

CS145: INTRODUCTION TO DATA MINING

09: Vector Data: Clustering Basics

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Announcement

- About Homework
 - Split HW3 into two: HW3 and HW4
- Optional HW6
 - We will pick the highest 5 homework scores

Methods to Learn

	Vector Data	Set Data	Sequence Data	Text Data
Classification	Logistic Regression; Decision Tree; KNN SVM; NN			Naïve Bayes for Text
Clustering	K-means; hierarchical clustering; DBSCAN; Mixture Models			PLSA
Prediction	Linear Regression GLM*			
Frequent Pattern Mining		Apriori; FP growth	GSP; PrefixSpan	
Similarity Search			DTW	

Vector Data: Clustering Basics

- Clustering Analysis: Basic Concepts 
- Partitioning methods
- Hierarchical Methods
- Density-Based Methods
- Summary

What is Cluster Analysis?

- Cluster: A collection of data objects
 - similar (or related) to one another within the same group
 - dissimilar (or unrelated) to the objects in other groups
- Cluster analysis (or *clustering*, *data segmentation*, ...)
 - Finding similarities between data according to the characteristics found in the data and grouping similar data objects into clusters
- **Unsupervised learning**: no predefined classes (i.e., *learning by observations* vs. learning by examples: supervised)
- Typical applications
 - As a **stand-alone tool** to get insight into data distribution
 - As a **preprocessing step** for other algorithms


Applications of Cluster Analysis

- Data reduction
 - Summarization: Preprocessing for regression, PCA, classification, and association analysis
 - Compression: Image processing: vector quantization
- Prediction based on groups
 - Cluster & find characteristics/patterns for each group
- Finding K-nearest Neighbors
 - Localizing search to one or a small number of clusters
- Outlier detection: Outliers are often viewed as those “far away” from any cluster

Clustering: Application Examples

- **Biology:** taxonomy of living things: kingdom, phylum, class, order, family, genus and species
- **Information retrieval:** document clustering
- **Land use:** Identification of areas of similar land use in an earth observation database
- **Marketing:** Help marketers discover distinct groups in their customer bases, and then use this knowledge to develop targeted marketing programs
- **City-planning:** Identifying groups of houses according to their house type, value, and geographical location
- **Earth-quake studies:** Observed earth quake epicenters should be clustered along continent faults
- **Climate:** understanding earth climate, find patterns of atmospheric and ocean

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Partitioning Algorithms: Basic Concept

- Partitioning method: Partitioning a dataset D of n objects into a set of k clusters, such that the sum of squared distances is minimized (where c_j is the centroid or medoid of cluster C_j)

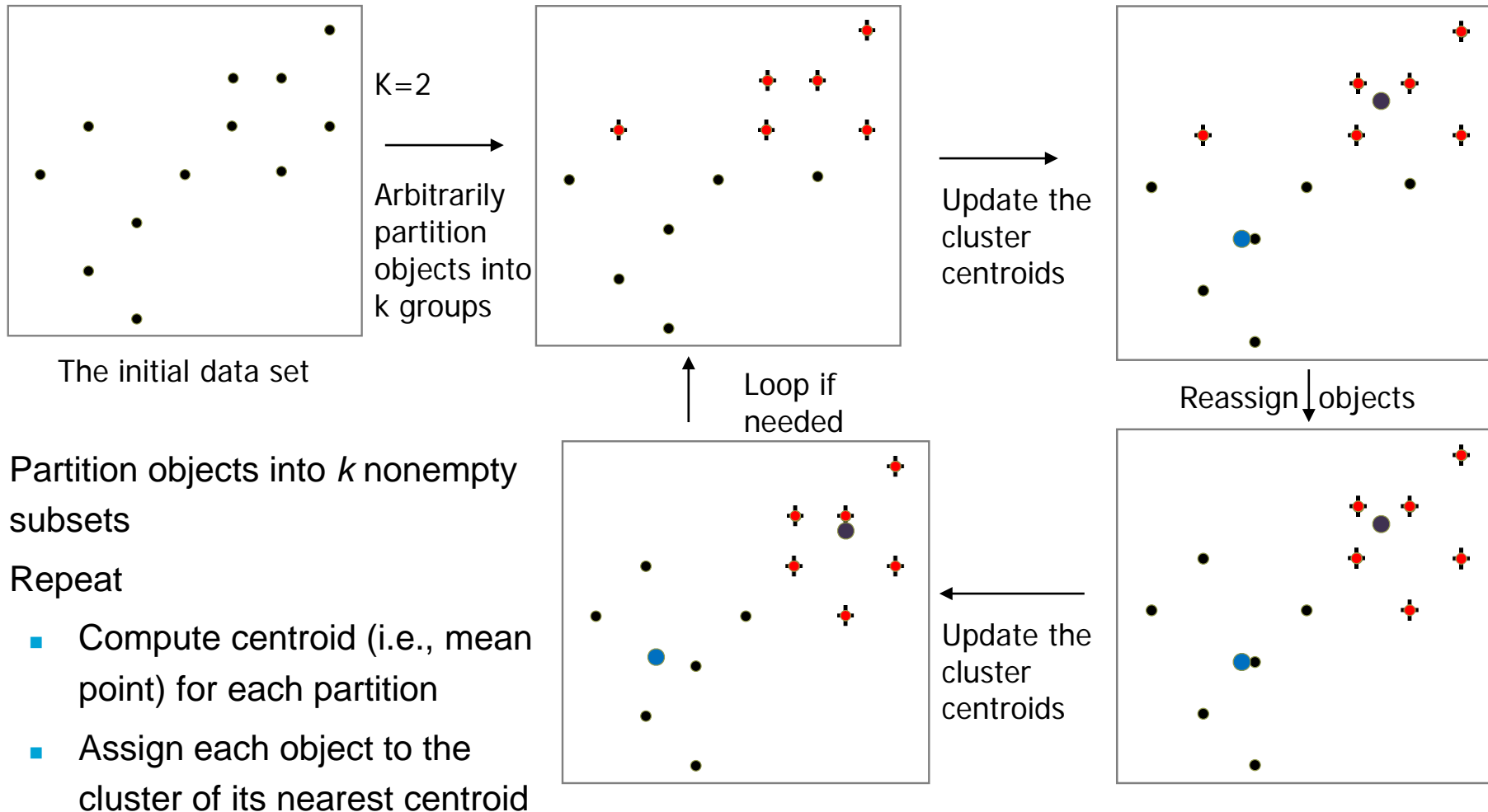
$$J = \sum_{j=1}^k \sum_{c(i)=j} d(x_i, c_j)^2$$

