# PSDDs for Tractable Learning in Structured and Unstructured Spaces


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# References


**Probabilistic Sentential Decision Diagrams**
Doga Kisa, Guy Van den Broeck, Arthur Choi and Adnan Darwiche
KR, 2014

**Learning with Massive Logical Constraints**
Doga Kisa, Guy Van den Broeck, Arthur Choi and Adnan Darwiche
ICML 2014 workshop

**Tractable Learning for Structured Probability Spaces**
Arthur Choi, Guy Van den Broeck and Adnan Darwiche
IJCAI, 2015

**Tractable Learning for Complex Probability Queries**
Jessa Bekker, Jesse Davis, Arthur Choi, Adnan Darwiche, Guy Van den Broeck.
NIPS, 2015

**Learning the Structure of PSDDs**
Jessa Bekker, Yitao Liang and Guy Van den Broeck
Under review, 2017

**Towards Compact Interpretable Models: Learning and Shrinking PSDDs**
Yitao Liang and Guy Van den Broeck
Under review, 2017

*Structured vs. unstructured probability spaces?*

# Running Example

Courses:

- Logic (L)
- Knowledge Representation (K)
- Probability (P)
- Artificial Intelligence (A)

## Data

| L | K | P | A | Students |
|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 6 |
| 0 | 0 | 1 | 1 | 54 |
| 0 | 1 | 1 | 1 | 10 |
| 1 | 0 | 0 | 0 | 5 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 17 |
| 1 | 1 | 1 | 0 | 4 |
| 1 | 1 | 1 | 1 | 3 |

## Constraints

- Must take at least one of Probability or Logic.
- Probability is a prerequisite for AI.
- The prerequisites for KR is either AI or Logic.

# Probability Space

unstructured

| L | K | P | A |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 |

# Structured Probability Space

unstructured

| L | K | P | A |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 |

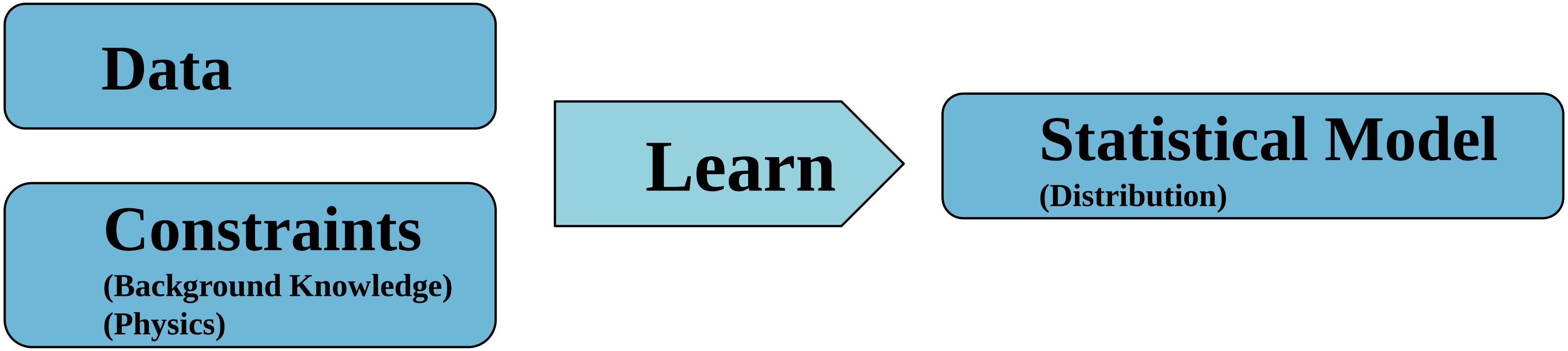
- Must take at least one of Probability or Logic.
- Probability is a prerequisite for AI.
- The prerequisites for KR is either AI or Logic.

**7 out of 16 instantiations are impossible**

structured

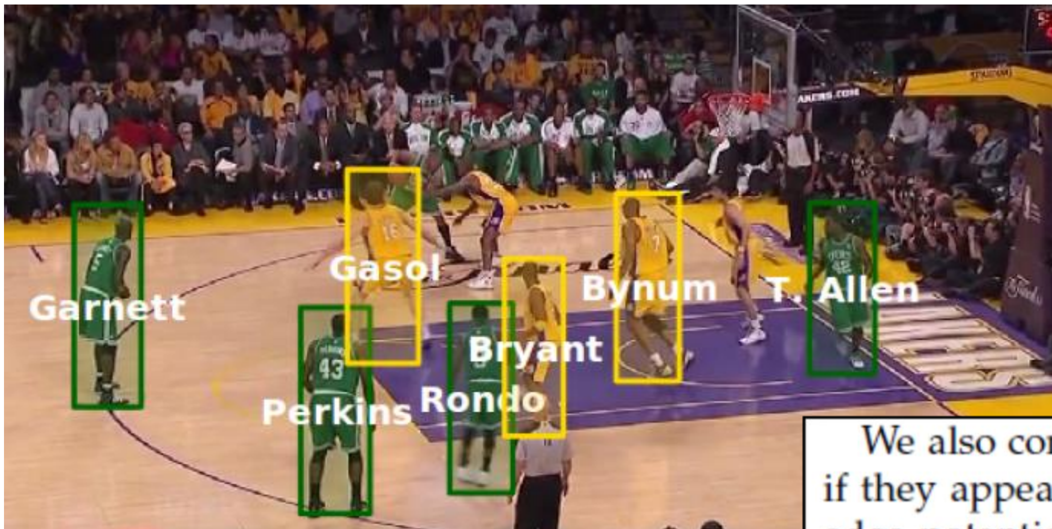
| L | K | P | A |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 |

# Learning with Constraints




Learn a statistical model that assigns
**zero probability**
to instantiations that violate the constraints.

# Example: Video




We also connect all pairs of identity nodes $y_{t,i}$ and $y_{t,j}$ if they appear in the same time $t$. We then introduce an edge potential that enforces mutual exclusion:

$$\psi_{\text{mutex}}(y_{t,i}, y_{t,j}) = \begin{cases} 1 & \text{if } y_{t,i} \neq y_{t,j} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

This potential specifies the constraint that a player can be appear only *once* in a frame. For example, if the $i$-th detection $y_{t,i}$ has been assign to Bryant, $y_{t,j}$ cannot have the same identity because Bryant is impossible to appear twice in a frame.

[Lu, W. L., Ting, J. A., Little, J. J., & Murphy, K. P. (2013). Learning to track and identify players from broadcast sports videos.]

# Example: Language

- Non-local dependencies:

  *At least one verb in each sentence*

- Sentence compression

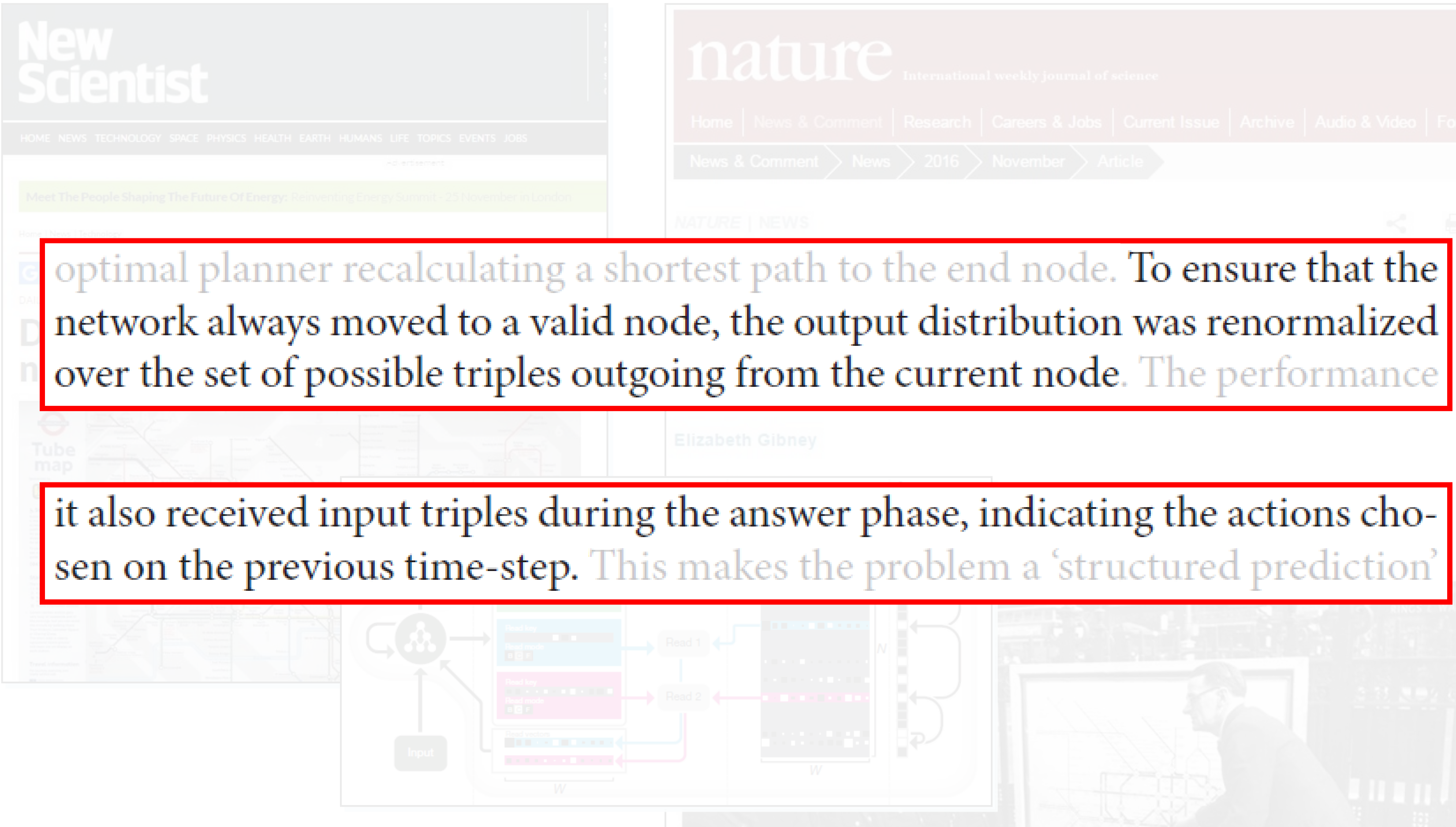
  *If a modifier is kept, its subject is also kept*

- Information extraction
- Semantic role labeling
- … and many more!

| Citations | |
|---|---|
| Start | The citation must start with author or editor. |
| AppearsOnce | Each field must be a consecutive list of words, and can appear at most once in a citation. |
| Punctuation | State transitions must occur on punctuation marks. |
| BookJournal | The words *proc, journal, proceedings, ACM* are *JOURNAL* or *BOOKTITLE*. |
| ... | ... |
| TechReport | The words *tech, technical* are *TECH_REPORT*. |
| Title | Quotations can appear only in titles. |
| Location | The words *CA, Australia, NY* are *LOCATION*. |

[Chang, M., Ratinov, L., & Roth, D. (2008). Constraints as prior knowledge],…, [Chang, M. W., Ratinov, L., & Roth, D. (2012). Structured learning with constrained conditional models.], [https://en.wikipedia.org/wiki/Constrained_conditional_model]

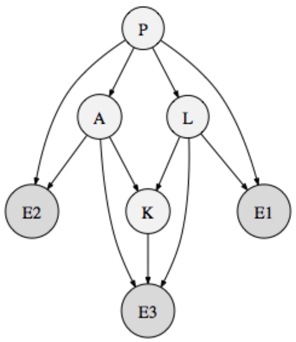
# Example: Deep Learning




[Graves, A., Wayne, G., Reynolds, M., Harley, T., Danihelka, I., Grabska-Barwińska, A., et al.. (2016). Hybrid computing using a neural network with dynamic external memory. *Nature*, *538*(7626), 471-476.]

