
- first-class observations

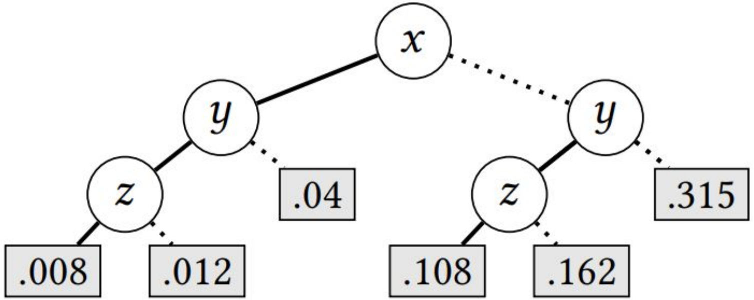
Probabilistic Program Inference

Path enumeration: find all of them!



Probabilistic Program Inference

Compact representation of paths in algebraic decision diagrams
[Storm, PRISM, Claret et al.]



Much better when small number of distinct probabilities!

- Great when there’s parameter symmetry (lifted inference)
- Scales exponentially with distinct parameters

  <u>Example</u>: *N coin flips with slightly different bias*

Probabilistic Program Inference

Path-based methods miss key ingredient: **factorization** …. aka the product nodes

```
let x = flip₁ 0.1 in
let y = if x then flip₂ 0.2 else
    flip₃ 0.3 in
let z = if y then flip₄ 0.4 else
    flip₅ 0.5 in z
```

$$\underbrace{0.1}_{x=T} \cdot \underbrace{0.2}_{y=T} \cdot \underbrace{0.4}_{z=T} + \underbrace{0.1}_{x=T} \cdot \underbrace{0.8}_{y=F} \cdot \underbrace{0.5}_{z=T} + \underbrace{0.9}_{x=F} \cdot \underbrace{0.3}_{y=T} \cdot \underbrace{0.4}_{z=T} + \underbrace{0.9}_{x=F} \cdot \underbrace{0.7}_{y=F} \cdot \underbrace{0.5}_{z=T}$$

$$\Rightarrow \underbrace{0.1}_{x=T} \cdot \left(\underbrace{0.2}_{y=T} \cdot \underbrace{0.4}_{z=T} + \underbrace{0.8}_{y=F} \cdot \underbrace{0.5}_{z=T} \right) + \underbrace{0.9}_{x=F} \cdot \left(\underbrace{0.3}_{y=T} \cdot \underbrace{0.4}_{z=T} + \underbrace{0.7}_{y=F} \cdot \underbrace{0.5}_{z=T} \right)$$

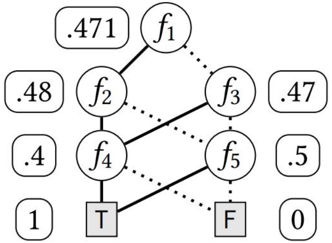
Symbolic Compilation in Dice

- Construct Boolean formula
- Satisfying assignments ≈ paths
- Variables are flips
- Associate weights with flips
- Compile factorized circuit

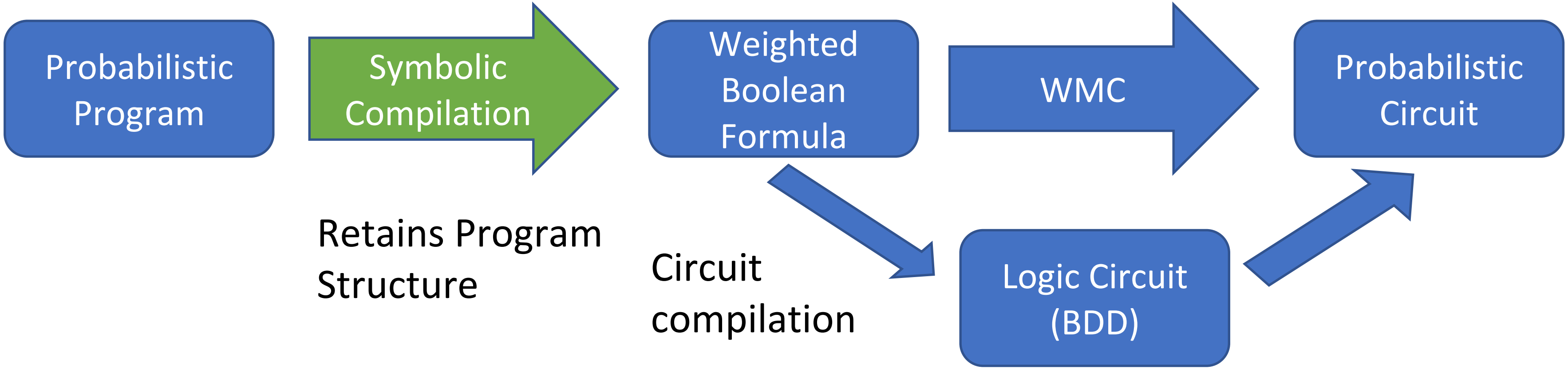
```
let x = flip₁ 0.1 in
let y = if x then flip₂ 0.2 else
    flip₃ 0.3 in
let z = if y then flip₄ 0.4 else
    flip₅ 0.5 in z
```