- Given k , find a partition of k clusters that optimizes the chosen partitioning criterion
 - Global optimal: exhaustively enumerate all partitions
 - Heuristic methods: *k-means* and *k-medoids* algorithms
 - *k-means* (MacQueen'67, Lloyd'57/'82): Each cluster is represented by the center of the cluster
 - *k-medoids* or PAM (Partition around medoids) (Kaufman & Rousseeuw'87): Each cluster is represented by one of the objects in the cluster

The *K-Means* Clustering Method

- Given k , the *k-means* algorithm is implemented in four steps:
 - Step 0: Partition objects into k nonempty subsets
 - Step 1: Compute seed points as the centroids of the clusters of the current partitioning (the centroid is the center, i.e., *mean point*, of the cluster)
 - Step 2: Assign each object to the cluster with the nearest seed point
 - Step 3: Go back to Step 1, stop when the assignment does not change

An Example of *K-Means* Clustering



- Partition objects into k nonempty subsets
- Repeat
 - Compute centroid (i.e., mean point) for each partition
 - Assign each object to the cluster of its nearest centroid

■ Until no change

Theory Behind K-Means

- Objective function
 - $J = \sum_{j=1}^k \sum_{C(i)=j} \|x_i - c_j\|^2$
- Re-arrange the objective function
 - $J = \sum_{j=1}^k \sum_i w_{ij} \|x_i - c_j\|^2$
 - $w_{ij} \in \{0,1\}$
 - $w_{ij} = 1$, if x_i belongs to cluster j ; $w_{ij} = 0$, otherwise
 - Looking for:
 - The best assignment w_{ij}
 - The best center c_j

Solution of K-Means

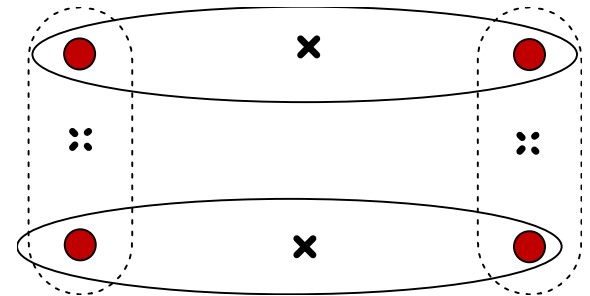
- Iterations $J = \sum_{j=1}^k \sum_i w_{ij} \|x_i - c_j\|^2$
 - Step 1: Fix centers c_j , find assignment w_{ij} that minimizes J
 - $\Rightarrow w_{ij} = 1$, if $\|x_i - c_j\|^2$ is the smallest
 - Step 2: Fix assignment w_{ij} , find centers that minimize J
 - \Rightarrow first derivative of $J = 0$
 - $\Rightarrow \frac{\partial J}{\partial c_j} = -2 \sum_i w_{ij} (x_i - c_j) = 0$
 - $\Rightarrow c_j = \frac{\sum_i w_{ij} x_i}{\sum_i w_{ij}}$
 - Note $\sum_i w_{ij}$ is the total number of objects in cluster j

Comments on the *K-Means* Method

- Strength: *Efficient*: $O(tkn)$, where n is # objects, k is # clusters, and t is # iterations. Normally, $k, t \ll n$.
- Comment: Often terminates at a *local optimal*
- Weakness
 - Applicable only to objects in a continuous n -dimensional space
 - Using the k -modes method for categorical data
 - In comparison, k -medoids can be applied to a wide range of data
 - Need to specify k , the *number* of clusters, in advance (there are ways to automatically determine the best k (see Hastie et al., 2009))
 - Sensitive to noisy data and *outliers*
 - Not suitable to discover clusters with *non-convex shapes*

