

**UCLA**

**Computer  
Science**

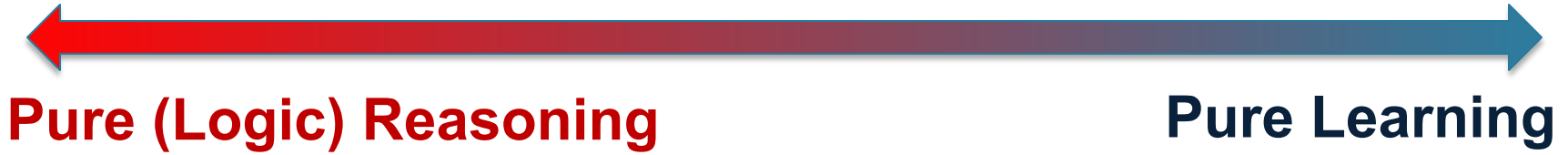


# Bridging Data and Knowledge in Neuro-Symbolic Learning

Guy Van den Broeck

Summer school on Neurosymbolic Programming - Jul 11 2022

# The AI Dilemma



# The AI Dilemma



**Pure (Logic) Reasoning**

**Pure Learning**

- Slow thinking: deliberative, cognitive, model-based, extrapolation
- Amazing achievements until this day
- “*Pure logic is brittle*”  
noise, uncertainty, incomplete knowledge, ...



# The AI Dilemma



**Pure (Logic) Reasoning**

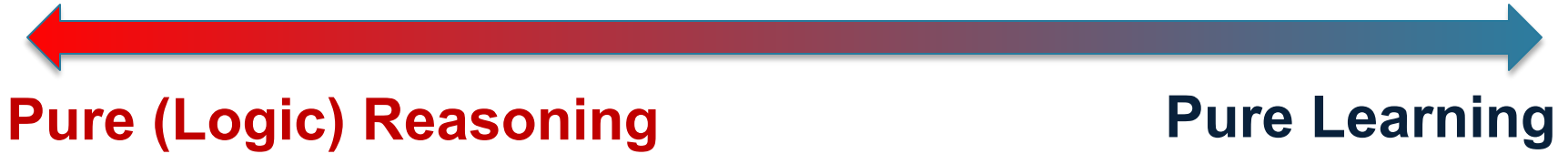
**Pure Learning**

- Fast thinking: instinctive, perceptive, model-free, interpolation
- Amazing achievements recently
- *“Pure learning is brittle”*

bias, algorithmic fairness, interpretability, explainability, adversarial attacks, unknown unknowns, calibration, verification, missing features, missing labels, data efficiency, shift in distribution, general robustness and safety fails to incorporate a sensible model of the world



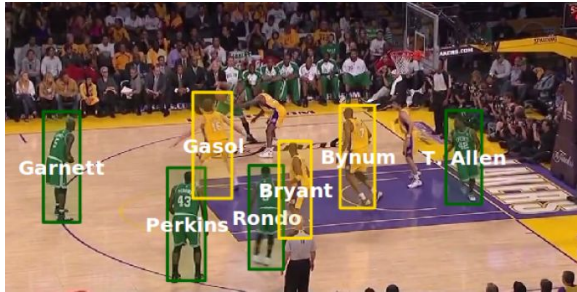
# The AI Dilemma



Integrate reasoning into modern learning algorithms

*Today: Deep learning with structured output constraints*  
*Learning monotonic neural networks*

# Knowledge in Vision, Robotics, NLP, Activity Recognition

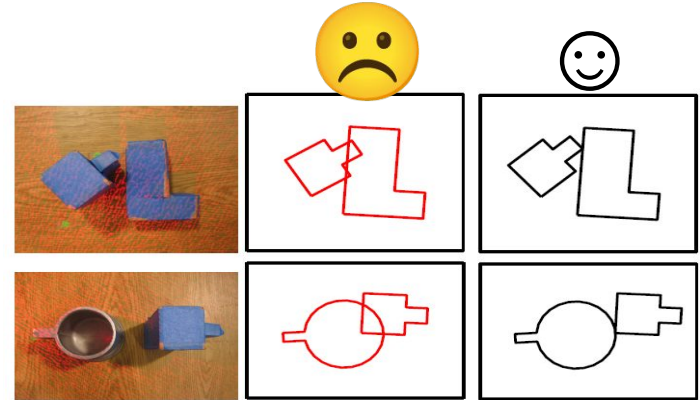


People appear at most once in a frame

A= At least one verb  
in each sentence.  
If X and Y are married,  
then they are people.



Cut the orange before squeezing the orange

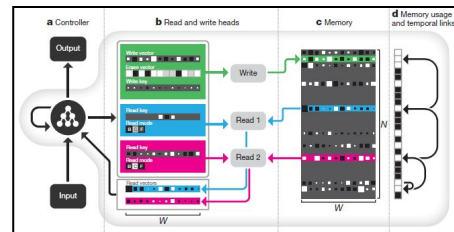


Rigid objects don't overlap

# Motivation: Deep Learning

The image shows a screenshot of a New Scientist article. The header features the 'New Scientist' logo and navigation links for Home, News, Technology, Space, Physics, Health, Earth, Humans, Life, Topics, Events, and Jobs. The main headline reads 'Meet The People Shaping The Future Of Energy: Reinventing Energy Summit - 25 November in London'. Below this, there is a social media sharing bar with icons for Google+, Facebook, Twitter, and a plus sign, along with a '26' notification. The article title is 'DeepMind's AI has learned to navigate the Tube using memory', dated 'DAILY NEWS 12 October 2016'. The main image shows a person with long blonde hair looking at a large London Underground map.

The image shows a screenshot of a Nature article. The header features the 'nature' logo and the text 'International weekly journal of science'. Navigation links include Home, News & Comment, Research, Careers & Jobs, Current Issue, Archive, Audio & Video, and For. The article title is 'Google's AI reasons its way around the London Underground', dated '2016 November Article'. The main text reads: 'DeepMind's latest technique uses external memory to solve tasks that require logic and reasoning — a step toward more human-like AI.'



[Graves, A., Wayne, G., Reynolds, M., Harley, T., Danihelka, I., Grabska-Barwińska, A., et al.. (2016). Hybrid computing using a neural network with dynamic external memory. *Nature*, 538(7626), 471-476.]

# Motivation: Deep Learning

DeepMind's latest technique uses external memory to solve tasks that require **logic** and **reasoning** — a step toward more human-like AI.

... but ...