$$\underbrace{0.1}_{x=T}\cdot\underbrace{0.2}_{y=T}\cdot\underbrace{0.4}_{z=T}+\underbrace{0.1}_{x=T}\cdot\underbrace{0.8}_{y=F}\cdot\underbrace{0.5}_{z=T}+\underbrace{0.9}_{x=F}\cdot\underbrace{0.3}_{y=T}\cdot\underbrace{0.4}_{z=T}+\underbrace{0.9}_{x=F}\cdot\underbrace{0.7}_{y=F}\cdot\underbrace{0.5}_{z=T}$$

$$f_1 f_2 f_4 \vee f_1 \bar{f}_2 f_5 \vee \bar{f}_1 f_3 f_4 \vee \bar{f}_1 \bar{f}_3 f_5$$



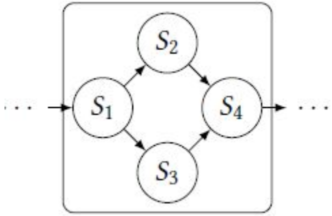
Symbolic Compilation to Probabilistic Circuits



Retains Program Structure

Circuit compilation

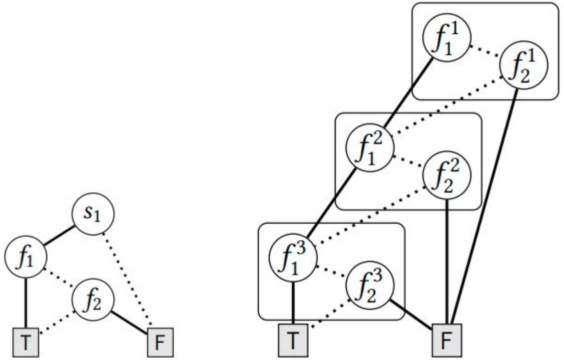
Factorized Inference in Dice



(a) Network diagram.

```
fun diamond(s₁:Bool):Bool {
  let route = flip₁ 0.5 in
  let s₂ = if route then s₁ else F in
  let s₃ = if route then F else s₁ in
  let drop = flip₂ 0.0001 in
  s₂ ∨ (s₃ ∧ ¬drop)}
diamond(diamond(diamond(T)))
```

(b) Probabilistic program defining the network.



(c) diamond function. (d) Final BDD.

Network Verification

First-Class Observations

```
fun EncryptChar(key:int, obs:char):Bool {
  let randomChar = ChooseChar() in
  let ciphertext = (randomChar + key) % 26 in
  let _ = observe ciphertext = obs in
  true}
let k = UniformInt(0, 25) in
let _ = EncryptChar(k, 'H') in ···
let _ = EncryptChar(k, 'D') in k
```

Frequency Analyzer for a Caesar cipher in Dice

PPL benchmarks from PL community

Benchmark	Psi (ms)	DP (ms)	Dice (ms)	# Paths	BDD Size
Grass	167	57	**1.0**	9.5×10^1	15
Burglar Alarm	98	10	**1.1**	2.5×10^2	11
Coin Bias	94	23	**1.0**	4	13
Noisy Or	81	152	**1.0**	1.6×10^4	35
Evidence1	48	32	**1.0**	9	5
Evidence2	59	28	**1.0**	9	6
Murder Mystery	193	75	**1.0**	1.6×10^1	6

Scalable Inference

Benchmark	Psi (ms)	DP (ms)	Dice (ms)	# Parameters	# Paths	BDD Size
Cancer [48]	772	46	**1.0**	10	1.1×10^{3}	28
Survey [73]	2477	152	**2.0**	21	1.3×10^{4}	73
Alarm [5]	✗	✗	**9.0**	509	1.0×10^{36}	1.3×10^{3}
Insurance [7]	✗	✗	**75.0**	984	1.2×10^{40}	1.0×10^{5}
Hepar2 [63]	✗	✗	**54.0**	1453	2.9×10^{69}	1.3×10^{3}
Hailfinder [1]	✗	✗	**526.0**	2656	2.0×10^{76}	6.5×10^{4}
Pigs	✗	✗	**32.0**	5618	7.3×10^{492}	35
Water [43]	✗	✗	**2926.0**	1.0×10^{4}	3.2×10^{54}	5.1×10^{4}
Munin [3]	✗	✗	**1945.0**	8.1×10^{5}	2.1×10^{1622}	1.1×10^{4}