Variations of the *K-Means* Method*


- Most of the variants of the *k-means* which differ in
 - Selection of the initial *k* means
 - Dissimilarity calculations
 - Strategies to calculate cluster means
- Handling categorical data: *k-modes*
 - Replacing means of clusters with modes
 - Using new dissimilarity measures to deal with categorical objects
 - Using a frequency-based method to update modes of clusters
 - A mixture of categorical and numerical data: *k-prototype* method



The K-Medoid Clustering Method*

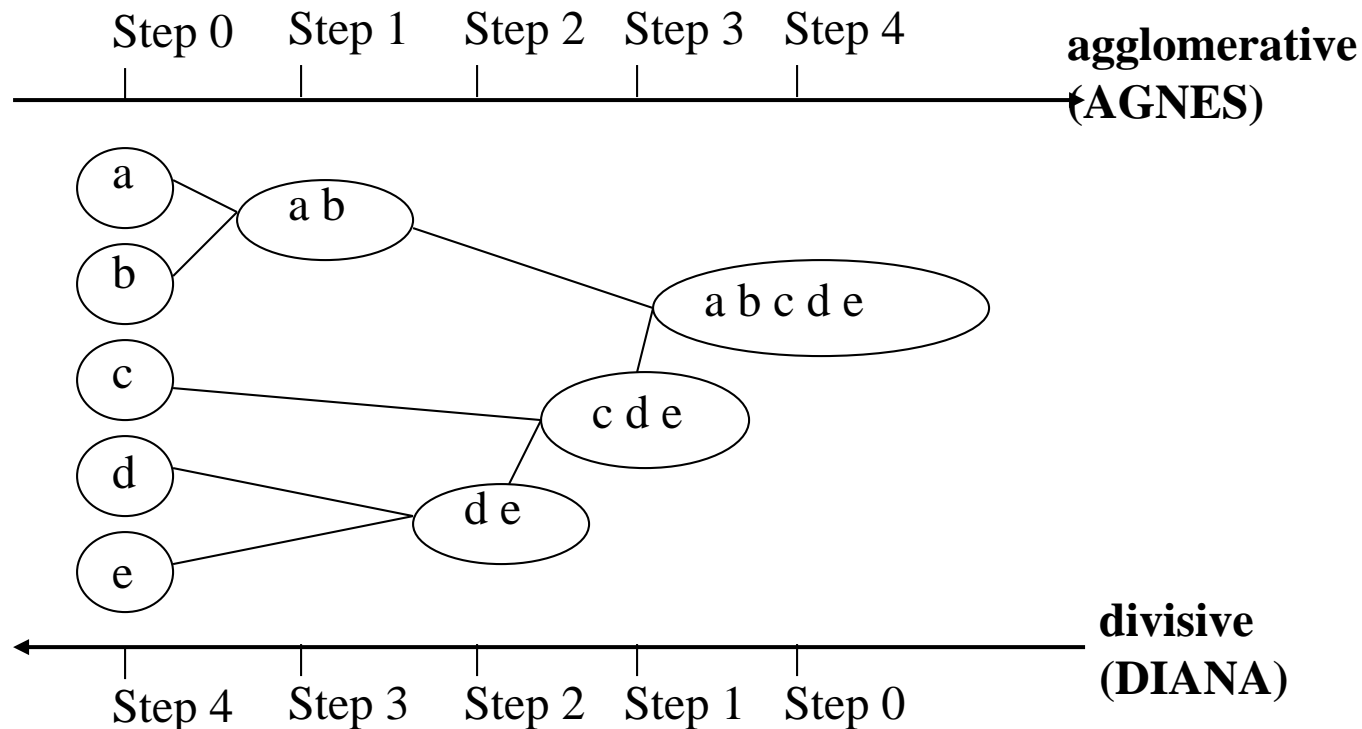
- *K-Medoids* Clustering: Find *representative* objects (medoids) in clusters
 - *PAM* (Partitioning Around Medoids, Kaufmann & Rousseeuw 1987)
 - Starts from an initial set of medoids and iteratively replaces one of the medoids by one of the non-medoids if it improves the total distance of the resulting clustering
 - *PAM* works effectively for small data sets, but does not scale well for large data sets (due to the computational complexity)
- Efficiency improvement on PAM
 - *CLARA* (Kaufmann & Rousseeuw, 1990): PAM on samples
 - *CLARANS* (Ng & Han, 1994): Randomized re-sampling

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- Density-Based Methods
- Summary