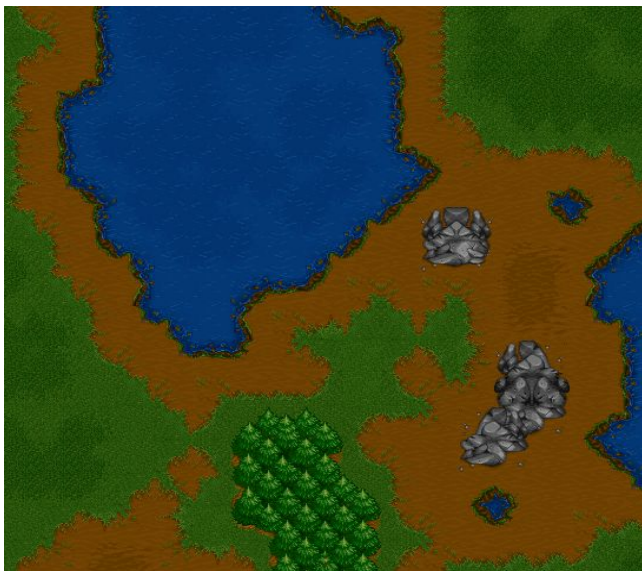
optimal planner recalculating a shortest path to the end node. To ensure that the network always moved to a valid node, the output distribution was renormalized over the set of possible triples outgoing from the current node. The performance

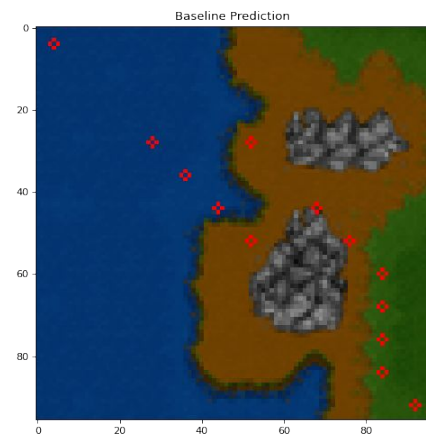
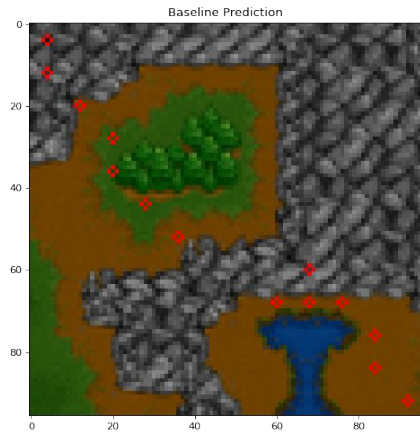
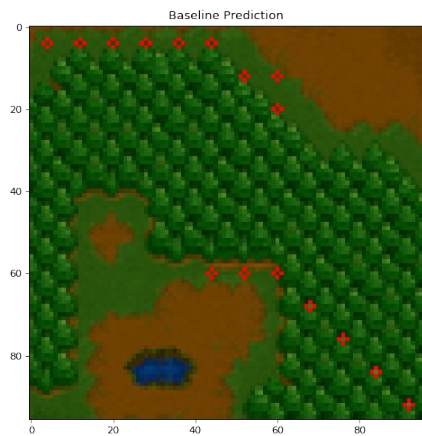
it also received input triples during the answer phase, indicating the actions chosen on the previous time-step. This makes the problem a 'structured prediction'



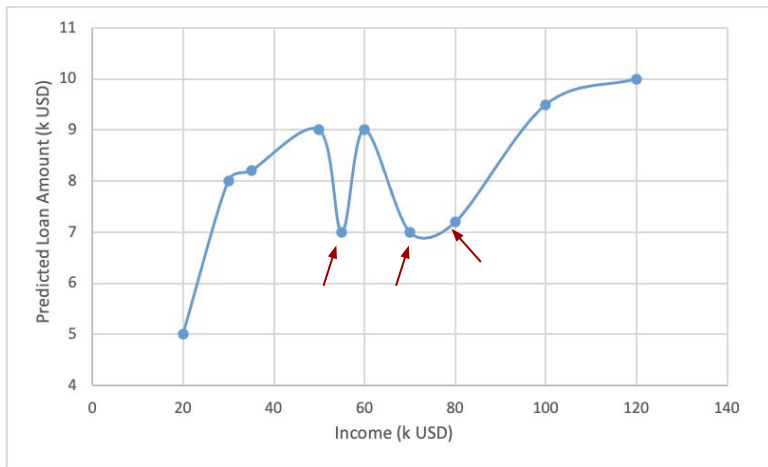
# Warcraft Shortest Path

Predicting the minimum-cost path





# Predict Loan Amount



Neural Network Model: **Increasing income can decrease the approved loan amount**

Monotonicity (Prior Knowledge):

**Increasing income should increase the approved loan amount**

# Knowledge vs. Data

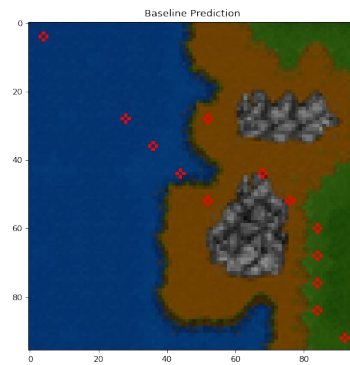
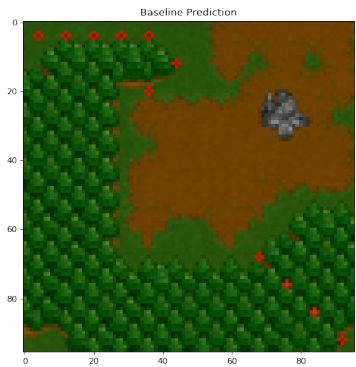
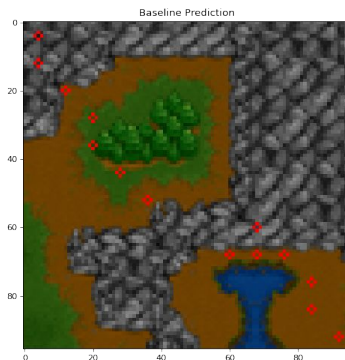
- Where did the world knowledge go?
  - Python scripts
    - Decode/encode/search cleverly
    - Fix inconsistent beliefs
  - Rule-based decision systems
  - Dataset design
  - “a big hack” (with author’s permission)
- In some sense we went backwards
  - Less principled, scientific, and intellectually satisfying ways of incorporating knowledge

# Deep Learning with Constraints

*without constraint*



*without constraint*



# Warcraft min-cost simple-path prediction results

Test accuracy %	Coherent	Incoherent	Constraint
ResNet-18	44.8	<b>97.7</b>	56.9

*Is prediction the shortest path?  
**This is the real task!***

*Are individual edge predictions correct?*

*Is output a path?*

# pylon

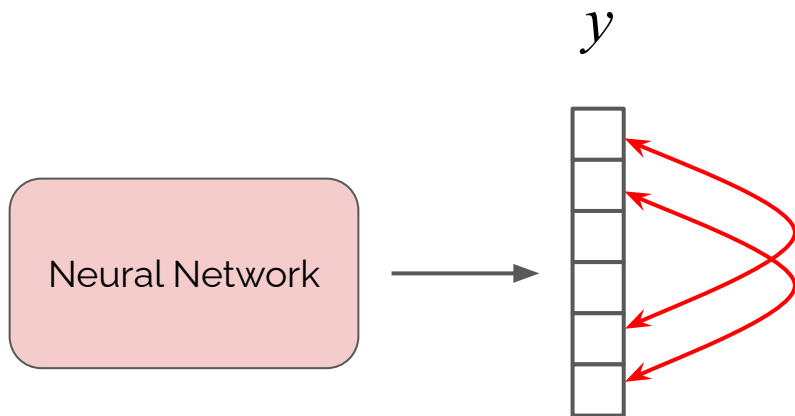
A PyTorch Framework for Learning with Constraints

Kareem Ahmed   Tao Li   Thy Ton   Quan Guo,  
Kai-Wei Chang   Parisa Kordjamshidi   Vivek Srikumar  
Guy Van den Broeck   Sameer Singh

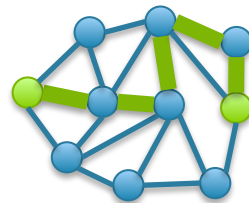
<http://pylon-lib.github.io>



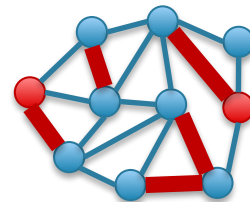
# Declarative Knowledge of the Output



How is the output structured?  
Are all possible outputs valid?



vs.