# What are people doing now?

- Ignore constraints
- Handcraft into models
- Use specialized distributions
- Find non-structured encoding
- Try to learn constraints
- Hack your way around




Accuracy ?
Specialized skill ?
Intractable inference ?
Intractable learning ?
Waste parameters ?
Risk predicting out of space ?

+

**you are on your own ☹**

# Structured Probability Spaces

- Everywhere in ML!
  - Configuration problems, inventory, video, text, deep learning
  - Planning and diagnosis (physics)
  - Causal models: cooking scenarios (interpreting videos)
  - Combinatorial objects: parse trees, rankings, directed acyclic graphs, trees, simple paths, game traces, etc.
- Some representations: constrained conditional models, mixed networks, probabilistic logics.

**No statistical ML boxes out there that take constraints as input! ☹**

Goal: Constraints as important as data! General purpose!

# *Specification Language: Logic*

# Structured Probability Space

unstructured

| L | K | P | A |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
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| 1 | 1 | 1 | 1 |

- Must take at least one of Probability or Logic.
- Probability is a prerequisite for AI.
- The prerequisites for KR is either AI or Logic.

**7 out of 16 instantiations are impossible**

structured

| L | K | P | A |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 |

# Boolean Constraints

unstructured

| L | K | P | A |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 |

$$P \vee L$$
$$A \Rightarrow P$$
$$K \Rightarrow (P \vee L)$$

**7 out of 16 instantiations are impossible**

structured

| L | K | P | A |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 |

# Combinatorial Objects: Rankings

| rank | sushi |
|---|---|
| 1 | fatty tuna |
| 2 | sea urchin |
| 3 | salmon roe |
| 4 | shrimp |
| 5 | tuna |
| 6 | squid |
| 7 | tuna roll |
| 8 | see eel |
| 9 | egg |
| 10 | cucumber roll |

| rank | sushi |
|---|---|
| 1 | shrimp |
| 2 | sea urchin |
| 3 | salmon roe |
| 4 | fatty tuna |
| 5 | tuna |
| 6 | squid |
| 7 | tuna roll |
| 8 | see eel |
| 9 | egg |
| 10 | cucumber roll |

**10 items**: 3,628,800 rankings

**20 items**: 2,432,902,008,176,640,000 rankings

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|---|---|
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| 5 | tuna |
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| 9 | egg |
| 10 | cucumber roll |

**$A_{ij}$ item $i$ at position $j$**
**($n$ items require $n^2$ Boolean variables)**

An item may be assigned to more than one position

A position may contain more than one item

# Encoding Rankings in Logic

$A_{ij}$ : item $i$ at position $j$

| | pos 1 | pos 2 | pos 3 | pos 4 |
|---|---|---|---|---|
| item 1 | $A_{11}$ | $A_{12}$ | $A_{13}$ | $A_{14}$ |
| item 2 | $A_{21}$ | $A_{22}$ | $A_{23}$ | $A_{24}$ |
| item 3 | $A_{31}$ | $A_{32}$ | $A_{33}$ | $A_{34}$ |
| item 4 | $A_{41}$ | $A_{42}$ | $A_{43}$ | $A_{44}$ |

constraint: each item $i$ assigned to a unique position ($n$ constraints)

$$\bigvee_j A_{ij} \wedge \left( \bigwedge_{k \neq j} \neg A_{ik} \right)$$

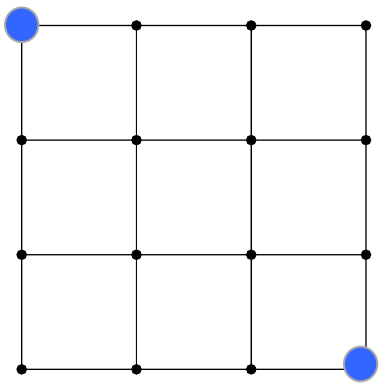
constraint: each position $j$ assigned a unique item ($n$ constraints)

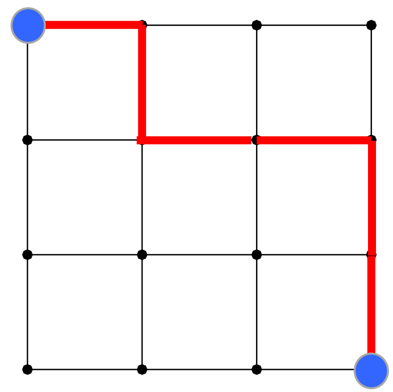
$$\bigvee_i A_{ij} \wedge \left( \bigwedge_{k \neq i} \neg A_{kj} \right)$$

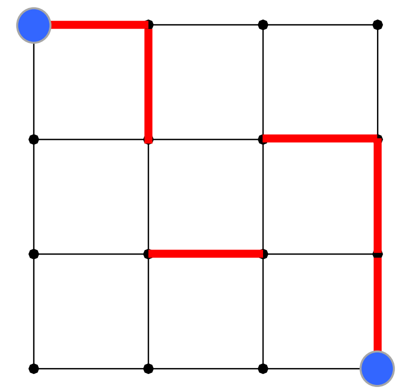
| | |
|---|---|
| total constraints | $2n$ |
| unstructured space | $2^{n^2}$ |
| structured space | $n!$ |

# Structured Space for Paths

cf. Nature paper





**Good variable assignment (represents route)**

**184**

**Bad variable assignment (does not represent route)**

**16,777,032**

Space easily encoded in logical constraints ☺

See [Choi, Tavabi, Darwiche, AAAI 2016]

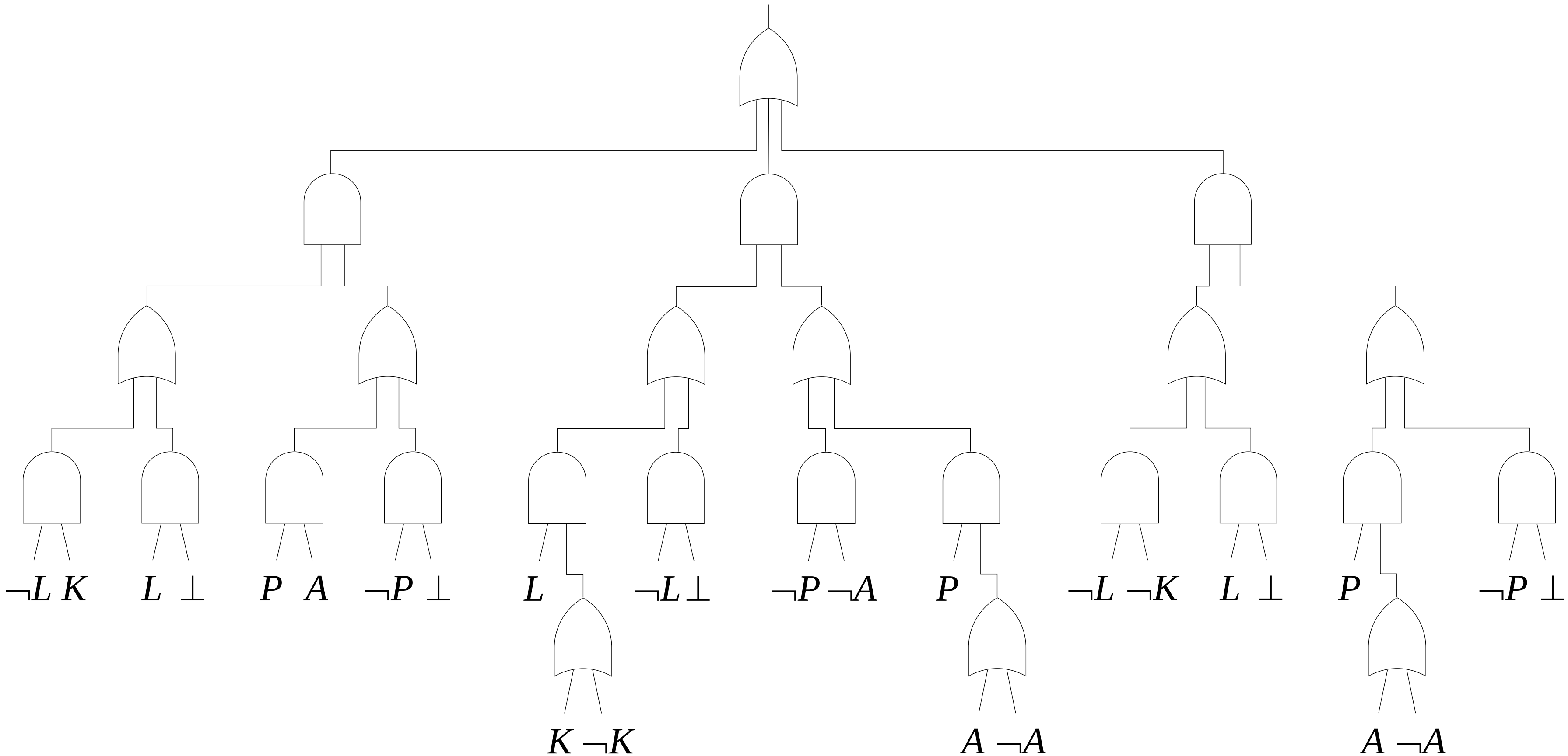
Unstructured probability space: 184+16,777,032 = $2^{24}$