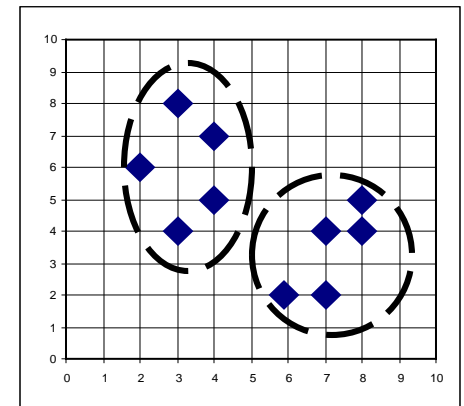
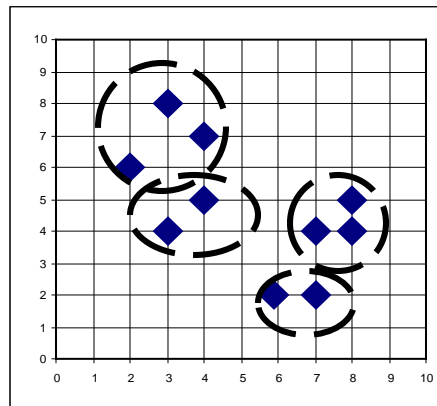
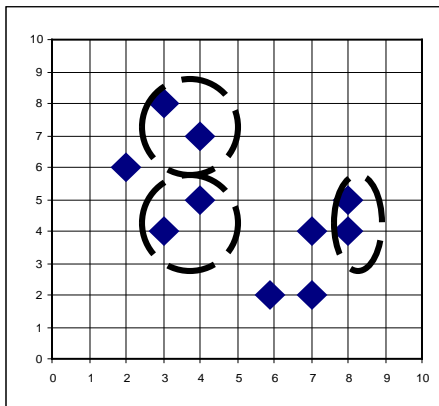
Hierarchical Clustering

- Use distance matrix as clustering criteria. This method does not require the number of clusters k as an input, but needs a termination condition



AGNES (Agglomerative Nesting)

- Introduced in Kaufmann and Rousseeuw (1990)
- Implemented in statistical packages, e.g., Splus
- Use the **single-link** method and the dissimilarity matrix
- Merge nodes that have the least dissimilarity
- Go on in a non-descending fashion
- Eventually all nodes belong to the same cluster