`import PL.*` – Denotational Semantics

- <u>Goal</u>: associate with every expression “e” a semantic object.
- <u>Notation</u>: semantic bracket: [[.]]
  - In Bayesian network: [[BN]] = Pr_{BN}(.)
  - In probabilistic programs: [[e]](.) for all expressions
  - Accepting and distributional semantics:

$$[\![\mathrm{e}]\!]_A \triangleq \sum_v [\![\mathrm{e}]\!](v), \qquad [\![\mathrm{e}]\!]_D(v) \triangleq \frac{1}{[\![\mathrm{e}]\!]_A}[\![\mathrm{e}]\!](v)$$

- <u>Idea</u>: don’t need to run ‘flip 0.4’ infinite times to know meaning

Denotational Semantics + Formal Inference Rules

$$[\![v_1]\!](v) \triangleq (\delta(v_1))(v) \qquad [\![\texttt{fst } (v_1, v_2)]\!](v) \triangleq (\delta(v_1))(v) \qquad [\![\texttt{snd } (v_1, v_2)]\!](v) \triangleq (\delta(v_2))(v)$$

$$[\![\texttt{if } v_g \texttt{ then } \mathsf{e}_1 \texttt{ else } \mathsf{e}_2]\!](v) \triangleq \begin{cases} [\![\mathsf{e}_1]\!](v) & \text{if } v_g = \mathsf{T} \\ [\![\mathsf{e}_2]\!](v) & \text{if } v_g = \mathsf{F} \\ 0 & \text{otherwise} \end{cases} \qquad [\![\texttt{flip } \theta]\!](v) \triangleq \begin{cases} \theta & \text{if } v = \mathsf{T} \\ 1-\theta & \text{if } v = \mathsf{F} \\ 0 & \text{otherwise} \end{cases}$$

$$[\![\texttt{observe } v_1]\!](v) \triangleq \begin{cases} 1 & \text{if } v_1 = \mathsf{T} \text{ and } v = \mathsf{T}, \\ 0 & \text{otherwise} \end{cases} \qquad [\![f(v_1)]\!](v) \triangleq \Big((T(f))(v_1)\Big)(v)$$

$$[\![\texttt{let } x = \mathsf{e}_1 \texttt{ in } \mathsf{e}_2]\!](v) \triangleq \sum_{v'} [\![\mathsf{e}_1]\!](v') \times [\![\mathsf{e}_2[x \mapsto v']]\!](v)$$

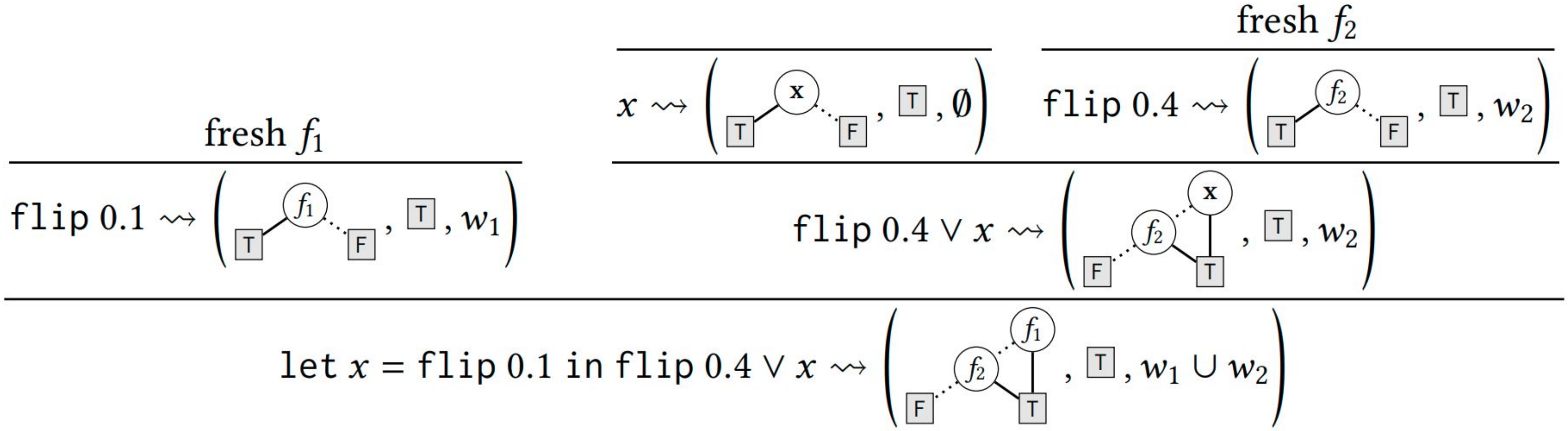
$$\frac{}{\mathsf{T} \rightsquigarrow (\mathsf{T}, \mathsf{T}, \emptyset)}\ (\text{C-True}) \qquad \frac{}{\mathsf{F} \rightsquigarrow (\mathsf{F}, \mathsf{T}, \emptyset)}\ (\text{C-False}) \qquad \frac{}{x \rightsquigarrow (\mathbf{x}, \mathsf{T}, \emptyset)}\ (\text{C-Ident})$$