# *“Deep Architecture”*

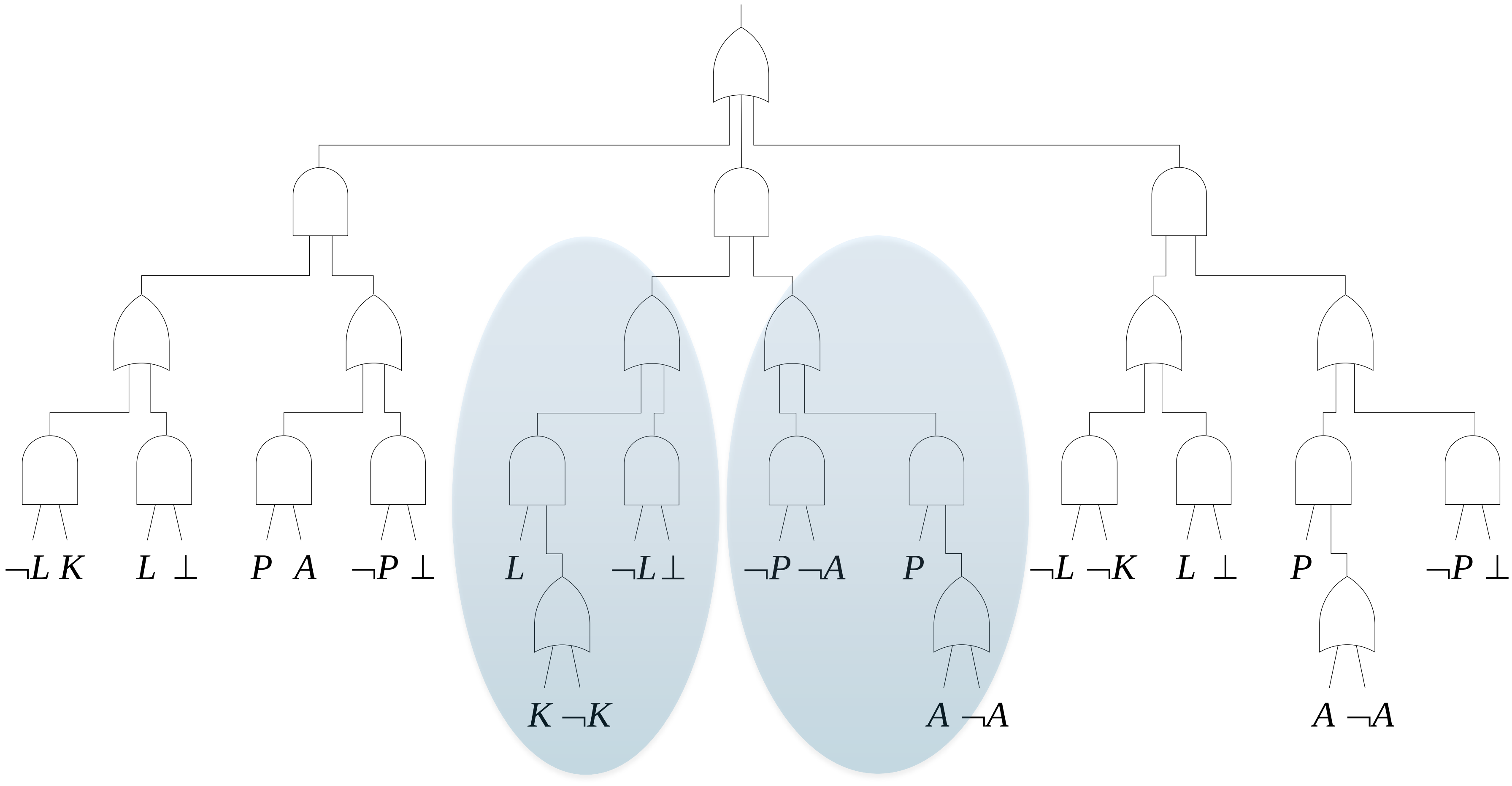
*Logic + Probability*

# Logical Circuits

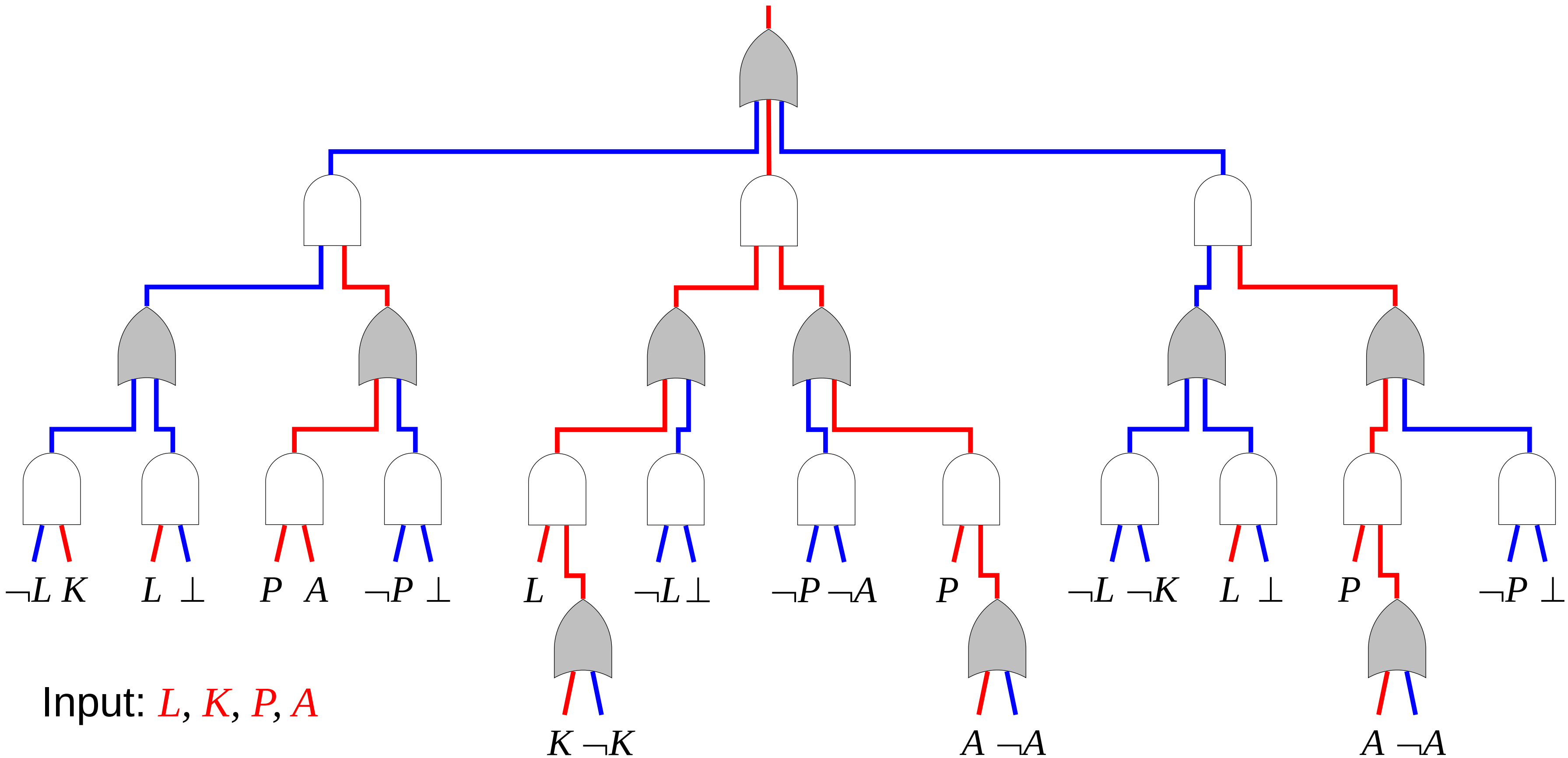
$P \vee L$
$A \Rightarrow P$
$K \Rightarrow (P \vee L)$



# Property: Decomposability

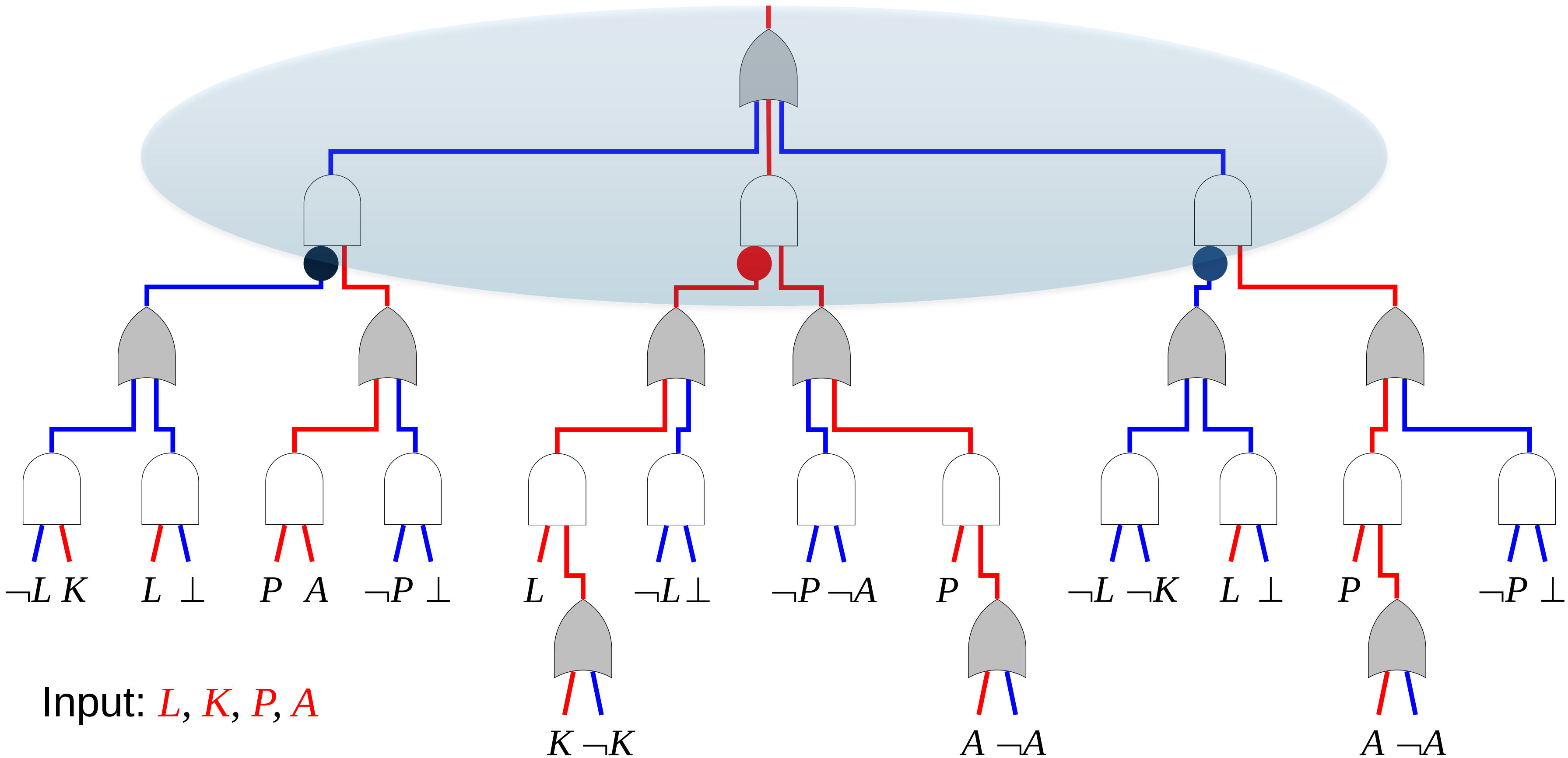


# Property: Determinism




Input: *L*, *K*, *P*, *A*

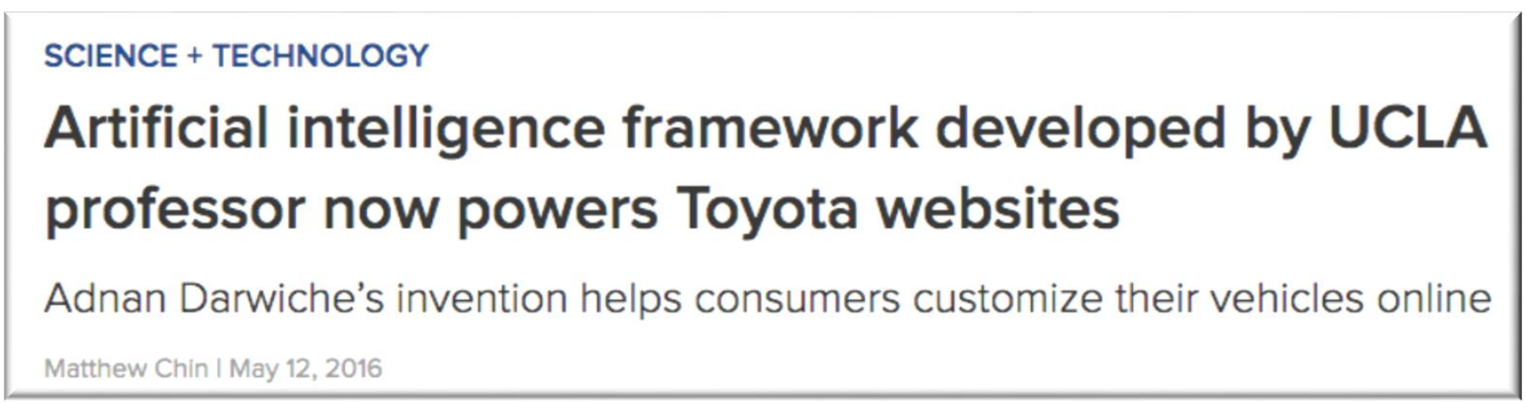
# Sentential Decision Diagram (SDD)