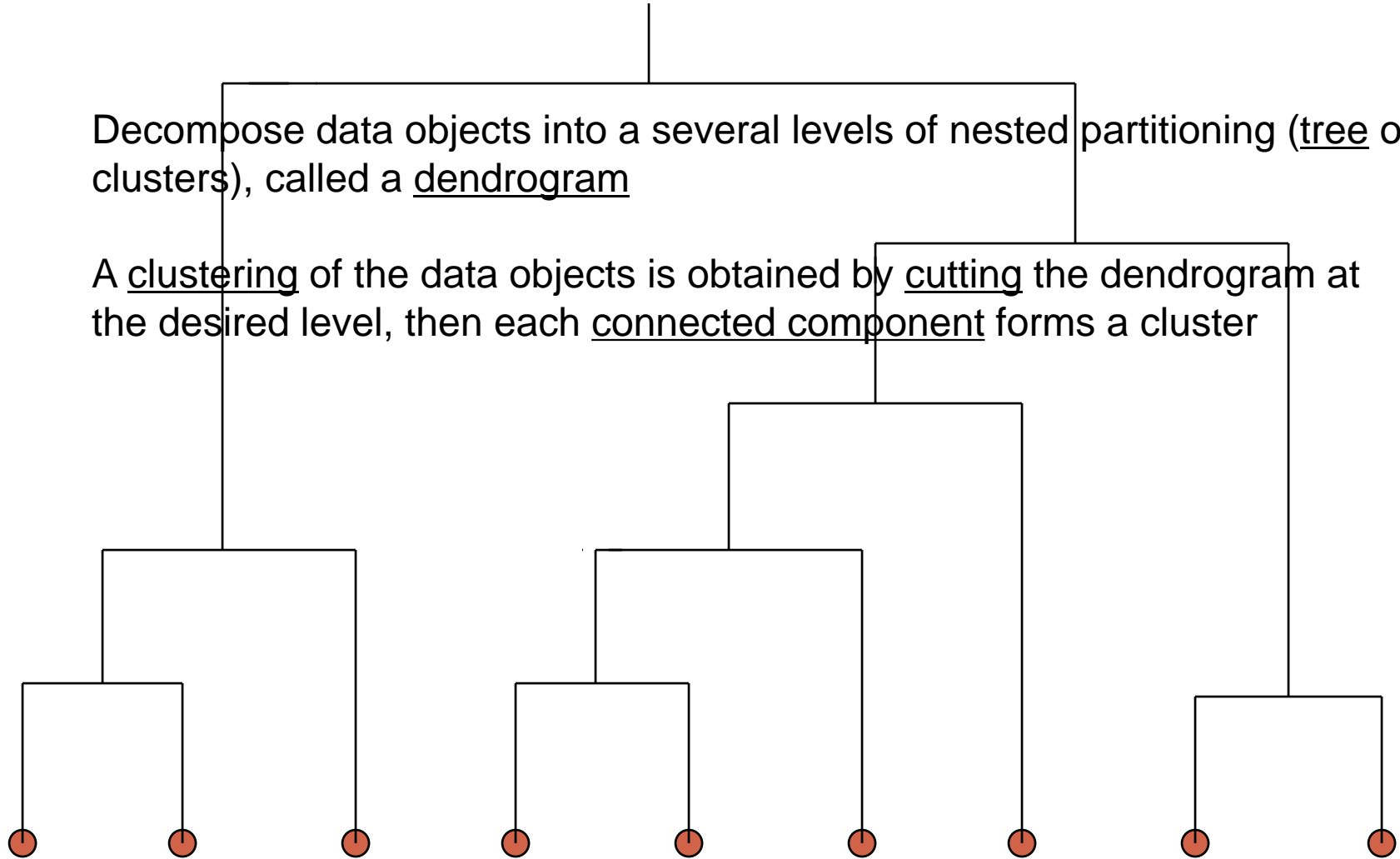
Pseudo Code

- Initialization: Place each data point into its own cluster and compute distance matrix between clusters
- Repeat:
 - Merge the two **closest** clusters
 - Update the distance matrix for the affected entries
- Until: all the data are merged into a single cluster

Dendrogram: Shows How Clusters are Merged

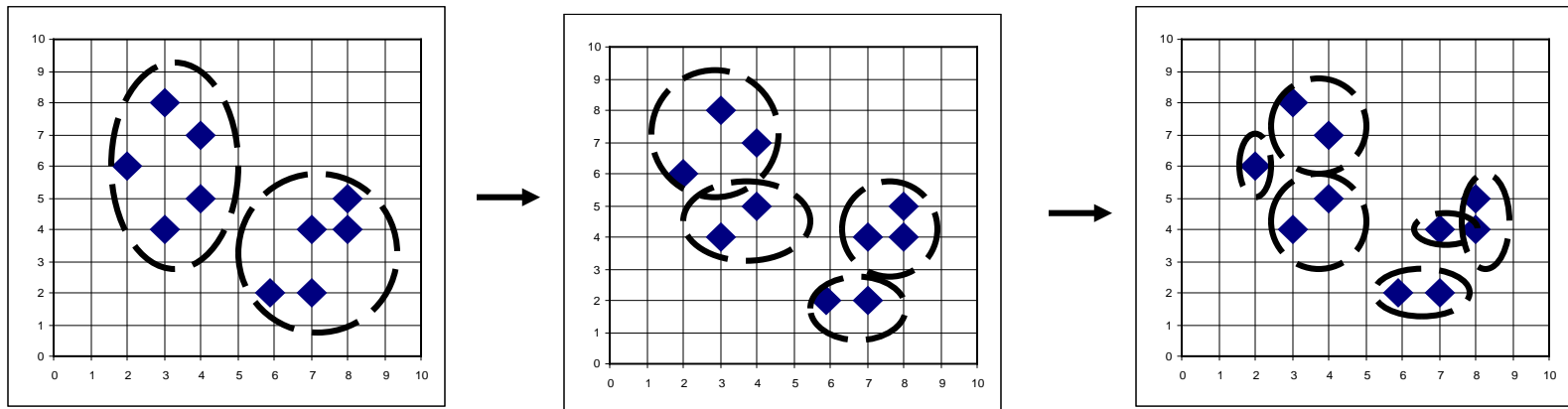
Decompose data objects into a several levels of nested partitioning (tree of clusters), called a dendrogram

A clustering of the data objects is obtained by cutting the dendrogram at the desired level, then each connected component forms a cluster

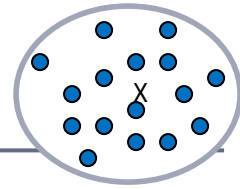
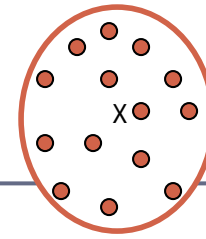


DIANA (Divisive Analysis)

- Introduced in Kaufmann and Rousseeuw (1990)
- Implemented in statistical analysis packages, e.g., Splus
- Inverse order of AGNES
- Eventually each node forms a cluster on its own

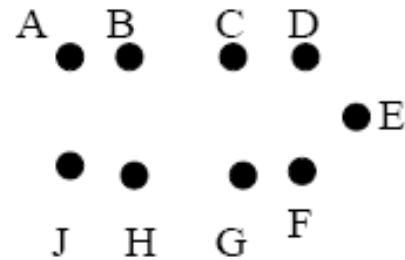


Distance between Clusters

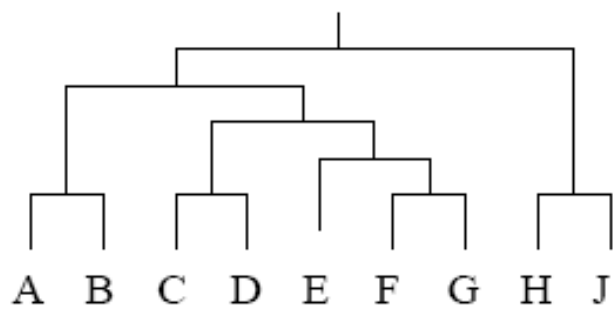
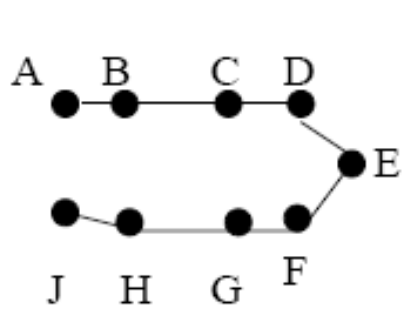