How are the outputs related to each other?

Learning this from data is inefficient  
Much easier to express this declaratively

# pylon

Library that extends PyTorch to allow injection of declarative knowledge

- **Easy to Express Knowledge:** users write **arbitrary constraints** on the output
- **Integrates with PyTorch:** **minimal change** to existing code
- **Efficient Training:** compiles into loss that can be **efficiently optimized**
  - Exact semantic loss (see later)
  - Monte-carlo estimate of loss
  - T-norm approximation
  - *your solver?*

# pylon

PyTorch Code

```
for i in range(train_iters):  
    ...  
    py = model(x)  
    ...  
    loss = CrossEntropy(py, ...)
```

1

Specify knowledge as a predicate

```
def check(y):  
    ...  
    return isValid
```

# pylon

PyTorch Code

```
for i in range(train_iters):  
    ...  
    py = model(x)  
    ...  
    loss = CrossEntropy(py, ...)  
    loss += constraint_loss(check)(py)
```

1

Specify knowledge as a predicate

```
def check(y):  
    ...  
    return isValid
```

2

Add as loss to training

```
loss += constraint_loss(check)
```

# pylon

PyTorch Code

```
for i in range(train_iters):  
    ...  
    py = model(x)  
    ...  
    loss = CrossEntropy(py, ...)  
    loss += constraint_loss(check)(py)
```

1 Specify knowledge as a predicate

```
def check(y):  
    ...  
    return isValid
```

2 Add as loss to training

```
loss += constraint_loss(check)
```

3 pylon derives the gradients  
(solves a combinatorial problem)

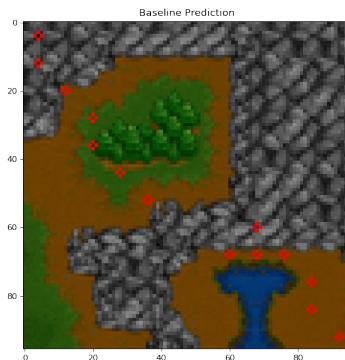
*without constraint*



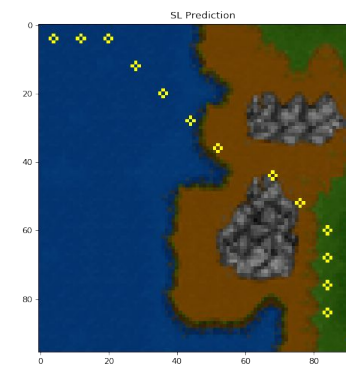
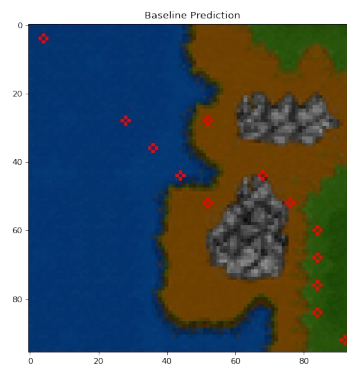
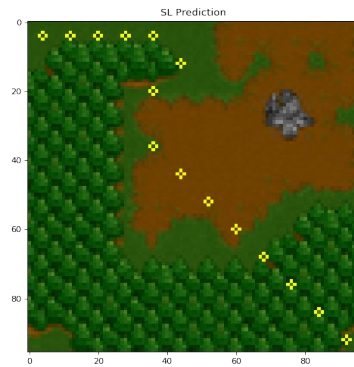
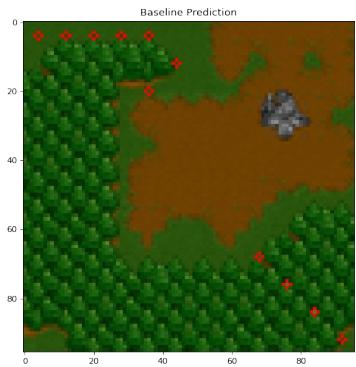
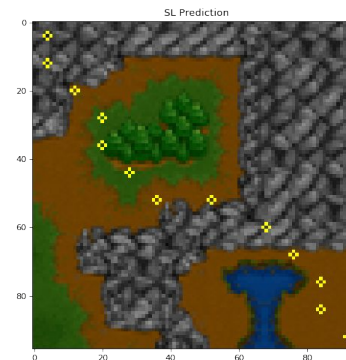
*with constraint*



*without constraint*



*with constraint*



## Warcraft min-cost simple-path prediction results

Test accuracy %	Coherent	Incoherent	Constraint
ResNet-18	44.8	<b>97.7</b>	56.9
+ Semantic loss	<b>50.9</b>	<b>97.7</b>	<b>67.4</b>

# Semantic Loss

Q: How close is output  $\mathbf{p}$  to satisfying constraint  $\alpha$ ?

A: Semantic loss function  $L(\alpha, \mathbf{p})$

- Axioms, for example:
  - If  $\alpha$  constrains to one label,  $L(\alpha, \mathbf{p})$  is cross-entropy
  - If  $\alpha$  implies  $\beta$  then  $L(\alpha, \mathbf{p}) \geq L(\beta, \mathbf{p})$  ( $\alpha$  more strict)
- Implied Properties:
  - If  $\alpha$  is equivalent to  $\beta$  then  $L(\alpha, \mathbf{p}) = L(\beta, \mathbf{p})$
  - If  $\mathbf{p}$  is Boolean and satisfies  $\alpha$  then  $L(\alpha, \mathbf{p}) = 0$

 **SEMANTIC**  
Loss!



# Axioms imply unique semantic loss:

$$L^s(\alpha, \mathbf{p}) \propto -\log \underbrace{\sum_{\mathbf{x} \models \alpha} \prod_{i: \mathbf{x} \models X_i} p_i \prod_{i: \mathbf{x} \models \neg X_i} (1 - p_i)}_{\text{Probability of satisfying constraint } \alpha \text{ after sampling from neural net output layer } \mathbf{p}}$$

Probability of satisfying constraint  $\alpha$  after sampling from neural net output layer  $\mathbf{p}$