Input: *L*, *K*, *P*, *A*

# Tractable for Logical Inference

- Is structured space empty? (SAT)
- Count size of structured space (#SAT)
- Check equivalence of spaces
- Algorithms linear in circuit size ☺
  (pass up, pass down, similar to backprop)

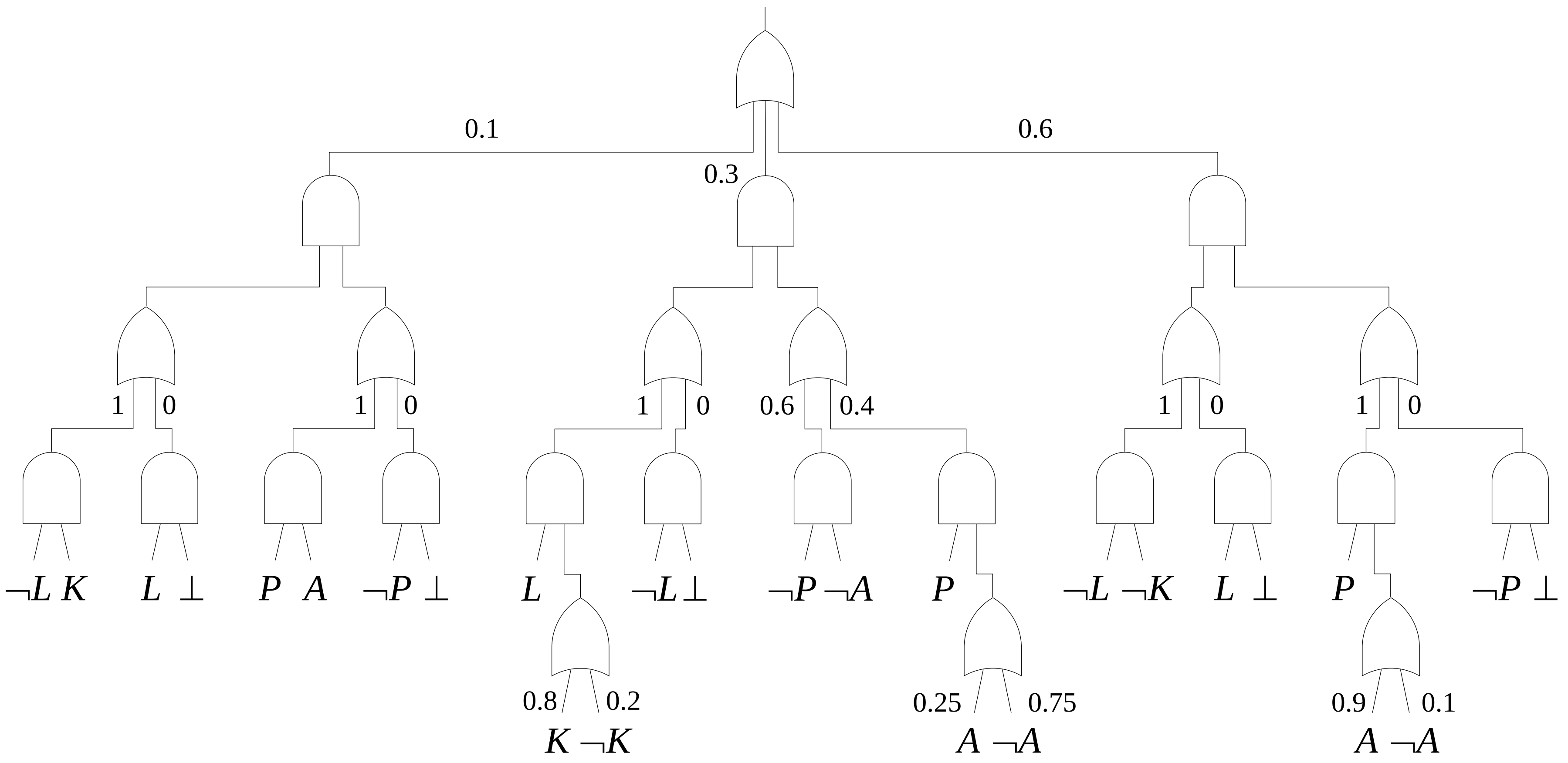

SCIENCE + TECHNOLOGY

**Artificial intelligence framework developed by UCLA professor now powers Toyota websites**

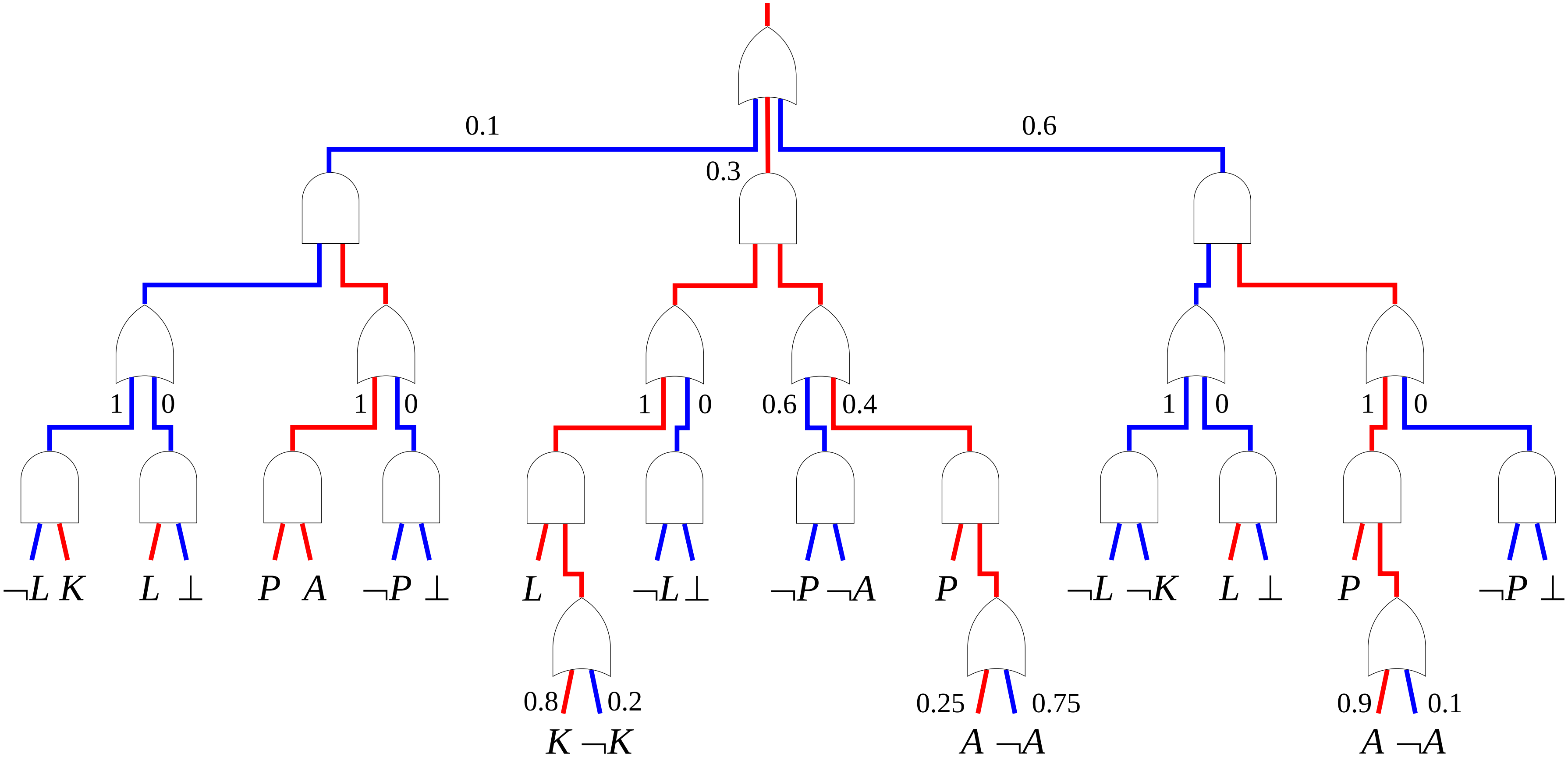
Adnan Darwiche's invention helps consumers customize their vehicles online