- **Single link:** smallest distance between an element in one cluster and an element in the other, i.e., $\text{dist}(K_i, K_j) = \min \text{dist}(t_{ip}, t_{jq})$
- **Complete link:** largest distance between an element in one cluster and an element in the other, i.e., $\text{dist}(K_i, K_j) = \max \text{dist}(t_{ip}, t_{jq})$
- **Average:** avg distance between an element in one cluster and an element in the other, i.e., $\text{dist}(K_i, K_j) = \text{avg dist}(t_{ip}, t_{jq})$
- **Centroid:** distance between the centroids of two clusters, i.e., $\text{dist}(K_i, K_j) = \text{dist}(C_i, C_j)$
- **Medoid:** distance between the medoids of two clusters, i.e., $\text{dist}(K_i, K_j) = \text{dist}(M_i, M_j)$
 - **Medoid:** a chosen, centrally located object in the cluster

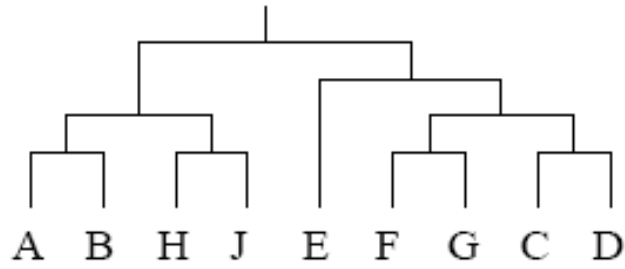
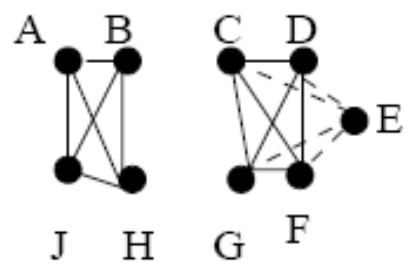
Example: Single Link vs. Complete Link



(a) Data set



(b) Clustering using single linkage




(c) Clustering using complete linkage

Extensions to Hierarchical Clustering

- Major weakness of agglomerative clustering methods
 - Can never undo what was done previously
 - Do not scale well: time complexity of at least $O(n^2)$, where n is the number of total objects
- Integration of hierarchical & distance-based clustering
 - *BIRCH (1996): uses CF-tree and incrementally adjusts the quality of sub-clusters
 - *CHAMELEON (1999): hierarchical clustering using dynamic modeling

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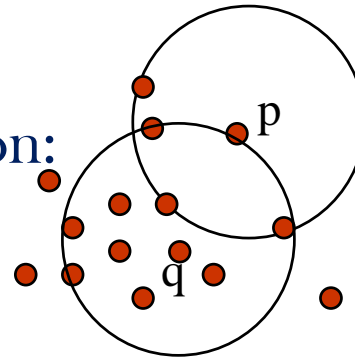
Density-Based Clustering Methods

- Clustering based on density (local cluster criterion), such as density-connected points
- Major features:
 - Discover clusters of arbitrary shape
 - Handle noise
 - One scan
 - Need density parameters as termination condition
- Several interesting studies:
 - DBSCAN: Ester, et al. (KDD'96)
 - OPTICS*: Ankerst, et al (SIGMOD'99).
 - DENCLUE*: Hinneburg & D. Keim (KDD'98)
 - CLIQUE*: Agrawal, et al. (SIGMOD'98) (more grid-based)

DBSCAN: Basic Concepts

- Two parameters:
 - *Eps*: Maximum radius of the neighborhood
 - *MinPts*: Minimum number of points in an *Eps*-neighborhood of that point
- $N_{Eps}(q)$: {p belongs to D | dist(p,q) ≤ Eps}
- **Directly density-reachable**: A point p is directly density-reachable from a point q w.r.t. *Eps*, *MinPts* if
 - p belongs to $N_{Eps}(q)$
 - q is a **core point**, **core point** condition:

$$|N_{Eps}(q)| \geq MinPts$$



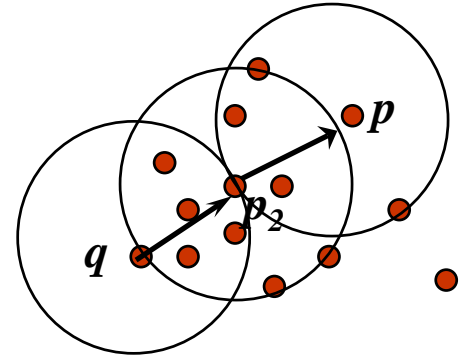
MinPts = 5

Eps = 1 cm

Density-Reachable and Density-Connected

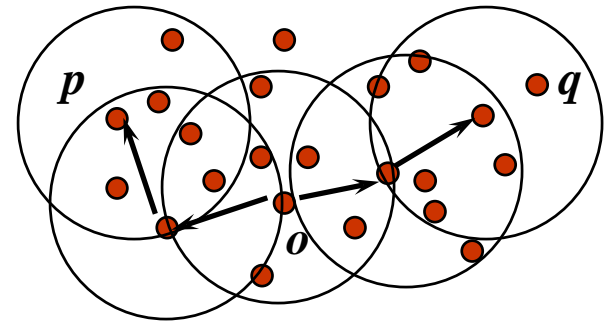
- Density-reachable:

- A point p is **density-reachable** from a point q w.r.t. Eps , $MinPts$ if there is a chain of points p_1, \dots, p_n , $p_1 = q$, $p_n = p$ such that p_{i+1} is directly density-reachable from p_i



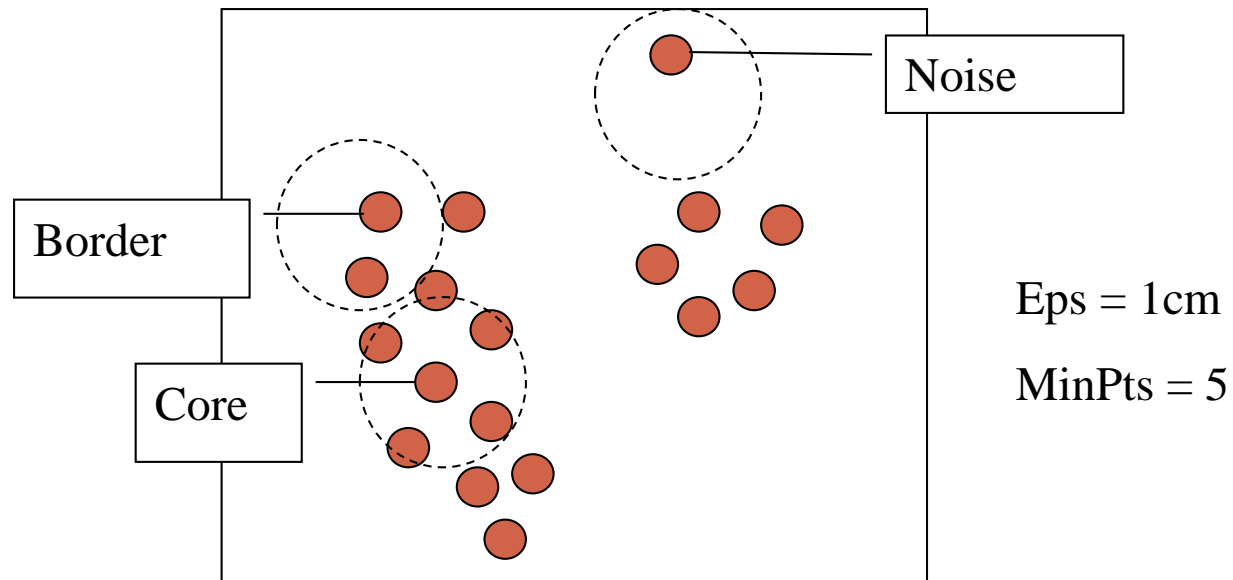
- Density-connected

- A point p is **density-connected** to a point q w.r.t. Eps , $MinPts$ if there is a point o such that both, p and q are density-reachable from o w.r.t. Eps and $MinPts$



DBSCAN: Density-Based Spatial Clustering of Applications with Noise

- Relies on a *density-based* notion of cluster: A *cluster* is defined as a maximal set of density-connected points
- **Noise**: object not contained in any cluster is noise
- Discovers clusters of arbitrary shape in spatial databases with noise



DBSCAN: The Algorithm

- (1) mark all objects as unvisited;
- (2) do
- (3) randomly select an unvisited object p ;
- (4) mark p as visited;
- (5) if the ϵ -neighborhood of p has at least $MinPts$ objects
- (6) create a new cluster C , and add p to C ;
- (7) let N be the set of objects in the ϵ -neighborhood of p ;
- (8) for each point p' in N
- (9) if p' is unvisited
- (10) mark p' as visited;
- (11) if the ϵ -neighborhood of p' has at least $MinPts$ points,
 add those points to N ;
- (12) if p' is not yet a member of any cluster, add p' to C ;
- (13) end for
- (14) output C ;
- (15) else mark p as noise;
- (16) until no object is unvisited;

- *If a spatial index is used, the computational complexity of DBSCAN is $O(n \log n)$, where n is the number of database objects. Otherwise, the complexity is $O(n^2)$*

DBSCAN: Sensitive to Parameters

Figure 8. DBScan results for DS1 with MinPts at 4 and Eps at (a) 0.5 and (b) 0.4.