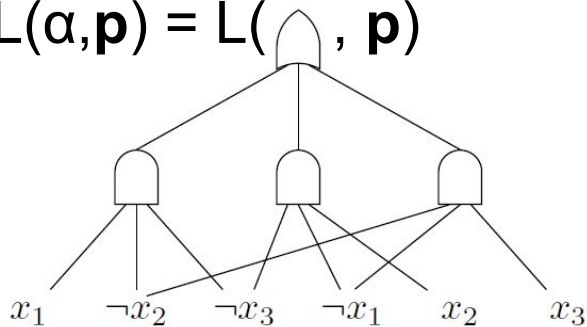
In general: #P-hard 😞

Do this probabilistic-logical reasoning during learning in a computation graph

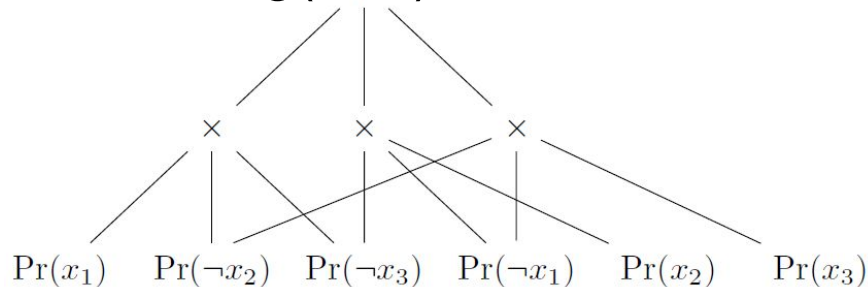
# Circuits = Computation Graphs

- Logical circuits that can count solutions (#SAT)  
also compute semantic loss efficiently in size of circuit

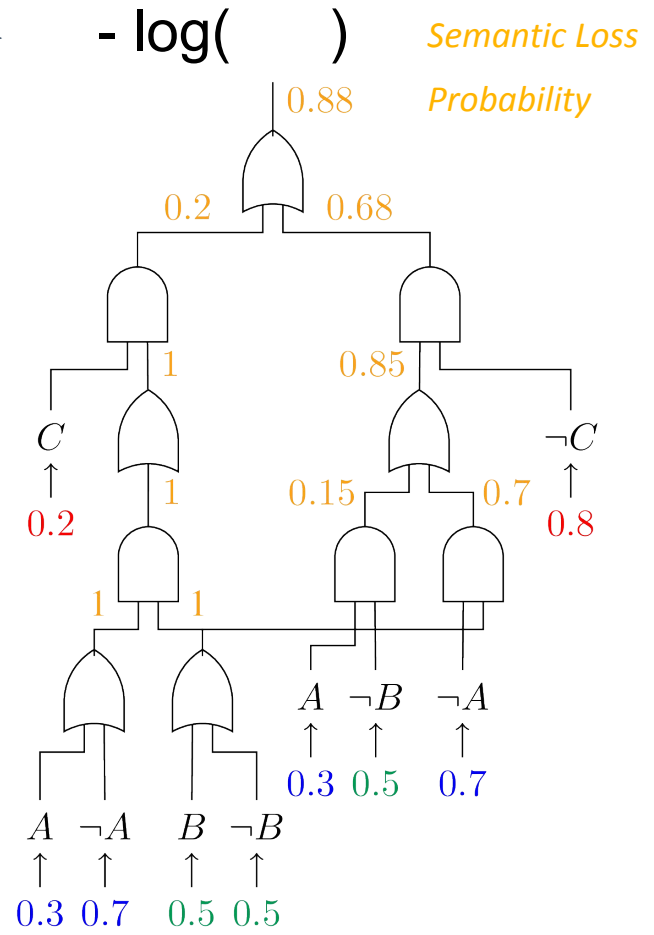
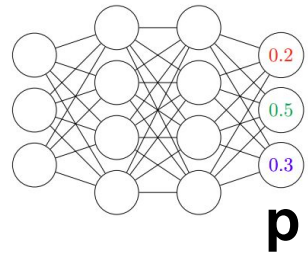
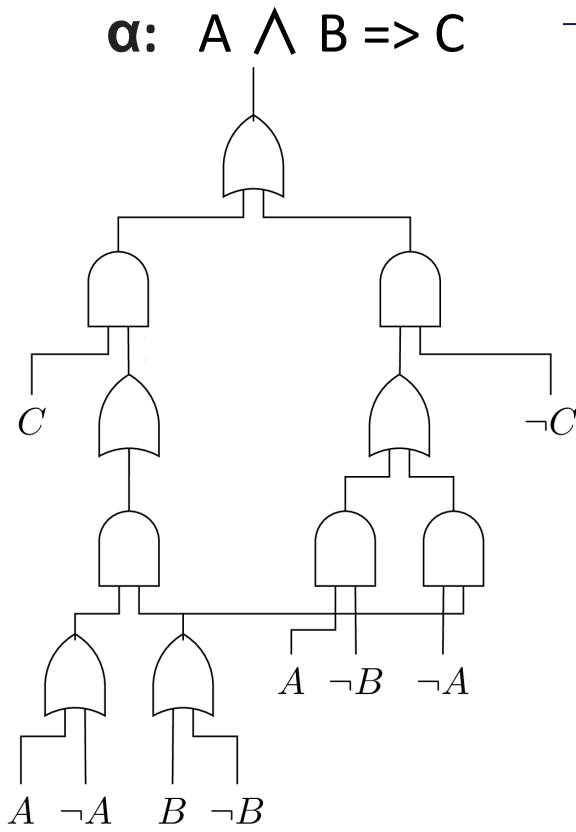
$$L(\alpha, \mathbf{p}) = L(\text{Circuit}, \mathbf{p})$$

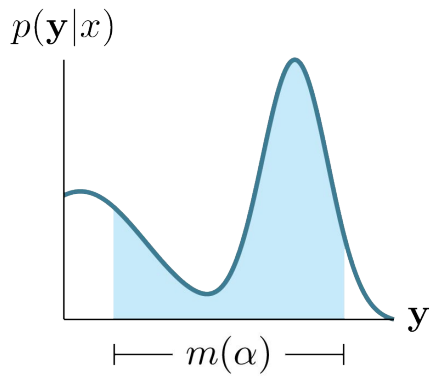


$$= -\log(\text{Circuit})$$

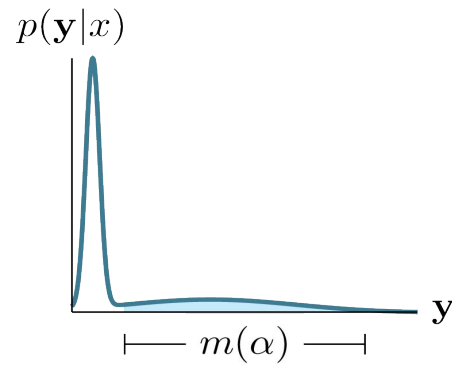


- Compilation into circuit by SAT solvers (once)
- Add circuit to neural network output in pytorch/tensorflow/...



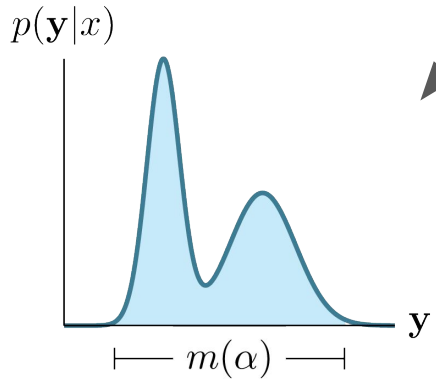


a) A network uncertain over both valid & invalid predictions

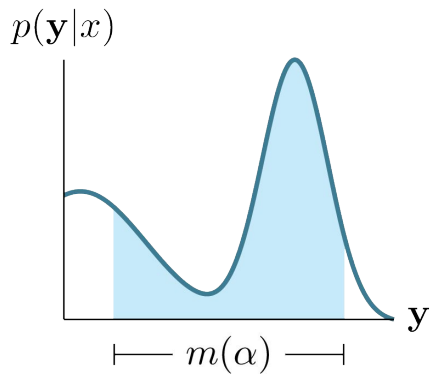


b) A network allocating most of its mass to an invalid prediction.