$$\frac{\text{fresh } \mathbf{f}}{\texttt{flip } \theta \rightsquigarrow \left(\mathbf{f}, \mathsf{T}, (\mathbf{f} \mapsto \theta, \mathsf{T}, \overline{\mathbf{f}} \mapsto 1-\theta)\right)}\ (\text{C-Flip}) \qquad \frac{\mathsf{aexp} \rightsquigarrow (\varphi, \mathsf{T}, \emptyset)}{\texttt{observe } \mathsf{aexp} \rightsquigarrow (\mathsf{T}, \varphi, \emptyset)}\ (\text{C-Obs})$$

$$\frac{\mathsf{aexp} \rightsquigarrow (\varphi_g, \mathsf{T}, \emptyset) \qquad \mathsf{e}_T \rightsquigarrow (\varphi_T, \gamma_T, w_T) \qquad \mathsf{e}_E \rightsquigarrow (\varphi_E, \gamma_E, w_E)}{\texttt{if } \mathsf{aexp} \texttt{ then } \mathsf{e}_\mathsf{T} \texttt{ else } \mathsf{e}_\mathsf{E} \rightsquigarrow \Big(\big((\varphi_g \wedge \varphi_T) \vee (\overline{\varphi}_g \wedge \varphi_E\big), \big((\varphi_g \wedge \gamma_T) \vee (\overline{\varphi}_g \wedge \gamma_E\big), w_T \cup w_E\Big)}\ (\text{C-Ite})$$

$$\frac{\mathsf{e}_1 \rightsquigarrow (\varphi_1, \gamma_1, w_1) \qquad \mathsf{e}_2 \rightsquigarrow (\varphi_2, \gamma_2, w_2)}{\texttt{let } x = \mathsf{e}_1 \texttt{ in } \mathsf{e}_2 \rightsquigarrow (\varphi_2[\mathbf{x} \mapsto \varphi_1], \gamma_1 \wedge \gamma_2[\mathbf{x} \mapsto \varphi_1], w_1 \cup w_2)}\ (\text{C-Let})$$

Provably Correct Inference!



Better Inference?

Exploit modularity

1. <u>AI modularity</u>:
   Discover contextual independencies and **factorize**
2. <u>PL modularity</u>:
   Compile procedure summaries and reuse at each call site

Reason about programs! Compiler optimizations.
Quick preview:

3. Flip hoisting optimization
4. Eager compilation

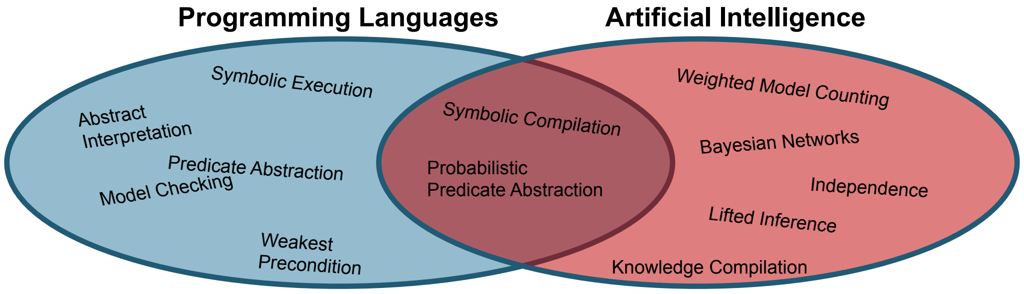
Compiler Optimizations (sneak preview)

Benchmark	Naive compilation	determinism	flip hoisting + determinism	Eager + flip lifting	*Ace baseline*
alarm	156	140	83	69	422
water	56,267	65,975	1509	941	605
insurance	140	100	100	128	492
hepar2	95	80	80	80	495
pigs	3,772	2490	2112	186	985
munin	>1,000,000	>1,000,000	109,687	16,536	3,500

Inference time in milliseconds

Conclusions

- Are we already in the age of computational abstractions?
- **Probabilistic circuits** for learning deep tractable probabilistic models
- **Probabilistic programs** as the new probabilistic knowledge representation language



Thanks