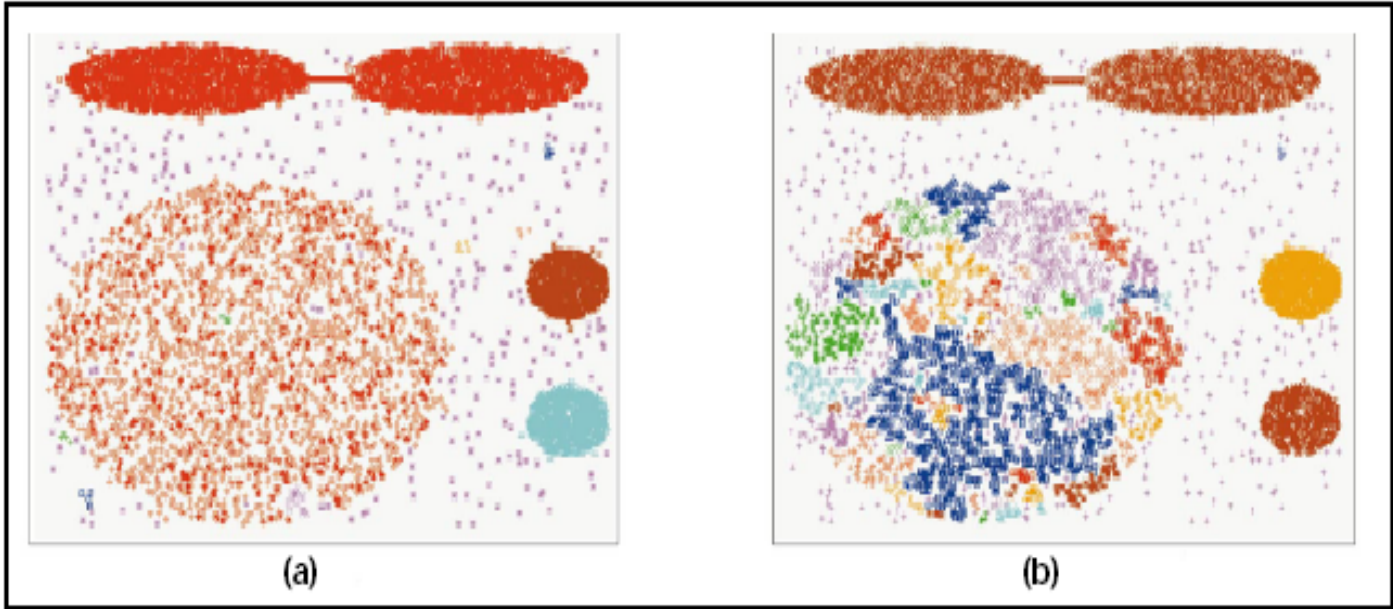
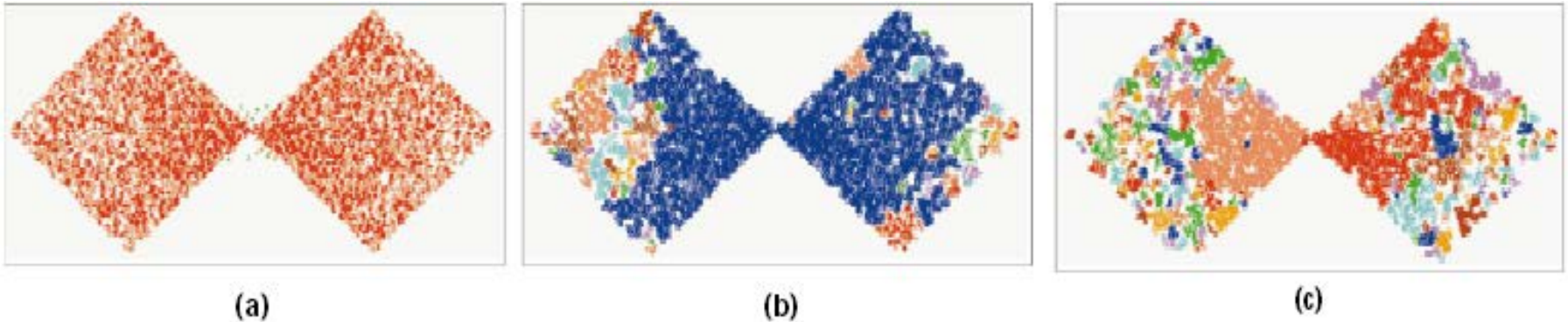


Figure 9. DBScan results for DS2 with MinPts at 4 and Eps at (a) 5.0, (b) 3.5, and (c) 3.0.




DBSCAN online Demo:

<http://webdocs.cs.ualberta.ca/~yaling/Cluster/Applet/Code/Cluster.html>

Questions about Parameters

- Fix Eps, increase MinPts, what will happen?
- Fix MinPts, decrease Eps, what will happen?

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Summary

- **Cluster analysis** groups objects based on their **similarity** and has wide applications; Measure of similarity can be computed for **various types of data**
- **K-means** and **K-medoids** algorithms are popular partitioning-based clustering algorithms
- **AGNES** and **DIANA** are interesting hierarchical clustering algorithms
- **DBSCAN**, **OPTICS***, and **DENCLUE*** are interesting density-based algorithms

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