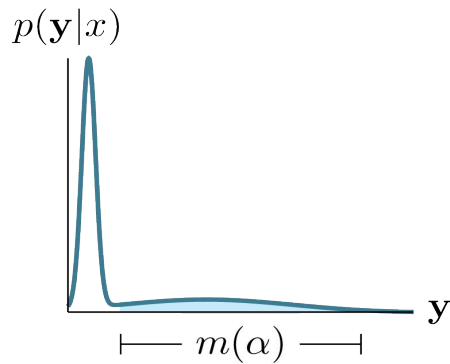
**Semantic Loss**



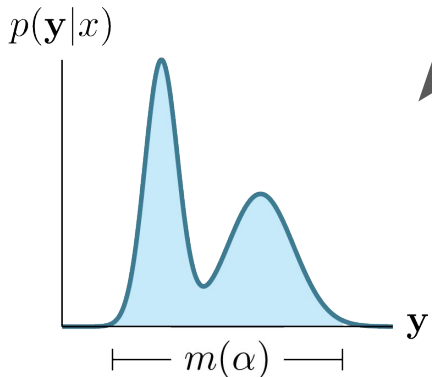
c) A network allocating most of its mass to models of the formula



a) A network uncertain over both valid & invalid predictions



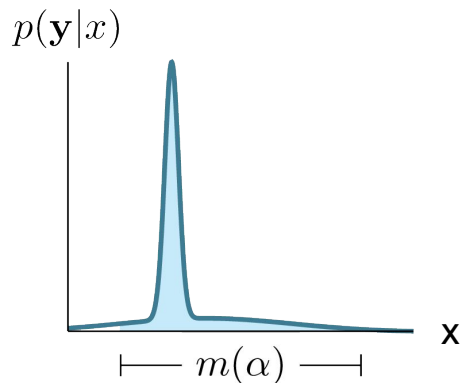
b) A network allocating most of its mass to an invalid prediction.



c) A network allocating most of its mass to models of the formula

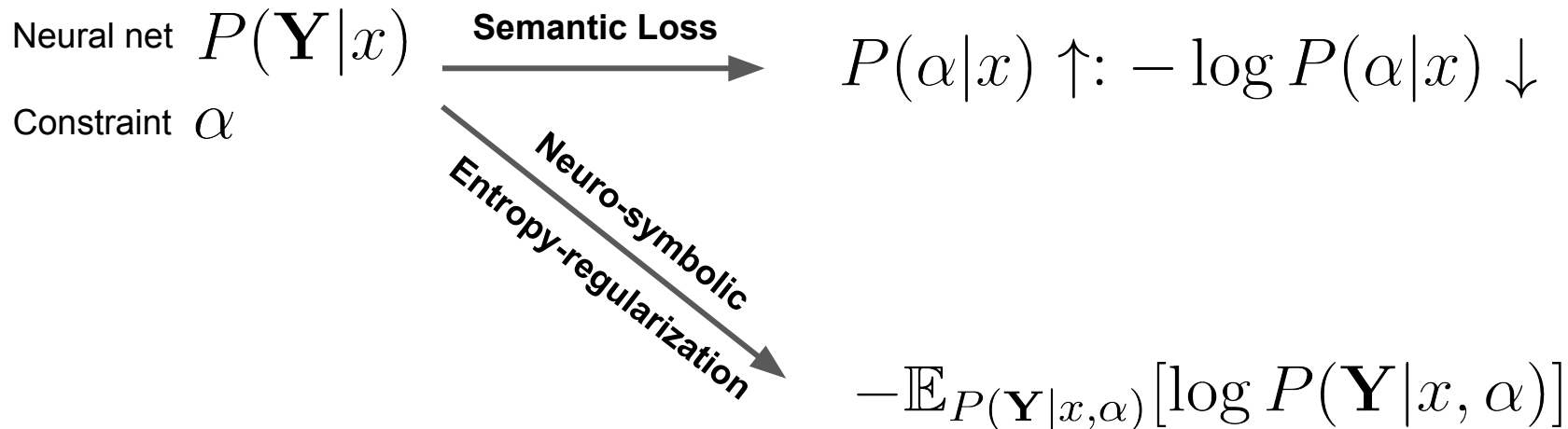
**Semantic Loss**

**Neuro-Symbolic Entropy Regularization**



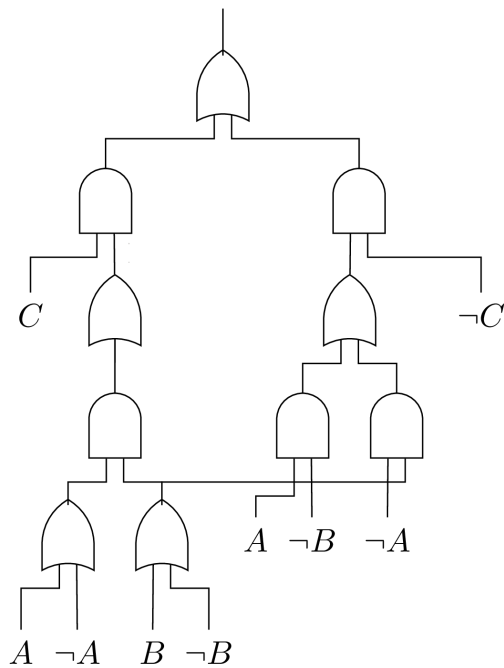
d) A network allocating most of mass to one model of formula