Matthew Chin | May 12, 2016

# PSDD: Probabilistic SDD

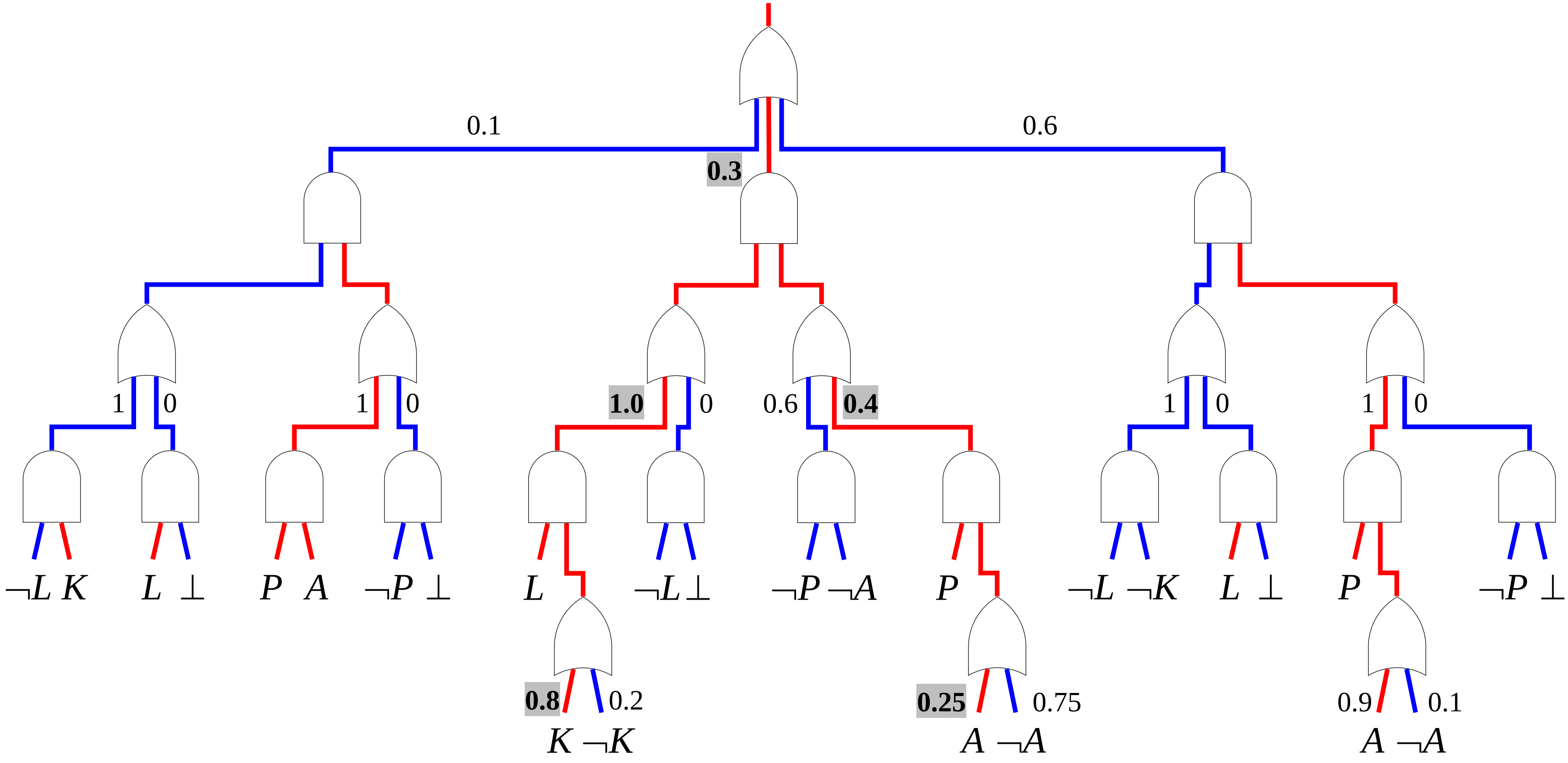


# PSDD: Probabilistic SDD



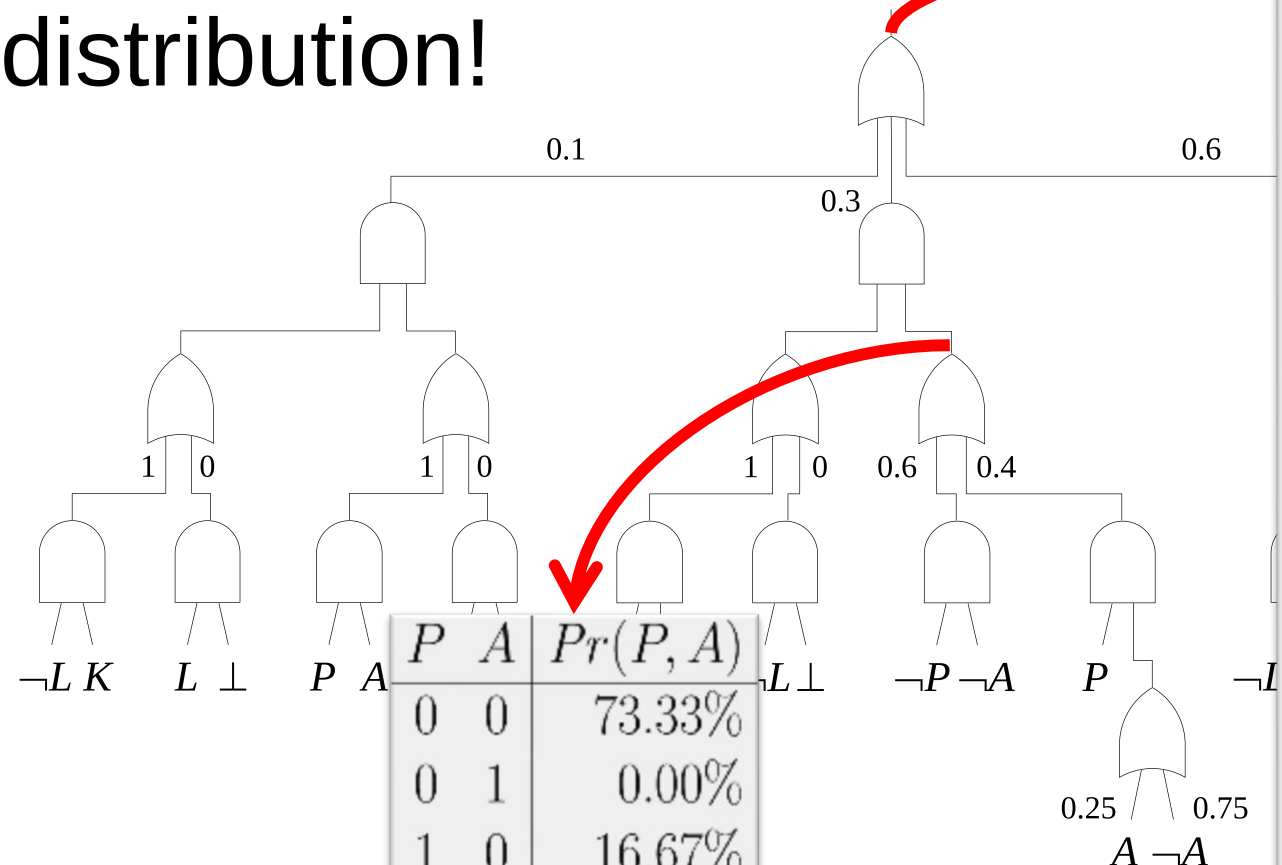

Input: *L*, *K*, *P*, *A*

# PSDD: Probabilistic SDD




Input: *L*, *K*, *P*, *A*  Pr(*L*,*K*,*P*,*A*) = 0.3 x 1.0 x 0.8 x 0.4 x 0.25 = 0.024

# PSDD nodes induce a normalized distribution!




| $P$ | $A$ | $Pr(P, A)$ |
|---|---|---|
| 0 | 0 | 73.33% |
| 0 | 1 | 0.00% |
| 1 | 0 | 16.67% |
| 1 | 1 | 10.00% |

| $L$ | $K$ | $P$ | $A$ | $Pr(L, K, P, A)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0.00% |
| 0 | 0 | 0 | 1 | 0.00% |
| 0 | 0 | 1 | 0 | 6.00% |
| 0 | 0 | 1 | 1 | 54.00% |
| 0 | 1 | 0 | 0 | 0.00% |
| 0 | 1 | 0 | 1 | 0.00% |
| 0 | 1 | 1 | 0 | 0.00% |
| 0 | 1 | 1 | 1 | 10.00% |
| 1 | 0 | 0 | 0 | 4.40% |
| 1 | 0 | 0 | 1 | 0.00% |
| 1 | 0 | 1 | 0 | 1.00% |
| 1 | 0 | 1 | 1 | 0.60% |
| 1 | 1 | 0 | 0 | 17.6% |
| 1 | 1 | 0 | 1 | 0.00% |
| 1 | 1 | 1 | 0 | 4.00% |
| 1 | 1 | 1 | 1 | 2.40% |

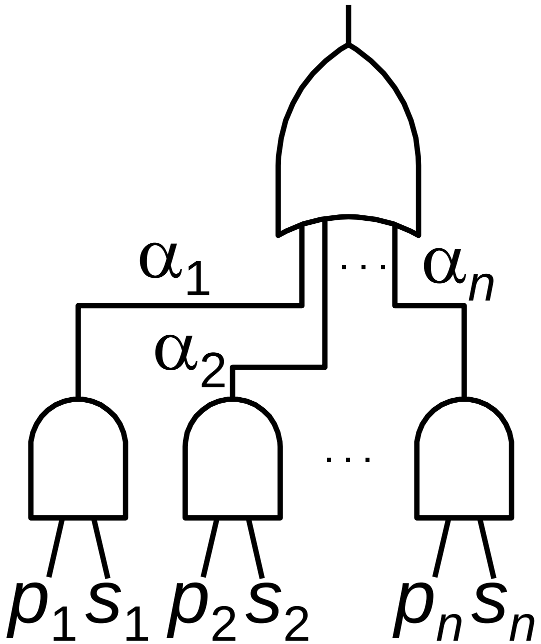
Can read probabilistic independences off the circuit structure

# Tractable for Probabilistic Inference

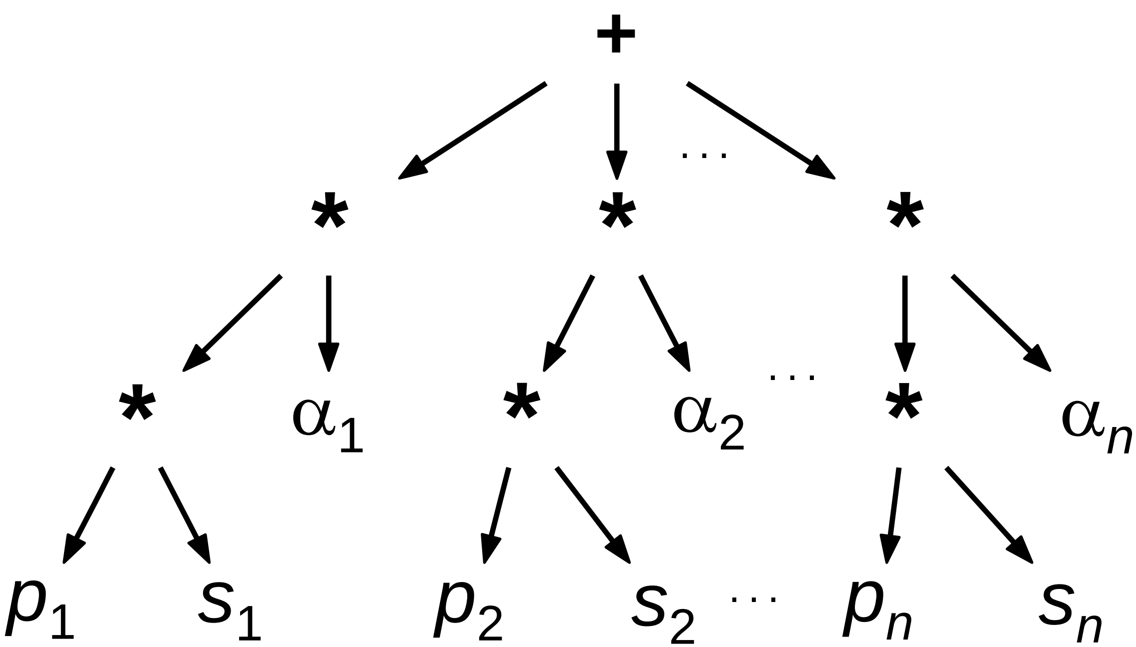
- **MAP inference**: Find most-likely assignment (otherwise NP-complete)
- Computing **conditional probabilities** Pr(x|y) (otherwise PP-complete)
- **Sample** from Pr(x|y)
- Algorithms linear in circuit size ☺
  (pass up, pass down, similar to backprop)

# PSDDs are Arithmetic Circuits

[Darwiche, JACM 2003]