# Two complementary neuro-symbolic losses

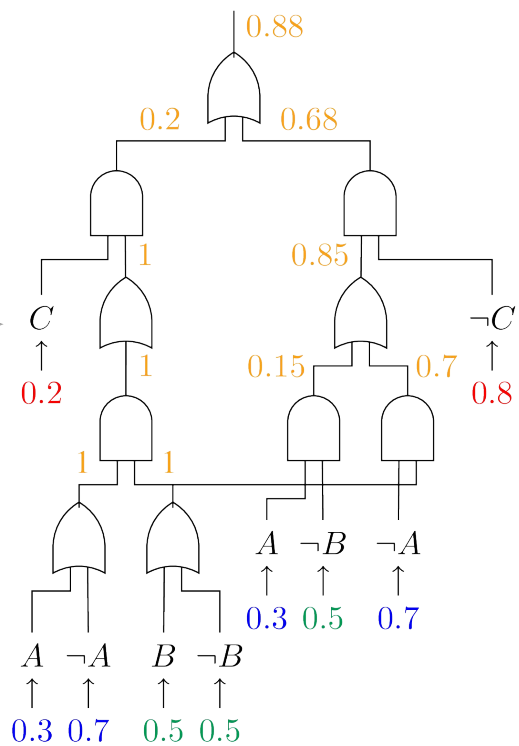


# Warcraft min-cost simple-path prediction results

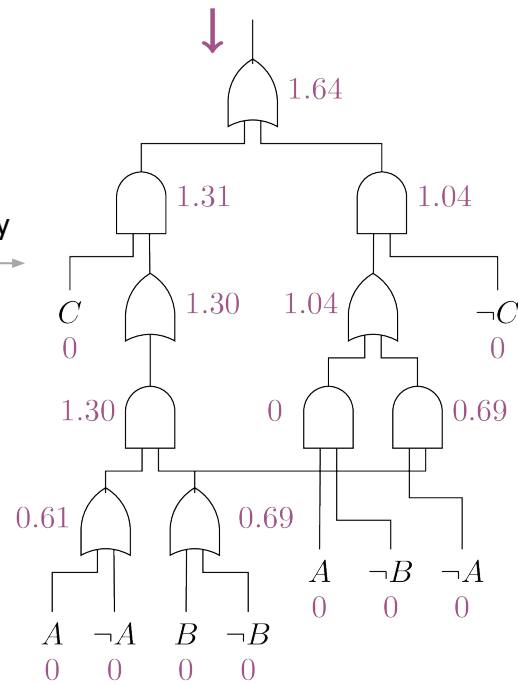
Test accuracy %	Coherent	Incoherent	Constraint
ResNet-18	44.8	<b>97.7</b>	56.9
Semantic loss	<b>50.9</b>	<b>97.7</b>	<b>67.4</b>
+ Entropy All	51.5	97.6	67.7
+ Entropy Circuit	<b>55.0</b>	<b>97.9</b>	<b>69.8</b>



Probability



Entropy



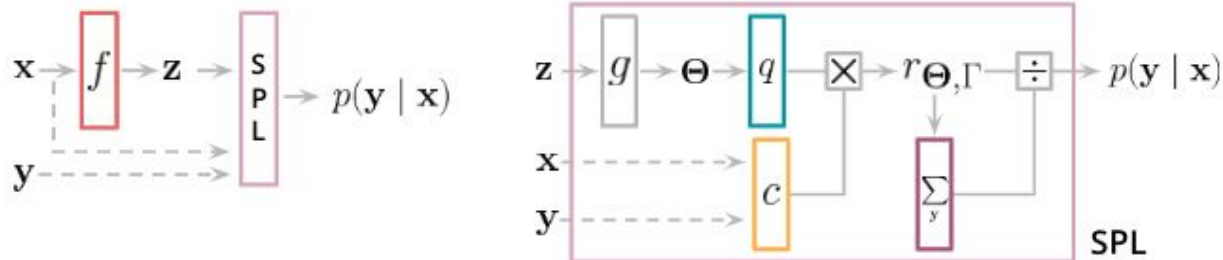


# Joint entity-relation extraction in natural language processing

#		3	5	10	15	25	50	75
ACE05	Baseline	4.92 ± 1.12	7.24 ± 1.75	13.66 ± 0.18	15.07 ± 1.79	21.65 ± 3.41	28.96 ± 0.98	33.02 ± 1.17
	Self-training	7.72 ± 1.21	12.83 ± 2.97	16.22 ± 3.08	17.55 ± 1.41	27.00 ± 3.66	32.90 ± 1.71	37.15 ± 1.42
	Product t-norm	8.89 ± 5.09	14.52 ± 2.13	19.22 ± 5.81	21.80 ± 7.67	30.15 ± 1.01	34.12 ± 2.75	37.35 ± 2.53
	Semantic Loss	12.00 ± 3.81	14.92 ± 3.14	22.23 ± 3.64	27.35 ± 3.10	30.78 ± 0.68	36.76 ± 1.40	38.49 ± 1.74
	+ Full Entropy	<b>14.80 ± 3.70</b>	15.78 ± 1.90	23.34 ± 4.07	28.09 ± 1.46	31.13 ± 2.26	36.05 ± 1.00	39.39 ± 1.21
	+ NeSy Entropy	14.72 ± 1.57	<b>18.38 ± 2.50</b>	<b>26.41 ± 0.49</b>	<b>31.17 ± 1.68</b>	<b>35.85 ± 0.75</b>	<b>37.62 ± 2.17</b>	<b>41.28 ± 0.46</b>
SciERC	Baseline	2.71 ± 1.10	2.94 ± 1.00	3.49 ± 1.80	3.56 ± 1.10	8.83 ± 1.00	12.32 ± 3.00	12.49 ± 2.60
	Self-training	3.56 ± 1.40	3.04 ± 0.90	4.14 ± 2.60	3.73 ± 1.10	9.44 ± 3.80	14.82 ± 1.20	13.79 ± 3.90
	Product t-norm	<b>6.50 ± 2.00</b>	8.86 ± 1.20	10.92 ± 1.60	13.38 ± 0.70	13.83 ± 2.90	19.20 ± 1.70	19.54 ± 1.70
	Semantic Loss	6.47 ± 1.02	<b>9.31 ± 0.76</b>	11.50 ± 1.53	12.97 ± 2.86	14.07 ± 2.33	20.47 ± 2.50	23.72 ± 0.38
	+ Full Entropy	6.26 ± 1.21	8.49 ± 0.85	11.12 ± 1.22	14.10 ± 2.79	17.25 ± 2.75	<b>22.42 ± 0.43</b>	24.37 ± 1.62
	+ NeSy Entropy	6.19 ± 2.40	8.11 ± 3.66	<b>13.17 ± 1.08</b>	<b>15.47 ± 2.19</b>	<b>17.45 ± 1.52</b>	22.14 ± 1.46	<b>25.11 ± 1.03</b>

# Semantic Probabilistic Layers

- How to give a 100% guarantee that Boolean constraints will be satisfied?
- Bake the constraint into the neural network as a special layer



- Secret sauce is again tractable circuits – computation graphs for reasoning

# Warcraft Shortest Path



GROUND TRUTH



RESNET-18



SEMANTIC LOSS



SPL (ours)

# Hierarchical Multi-Label Classification

“if the image is classified as a dog, it must also be classified as an animal”

“if the image is classified as an animal, it must be classified as either cat or dog”

DATASET	EXACT MATCH	
	HMCNN	MLP+SPL
CELLCYCLE	3.05 ± 0.11	<b>3.79 ± 0.18</b>
DERISI	1.39 ± 0.47	<b>2.28 ± 0.23</b>
EISEN	5.40 ± 0.15	<b>6.18 ± 0.33</b>
EXPR	4.20 ± 0.21	<b>5.54 ± 0.36</b>
GASCH1	3.48 ± 0.96	<b>4.65 ± 0.30</b>
GASCH2	3.11 ± 0.08	<b>3.95 ± 0.28</b>
SEQ	5.24 ± 0.27	<b>7.98 ± 0.28</b>
SPO	<b>1.97 ± 0.06</b>	<b>1.92 ± 0.11</b>
DIATOMS	48.21 ± 0.57	<b>58.71 ± 0.68</b>
ENRON	5.97 ± 0.56	<b>8.18 ± 0.68</b>
IMCLEF07A	79.75 ± 0.38	<b>86.08 ± 0.45</b>
IMCLEF07D	76.47 ± 0.35	<b>81.06 ± 0.68</b>

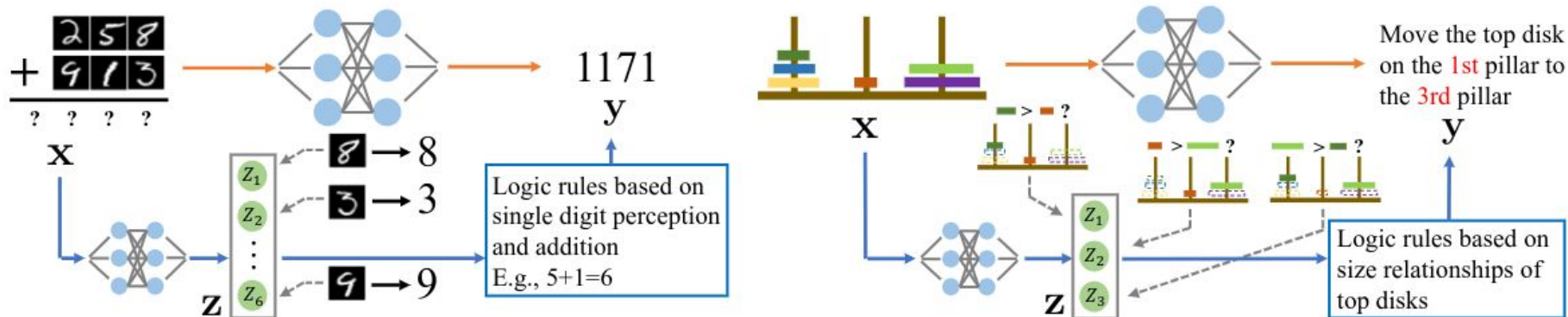
# Neuro-Symbolic Learning Settings

Learn

1. **neural network** given **symbols and constraints and data**
2. **neural network and constraints** given **symbols and data**
3. **neural network and constraints and symbols** given **data**

*Everyone is working on 1. Ongoing work on 2.*

# Neuro-Symbolic Joint Training



Learn invariant features using neural networks. Learn logic to tie it all together.

# Neuro-Symbolic Joint Training

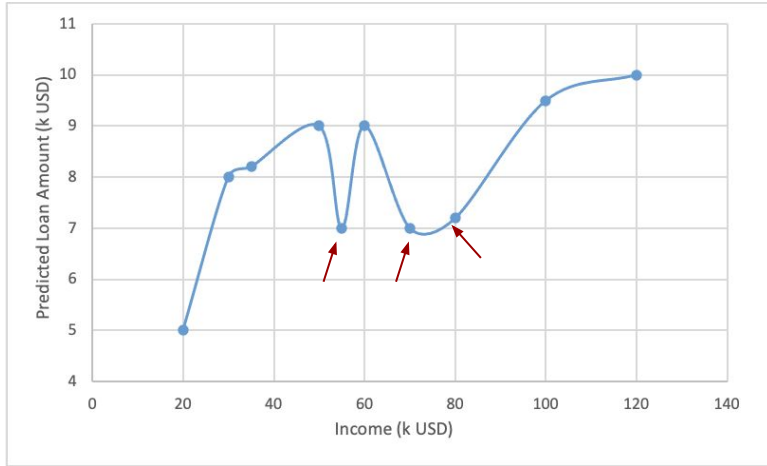
Model	Multi-digit addition [test seq length + train/test img]						Tower of Hanoi		
	5 w/ test	10 w/ test	20 w/ test	5 w/ train	10 w/ train	20 w/ train	Task #1	Task #2	Task #3
DeepProbLog <sup>†</sup>	88.30	77.46	timeout	94.92	89.74	timeout	89.28	97.96	89.33
LSTM	81.40	56.97	39.05	88.92	77.40	63.23	78.26	<b>98.32</b>	74.36
DNC	81.49	59.64	33.83	81.88	59.96	37.85	76.20	97.87	73.87
NToC(ours)	<b>89.82</b>	<b>77.97</b>	<b>63.55</b>	<b>89.97</b>	<b>86.07</b>	<b>71.96</b>	<b>85.16</b>	97.94	<b>85.49</b>

Learn invariant features using neural networks. Learn logic to tie it all together.

# Monotonicity Invariants for Neural Networks



# Predict Loan Amount

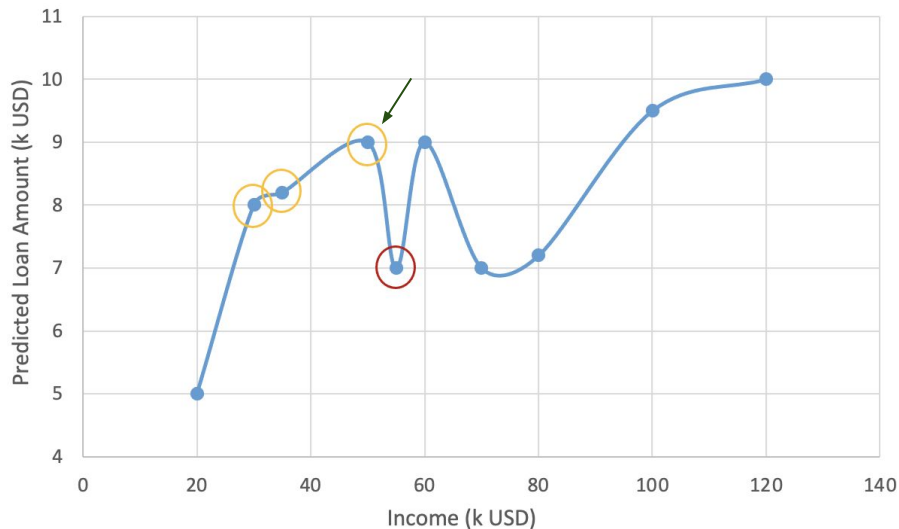


Neural Network Model: **Increasing income can decrease the approved loan amount**

Monotonicity (Prior Knowledge):

Increasing income should increase the approved loan amount

# Counterexamples

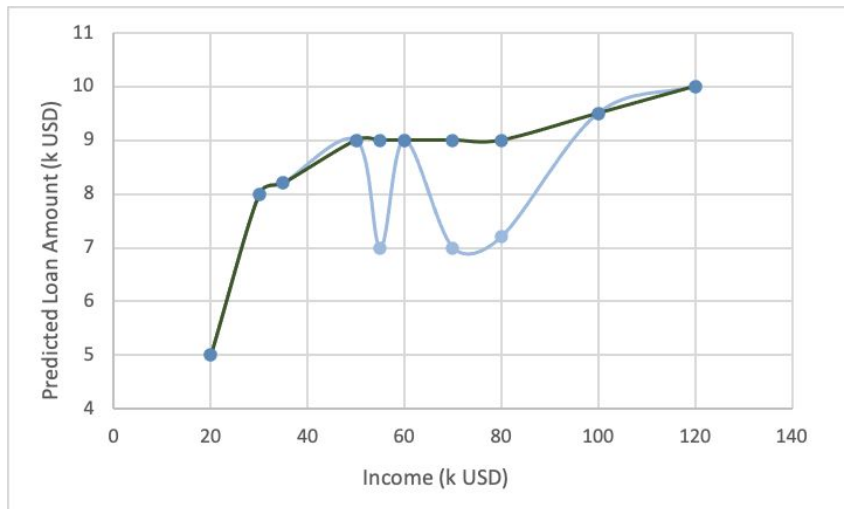


$$\exists x, y \ x \leq y \implies f(x) > f(y)$$

Computed using SMT(LRA)  
logical reasoning solver

Maximal counterexamples  
(largest violation) using OMT

# Counterexample-Guided Predictions



## Monotonic Envelope:

- Replace each prediction by its maximal counterexample
- Envelope construction is online (during prediction)
- Guarantees monotonic predictions for any ReLU neural net
  