**PSDD**




**AC**

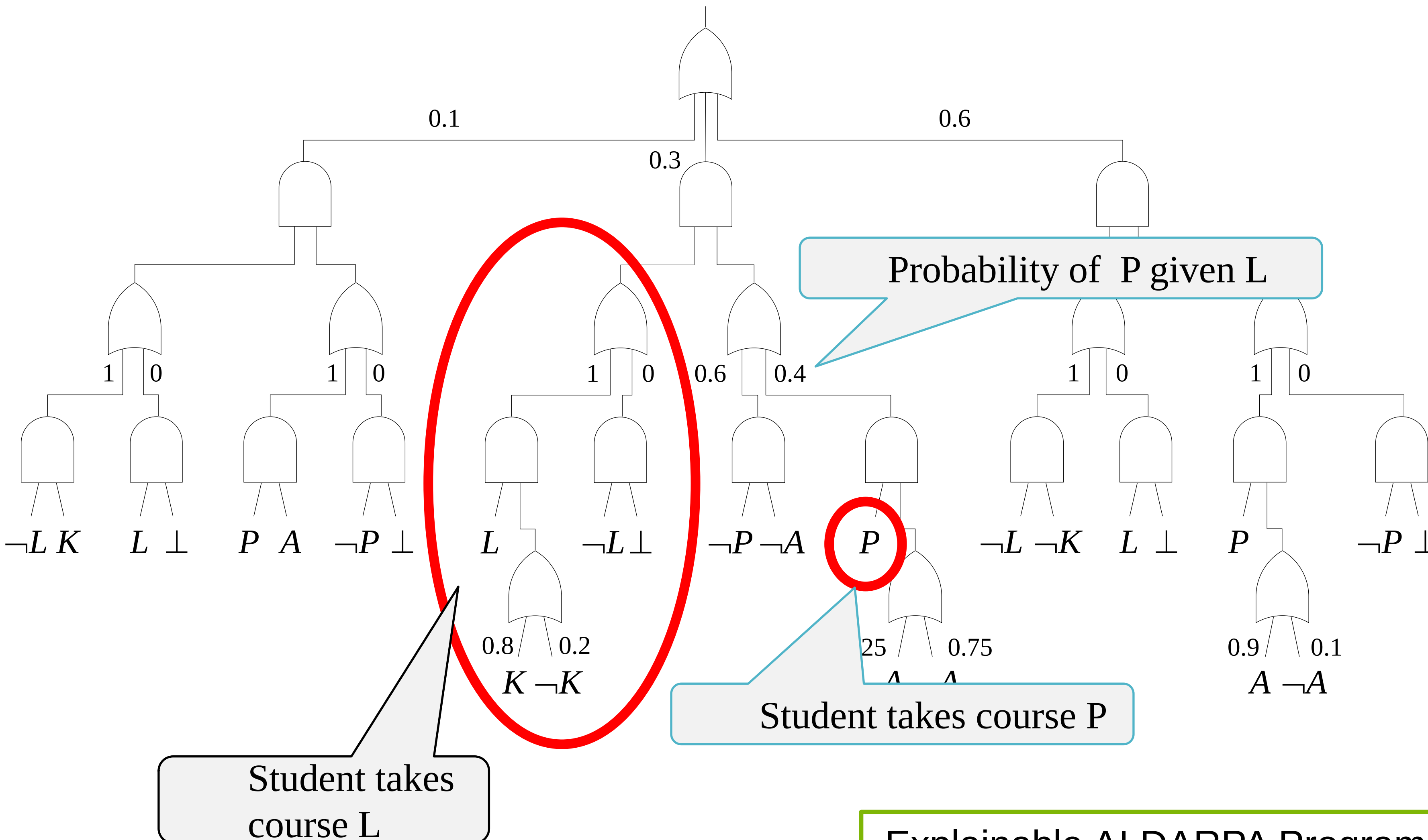
Known in the ML literature as SPNs
UAI 2011, NIPS 2012 best paper awards

[ICML 2014]
**(SPNs equivalent to ACs)**

# *Learning PSDDs*

*Logic + Probability + ML*

# Parameters are Interpretable




Explainable AI DARPA Program

# Learning Algorithms

- Parameter learning:

  Closed form max likelihood from complete data

  One pass over data to estimate Pr(x|y)

  Not a lot to say: very easy!

- Circuit learning (naïve):

  Compile constraints to SDD circuit

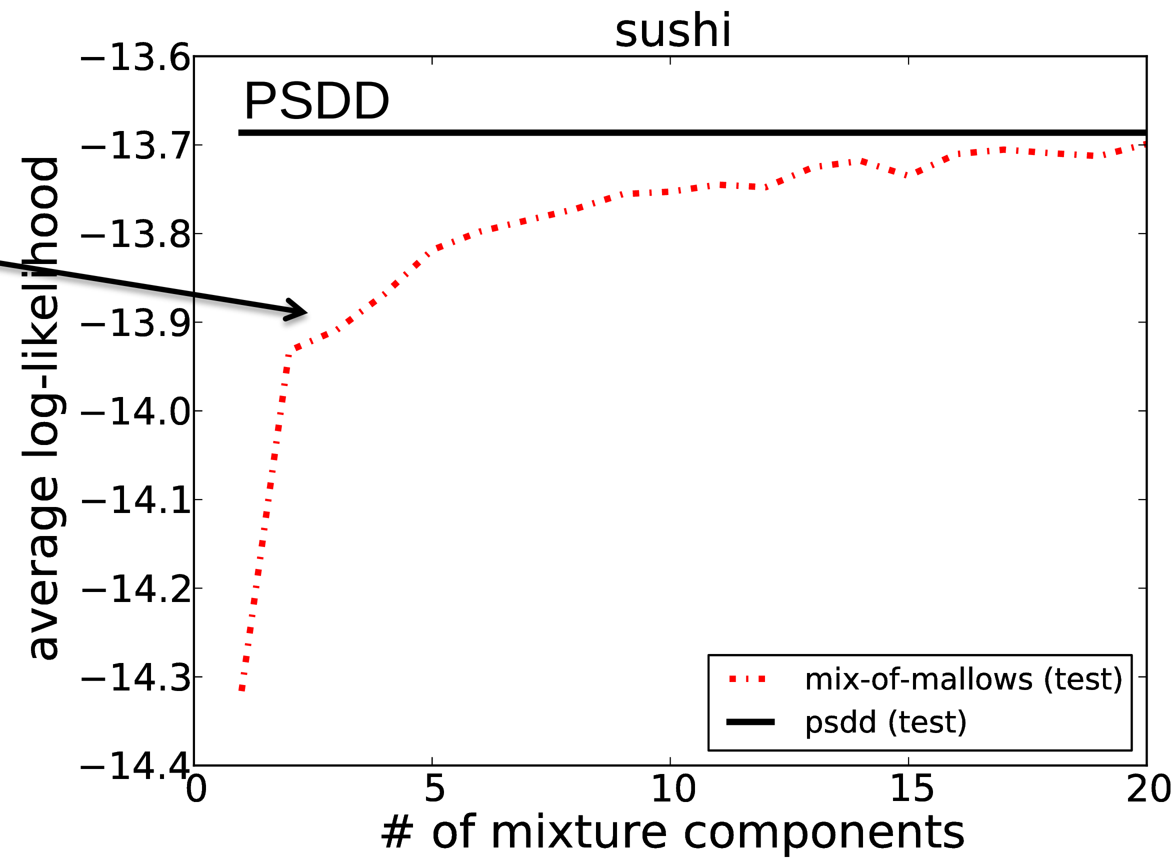
  – Use SAT solver technology

  – Circuit does not depend on data

# Learning Preference Distributions

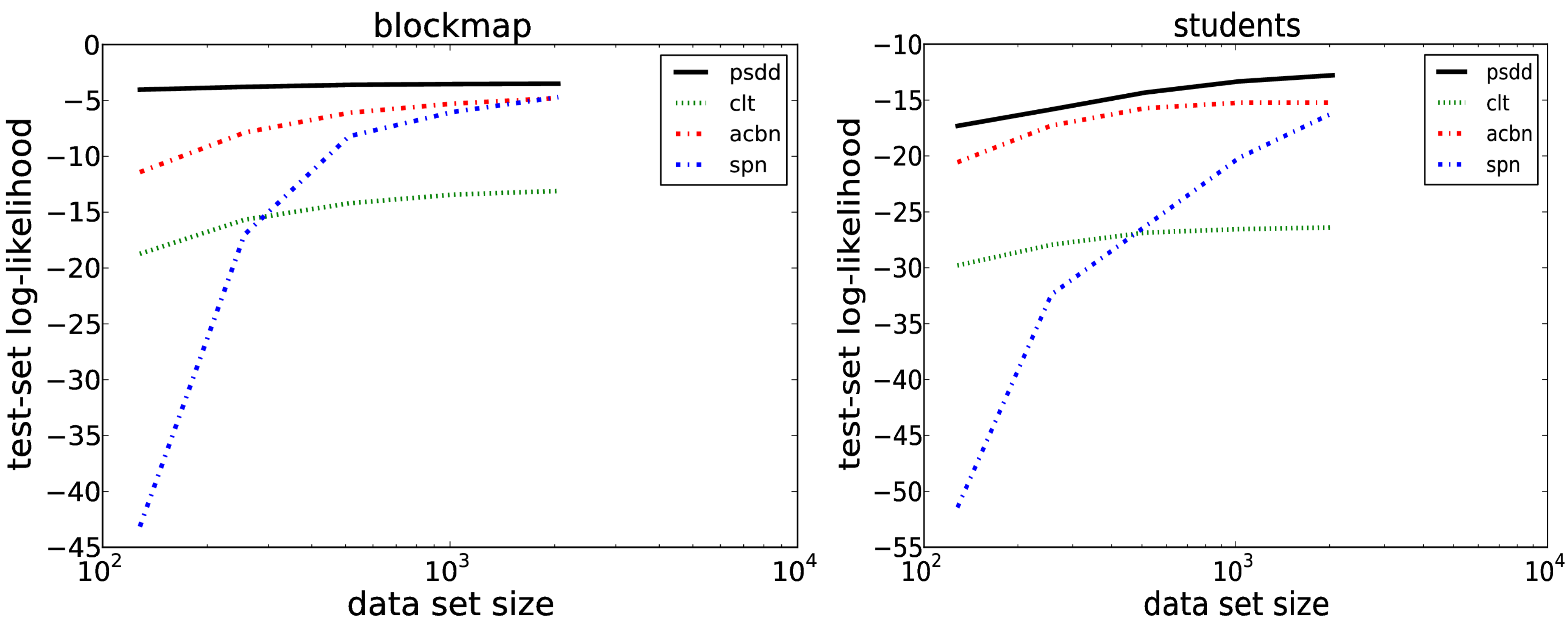
Special-purpose distribution: Mixture-of-Mallows

- # of components from 1 to 20
- EM with 10 random seeds
- implementation of Lu & Boutilier




This is the naive approach, circuit does not depend on data!