- Works for high-dimensional input
- Works for multiple monotonic features

# Monotonic Envelope: Performance

Dataset	Feature	NN <sub>b</sub>	Envelope
Auto-MPG	Weight	9.33±3.22	<b>9.19±3.41</b>
	Displ.	9.33±3.22	9.63±2.61
	W,D	9.33±3.22	9.63±2.61
	W,D,HP	9.33±3.22	9.63±2.61
Boston	Rooms	14.37±2.4	<b>14.19±2.28</b>
	Crime	14.37±2.4	<b>14.02±2.17</b>

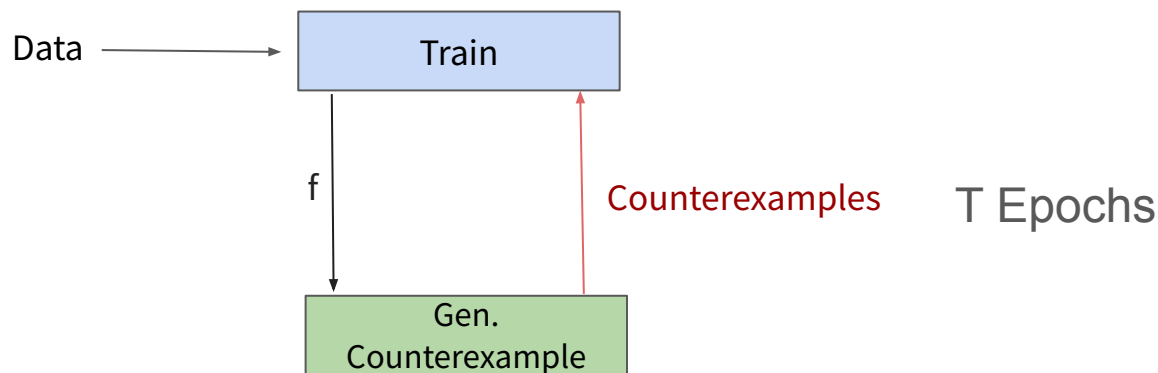
Dataset	Feature	NN <sub>b</sub>	Envelope
Heart	Trestbps	0.85±0.04	0.85±0.04
	Chol.	0.85±0.04	0.85±0.05
	T,C	0.85±0.04	0.85±0.05
Adult	Cap. Gain	0.84	0.84
	Hours	0.84	0.84

Guaranteed monotonicity at little to no cost

# Counterexample-Guided Learning

How to use monotonicity to improve model quality?

“Monotonicity as inductive bias”



# Counterexample-Guided Learning: Performance

Dataset	Feature	NN <sub>b</sub>	CGL
Auto-MPG	Weight	9.33±3.22	<b>9.04±2.76</b>
	Displ.	9.33±3.22	<b>9.08±2.87</b>
	W,D	9.33±3.22	<b>8.86±2.67</b>
	W,D,HP	9.33±3.22	<b>8.63±2.21</b>
Boston	Rooms	14.37±2.4	<b>12.24±2.87</b>
	Crime	14.37±2.4	<b>11.66±2.89</b>

Dataset	Feature	NN <sub>b</sub>	CGL
Heart	Trestbps	0.85±0.04	<b>0.86±0.02</b>
	Chol.	<b>0.85±0.04</b>	<b>0.85±0.05</b>
	T,C	0.85±0.04	<b>0.86±0.06</b>
Adult	Cap. Gain	<b>0.84</b>	<b>0.84</b>
	Hours	<b>0.84</b>	<b>0.84</b>

Monotonicity is a *great* inductive bias for learning

# COMET:

## Counterexample-Guided Monotonicity Enforced Training

Table 4: Monotonicity is an effective inductive bias. COMET outperforms Min-Max networks on all datasets. COMET outperforms DLN in regression datasets and achieves similar results in classification datasets.

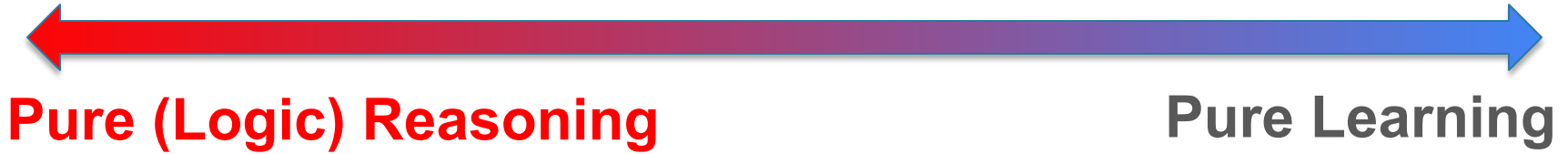
Dataset	Features	Min-Max	DLN	COMET
Auto-MPG	Weight	$9.91 \pm 1.20$	$16.77 \pm 2.57$	<b><math>8.92 \pm 2.93</math></b>
	Displ.	$11.78 \pm 2.20$	$16.67 \pm 2.25$	<b><math>9.11 \pm 2.25</math></b>
	W,D	$11.60 \pm 0.54$	$16.56 \pm 2.27$	<b><math>8.89 \pm 2.29</math></b>
	W,D,HP	$10.14 \pm 1.54$	$13.34 \pm 2.42$	<b><math>8.81 \pm 1.81</math></b>
Boston	Rooms	$30.88 \pm 13.78$	$15.93 \pm 1.40$	<b><math>11.54 \pm 2.55</math></b>
	Crime	$25.89 \pm 2.47$	$12.06 \pm 1.44$	<b><math>11.07 \pm 2.99</math></b>

Dataset	Features	Min-Max	DLN	COMET
Heart	Trestbps	$0.75 \pm 0.04$	$0.85 \pm 0.02$	<b><math>0.86 \pm 0.03</math></b>
	Chol.	$0.75 \pm 0.04$	$0.85 \pm 0.04$	<b><math>0.87 \pm 0.03</math></b>
	T,C	$0.75 \pm 0.04$	<b><math>0.86 \pm 0.02</math></b>	<b><math>0.86 \pm 0.03</math></b>
Adult	Cap. Gain	0.77	<b>0.84</b>	<b>0.84</b>
	Hours	0.73	<b>0.85</b>	0.84

COMET = Provable Guarantees + SotA Results

# The AI Dilemma



- Knowledge is (hidden) everywhere in ML
- A little bit of reasoning goes a long way!

*Deep learning with structured output constraints*  
*Learning monotonic neural networks*



# Thanks

*This was the work of many wonderful  
students/postdoc/collaborators!*

References: <http://starai.cs.ucla.edu/publications/>