# What happens if you **ignore** constraints?

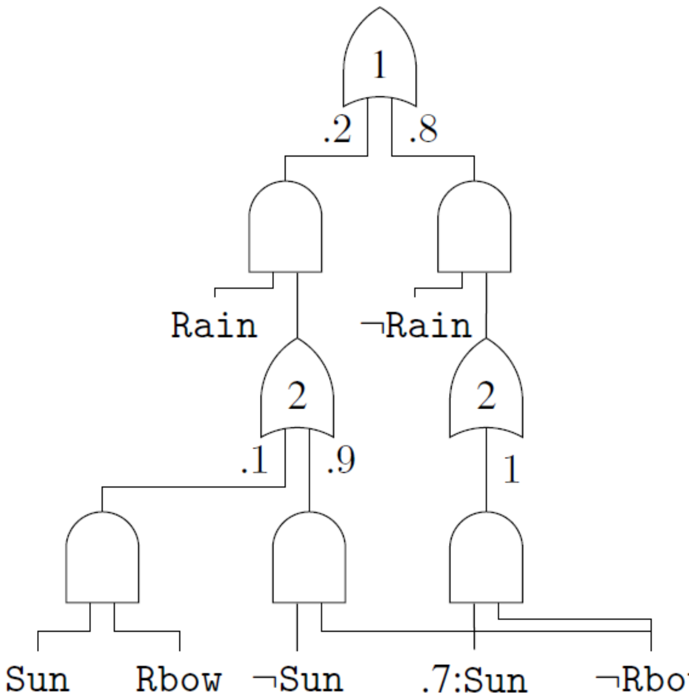


# *Learn Circuit from Data*

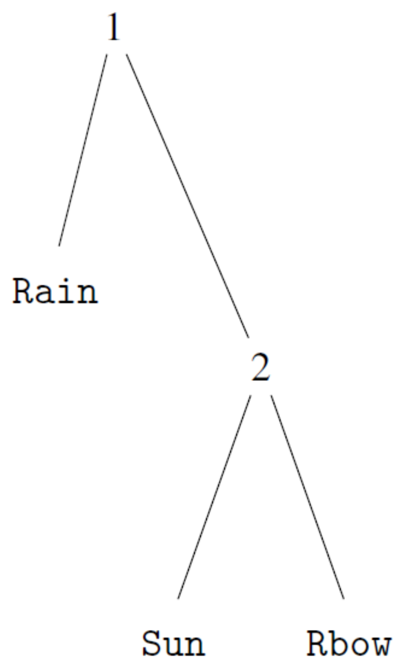
*Even in unstructured spaces*

# Variable Trees (vtrees)

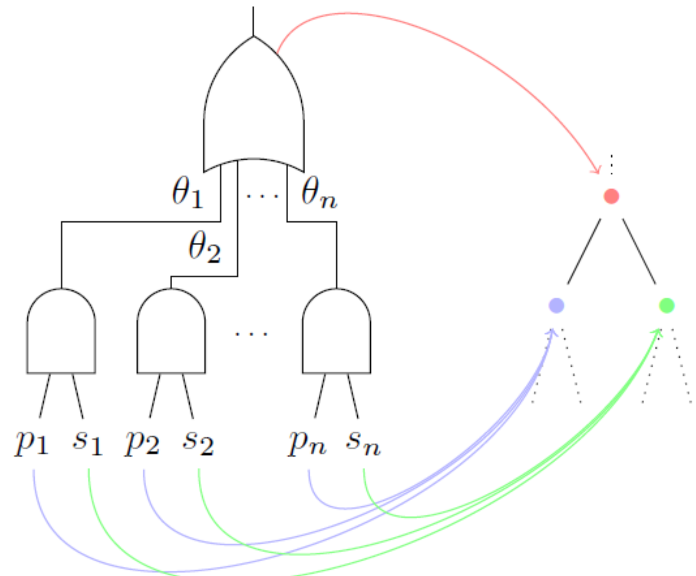
PSDD




Vtree




Correspondence

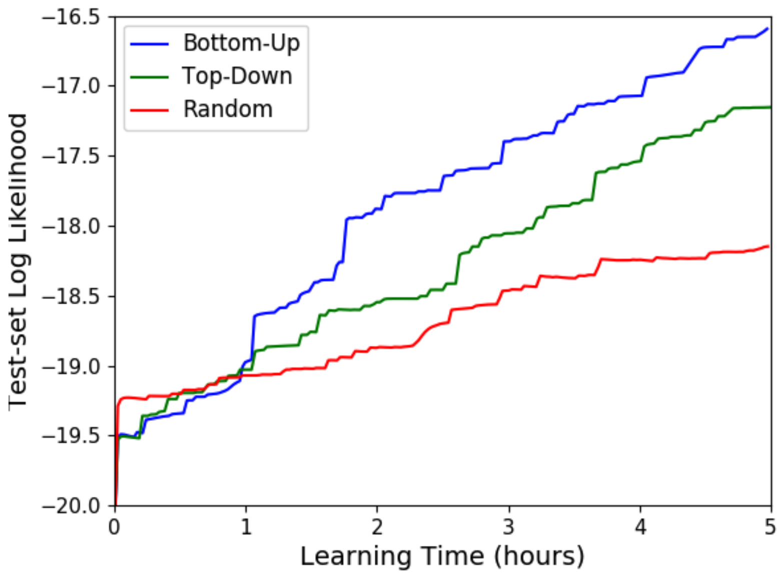


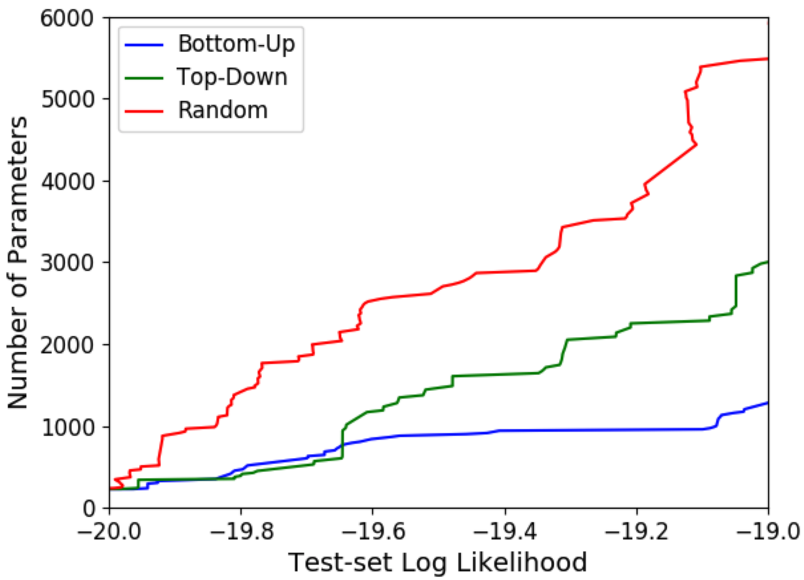
# Learning Variable Trees

- How much do vars depend on each other?

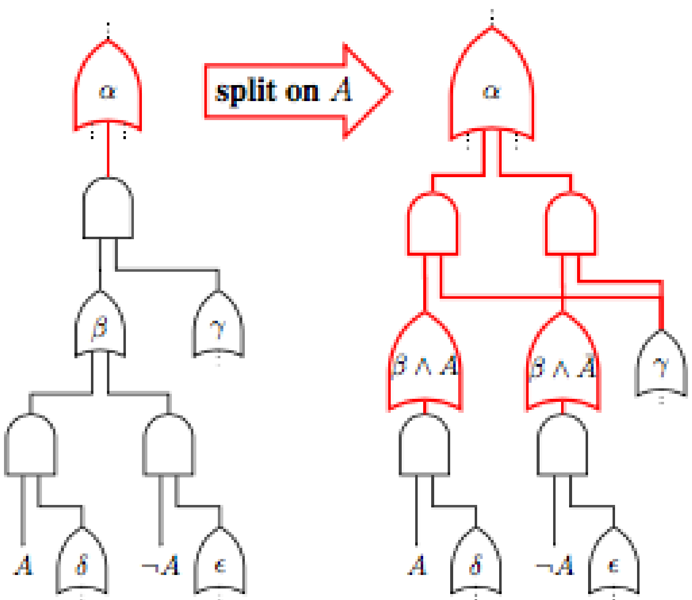
$$\mathrm{MI}(\mathbf{X},\mathbf{Y}) = \sum_{X\in\mathbf{X}}\sum_{Y\in\mathbf{Y}} \Pr(X,Y)\log\frac{\Pr(X,Y)}{\Pr(X)\Pr(Y)}$$

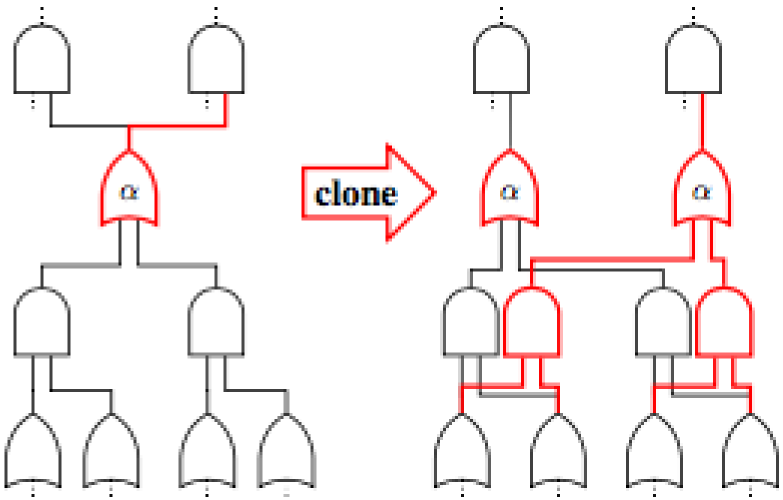
- Learn vtree by hierarchical clustering



# Learning Primitives



# Tractable Learning

- Circuit size is measurement of tractability
- Trade off size and quality of model

$$\text{score} = \frac{\ln \mathcal{L}(r' \mid \mathcal{D}) - \ln \mathcal{L}(r \mid \mathcal{D})}{\text{size}(r') - \text{size}(r)}$$

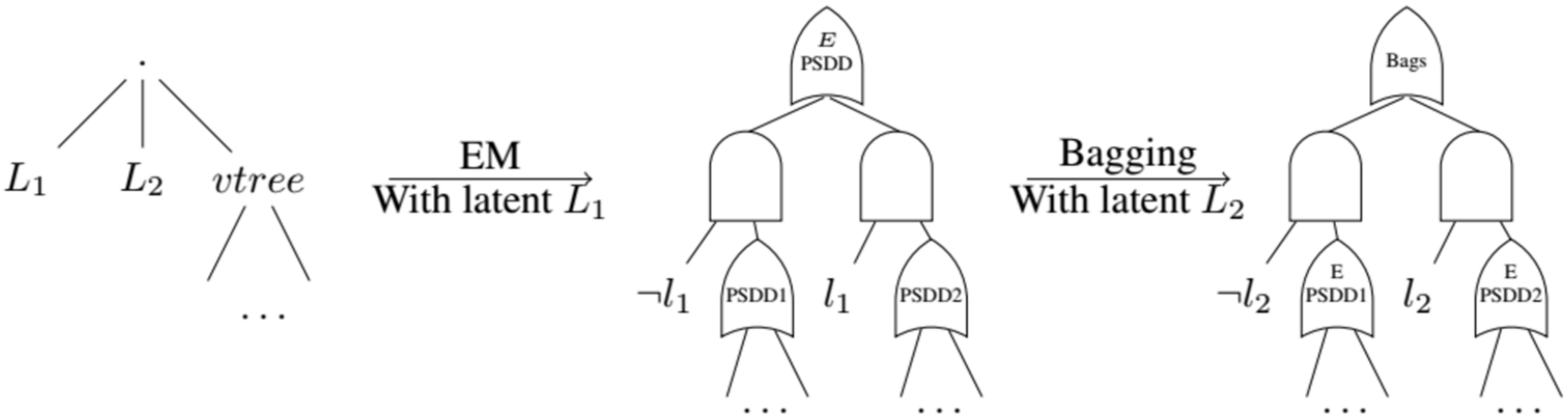
- Perform greedy local search

  Split and Clone

- Re-learn parameters in between

# Ensembles

- Performance boost
  - Add a few latent variables (L1,L2)
  - Perform expectation maximization
  - Perform bagging



# Experimental Results

| Dataset | $\|Var\|$ | LearnPSDD Ensemble | Best-to-Date |
|---|---|---|---|
| NLTCS | 16 | $-5.99^{\dagger}$ | $-6.00$ |
| MSNBC | 17 | $-6.04^{\dagger}$ | $-6.04^{\dagger}$ |
| KDD | 64 | $-2.11^{\dagger}$ | $-2.12$ |
| Plants | 69 | $-13.02$ | $-11.99^{\dagger}$ |
| Audio | 100 | $-39.94$ | $-39.49^{\dagger}$ |
| Jester | 100 | $-51.29$ | $-41.11^{\dagger}$ |
| Netflix | 100 | $-55.71^{\dagger}$ | $-55.84$ |
| Accidents | 111 | $-30.16$ | $-24.87^{\dagger}$ |
| Retail | 135 | $-10.72^{\dagger}$ | $-10.78$ |
| Pumsb-Star | 163 | $-26.12$ | $-22.40^{\dagger}$ |
| DNA | 180 | $-88.01$ | $-80.03^{\dagger}$ |
| Kosarek | 190 | $-10.52^{\dagger}$ | $-10.54$ |
| MSWeb | 294 | $-9.89$ | $-9.22^{\dagger}$ |
| Book | 500 | $-34.97$ | $-30.18^{\dagger}$ |
| EachMovie | 500 | $-58.01$ | $-51.14^{\dagger}$ |
| WebKB | 839 | $-161.09$ | $-150.10^{\dagger}$ |
| Reuters-52 | 889 | $-89.61$ | $-80.66^{\dagger}$ |
| 20NewsGrp. | 910 | $-155.97$ | $-150.88^{\dagger}$ |
| BBC | 1058 | $-253.19$ | $-233.26^{\dagger}$ |
| AD | 1556 | $-31.78$ | $-14.36^{\dagger}$ |

Surpasses the state of the art (SPNs, Cutset networks, ACs) on 6/20 datasets.

*Complex queries*

*and*

*Learning from constraints*

# Incomplete Data

a classical complete dataset

| id | X | Y | Z |
|---|---|---|---|
| 1 | $x_1$ | $y_2$ | $z_1$ |
| 2 | $x_2$ | $y_1$ | $z_2$ |
| 3 | $x_2$ | $y_1$ | $z_2$ |
| 4 | $x_1$ | $y_1$ | $z_1$ |
| 5 | $x_1$ | $y_2$ | $z_2$ |

closed-form (maximum-likelihood estimates are unique)

a classical incomplete dataset

| id | X | Y | Z |
|---|---|---|---|
| 1 | $x_1$ | $y_2$ | ? |
| 2 | $x_2$ | $y_1$ | ? |
| 3 | ? | ? | $z_2$ |
| 4 | ? | $y_1$ | $z_1$ |
| 5 | $x_1$ | $y_2$ | $z_2$ |

EM algorithm (on PSDDs)

a new type of incomplete dataset

| id | X | Y | Z |
|---|---|---|---|
| 1 | $X \equiv Z$ | | |
| 2 | $x_2$ and ($y_2$ or $z_2$) | | |
| 3 | $x_2 \Rightarrow y_1$ | | |
| 4 | $X \oplus Y \oplus Z \equiv 1$ | | |
| 5 | $x_1$ and $y_2$ and $z_2$ | | |

Missed in the ML literature

# Structured Datasets

a classical **complete** dataset
(e.g., total rankings)

| id | $1^{st}$ sushi | $2^{nd}$ sushi | $3^{rd}$ sushi | … |
|---|---|---|---|---|
| 1 | fatty tuna | sea urchin | salmon roe | … |
| 2 | fatty tuna | tuna | shrimp | … |
| 3 | tuna | tuna roll | sea eel | … |
| 4 | fatty tuna | salmon roe | tuna | … |
| 5 | egg | squid | shrimp | … |

a classical **incomplete** dataset
(e.g., top-*k* rankings)

| id | $1^{st}$ sushi | $2^{nd}$ sushi | $3^{rd}$ sushi | … |
|---|---|---|---|---|
| 1 | fatty tuna | sea urchin | ? | … |
| 2 | fatty tuna | ? | ? | … |
| 3 | tuna | tuna roll | ? | … |
| 4 | fatty tuna | salmon roe | ? | … |
| 5 | egg | ? | ? | … |

# Structured Datasets

a classical **complete** dataset
(e.g., total rankings)

| id | $1^{st}$ sushi | $2^{nd}$ sushi | $3^{rd}$ sushi | … |
|---|---|---|---|---|
| 1 | fatty tuna | sea urchin | salmon roe | … |
| 2 | fatty tuna | tuna | shrimp | … |
| 3 | tuna | tuna roll | sea eel | … |
| 4 | fatty tuna | salmon roe | tuna | … |
| 5 | egg | squid | shrimp | … |

a new type of **incomplete** dataset
(e.g., **partial** rankings)

| id | $1^{st}$ sushi | $2^{nd}$ sushi | $3^{rd}$ sushi | … |
|---|---|---|---|---|
| 1 | (fatty tuna > sea urchin) and (tuna > sea eel) | | | … |
| 2 | (fatty tuna is $1^{st}$) and (salmon roe > egg) | | | … |
| 3 | tuna > squid | | | … |
| 4 | egg is last | | | … |
| 5 | egg > squid > shrimp | | | … |

(represents constraints on
possible *total rankings*)

# Learning from Incomplete Data

- Movielens Dataset:
  - 3,900 movies, 6,040 users, 1m ratings
  - take ratings from 64 most rated movies
  - ratings 1-5 converted to pairwise prefs.
- PSDD for **partial** rankings
  - 4 tiers
  - 18,711 parameters

movies by expected tier

| rank | movie |
|---|---|
| 1 | The Godfather |
| 2 | The Usual Suspects |
| 3 | Casablanca |
| 4 | The Shawshank Redemption |
| 5 | Schindler’s List |
| 6 | One Flew Over the Cuckoo’s Nest |
| 7 | The Godfather: Part II |
| 8 | Monty Python and the Holy Grail |
| 9 | Raiders of the Lost Ark |
| 10 | Star Wars IV: A New Hope |

# PSDD Sizes

| items | tier size | Size | | |
|---|---|---|---|---|
| $n$ | $k$ | SDD | Structured Space | Unstructured Space |
| 8 | 2 | 443 | 840 | $1.84 \cdot 10^{19}$ |
| 27 | 3 | 4,114 | $1.18 \cdot 10^{9}$ | $2.82 \cdot 10^{219}$ |
| 64 | 4 | 23,497 | $3.56 \cdot 10^{18}$ | $1.04 \cdot 10^{1233}$ |
| 125 | 5 | 94,616 | $3.45 \cdot 10^{31}$ | $3.92 \cdot 10^{4703}$ |
| 216 | 6 | 297,295 | $1.57 \cdot 10^{48}$ | $7.16 \cdot 10^{14044}$ |
| 343 | 7 | 781,918 | $4.57 \cdot 10^{68}$ | $7.55 \cdot 10^{35415}$ |

# Structured Queries

| rank | movie |
| --- | --- |
| 1 | Star Wars V: The Empire Strikes Back |
| 2 | Star Wars IV: A New Hope |
| 3 | The Godfather |
| 4 | The Shawshank Redemption |
| 5 | The Usual Suspects |

- no other Star Wars movie in top-5
- at least one comedy in top-5

| rank | movie |
| --- | --- |
| 1 | Star Wars V: The Empire Strikes Back |
| 2 | American Beauty |
| 3 | The Godfather |
| 4 | The Usual Suspects |
| 5 | The Shawshank Redemption |

diversified recommendations via
***logical constraints***

# Conclusions

- Structured spaces are everywhere ☺
- PSDDs build on logical circuits
  1. Tractability
  2. Semantics
  3. Natural encoding of structured spaces
- Learning is effective
  1. From constraints encoding structured space

     State of the art preference distribution learning
  2. From standard unstructured datasets using search

     State of the art on standard tractable learning datasets
- Novel settings for inference and learning

  Structured spaces / learning from constraints / complex queries

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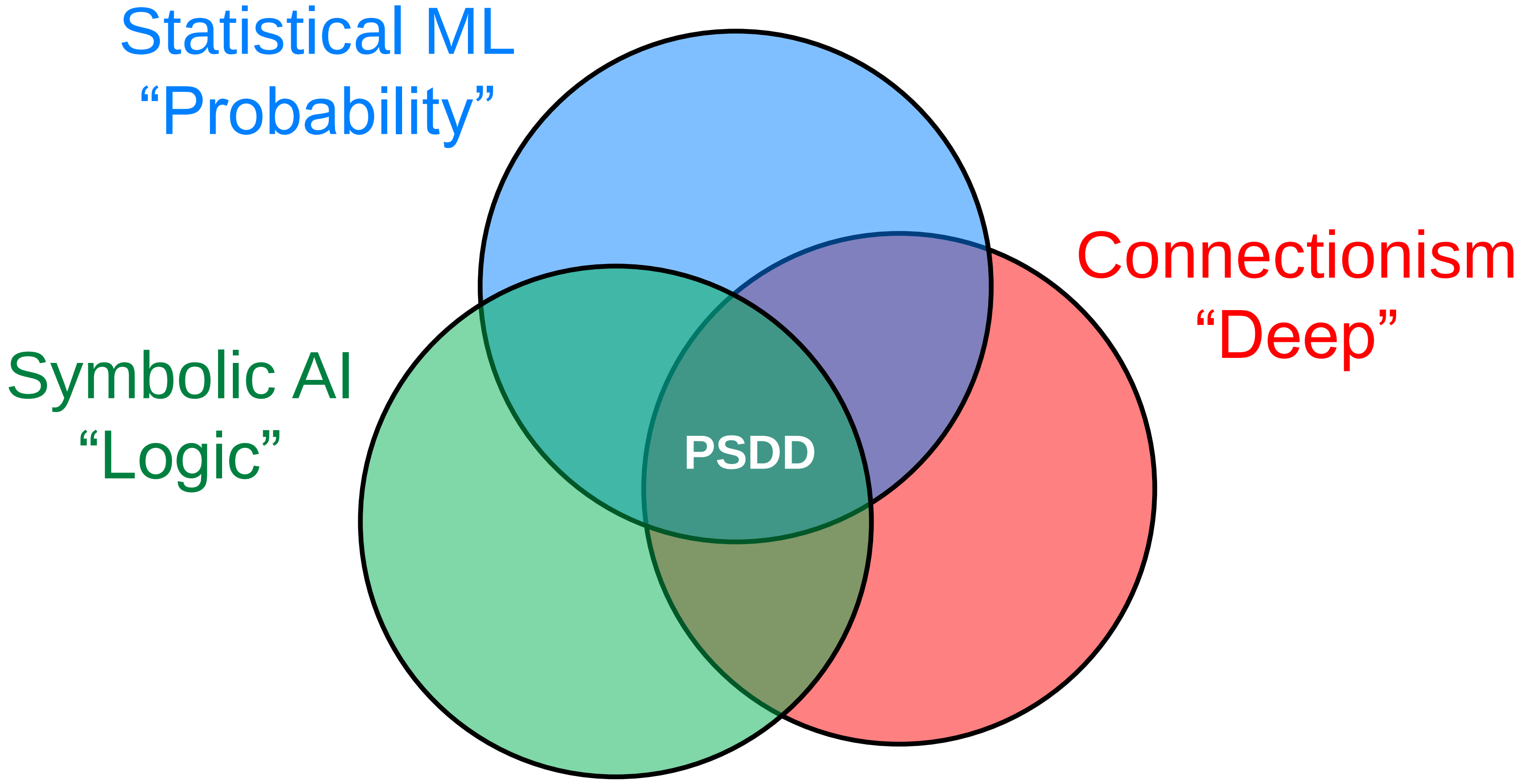
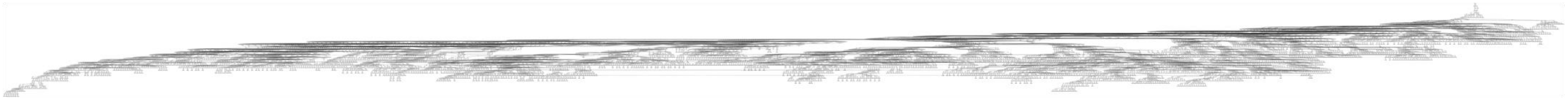
# Conclusions



# Questions?



*PSDD with 15,000 nodes*