

# Physics-Based Models for Image Analysis/Synthesis and Geometric Design

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## Abstract

*This invited paper reviews recently developed physics-based surface modeling techniques for geometric design, medical image analysis, and human facial modeling. I briefly motivate the problems of interest in each application area, describe the models that we have developed to address them, present sample results, and provide references to technical papers containing the full details.*

## 1 Deformable Models and the Segmentation of Medical Images

In recent years the role of medical imaging has expanded beyond the simple visualization and inspection of anatomic structures. It has become a tool for surgical planning and simulation, intra-operative navigation, radiotherapy planning, and for tracking the progress of disease. This increased role has opened an array of challenging problems centered on the computation of accurate geometric models of anatomic structures from medical images. Although modern imaging devices provide exceptional views of internal anatomy, the use of computers to quantify and analyze the embedded structures with accuracy and efficiency is limited. Accurate, repeatable, quantitative data must be efficiently extracted and compacted in order to support the spectrum of biomedical investigations and clinical activities.

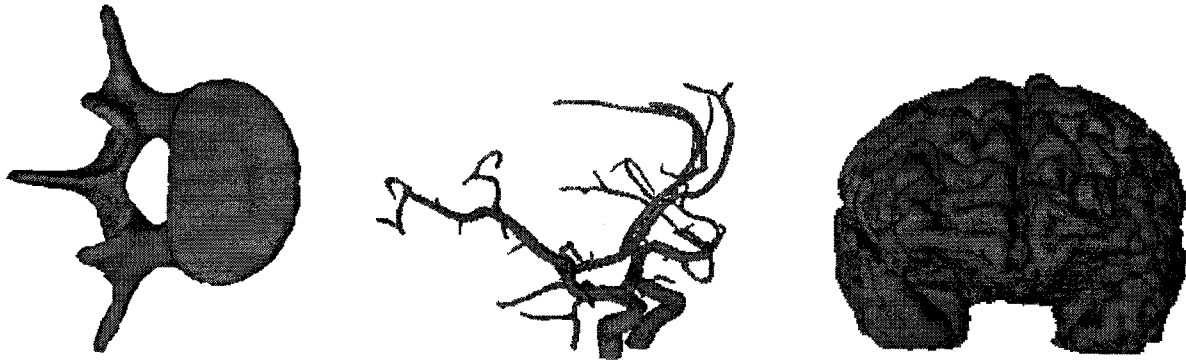
A promising and vigorously researched approach to tackling such problems is the use of deformable models (see the recent survey [4]). These powerful models have proven to be effective in segmenting, visualizing, matching, and tracking anatomical structures by exploiting (bottom-up) constraints derived from the image data together with (top-down) *a priori* knowledge about the location, size, shape, and smoothness of these structures. Deformable models provide a compact and analytical representation of object shape. Furthermore, deformable models support highly intuitive interaction mechanisms that allow medical scientists and practitioners to bring their expertise to bear on the image

interpretation task.

The mathematical foundations of deformable models [8, 9, 13] represent the confluence of geometry and physics with approximation and estimation theory. Geometry serves to represent object shape, physics imposes constraints on how the shape may vary over space and time, while approximation and estimation theory provides formal mechanisms for fitting the models to data.

Deformable model geometry usually attains broad shape coverage by employing geometric representations that involve many degrees of freedom, such as splines. The model remains manageable, however, because the degrees of freedom are generally not permitted to evolve independently, but are governed by physical principles that bestow intuitively meaningful behavior upon the geometric substrate. The name "deformable models" stems primarily from the use of elasticity theory at the physical level, generally within a Lagrangian dynamics setting. The physical interpretation of deformable models views them as elastic bodies which respond in a natural fashion to applied forces and constraints [10]. Typically, deformation energy functions defined in terms of the geometric degrees of freedom are associated with the deformable model. The energy grows monotonically as the model deforms away from a specified natural or "rest shape" and the energy expression often includes terms that constrain the smoothness or symmetry of the model. In the Lagrangian setting, the deformation energy gives rise to elastic (or, more generally, viscoelastic or plastic) forces internal to the model. Taking a physics-based view of classical optimal approximation, external potential energy functions are defined in terms of the data of interest to which the model is to be fitted. These potential energies give rise to external forces which deform the model such that it fits the data.

In [3], we introduced a new class of deformable models for medical image analysis, known as *topologically adaptable deformable models*. These models exploit an Affine Cell Decomposition of the image domain, creating a theoretically sound framework that significantly extends the abilities of classical deformable models. Embedding deformable models in an ACD framework allows the models



**Figure 1. T-surface segmentation of (a) human vertebra phantom from CT image volume, (b) cerebral vasculature from MRA image volume, (c) cerebral cortex from preprocessed MR image volume.**

to extract and reconstruct even the most complex biological structures. The ACD framework, combined with a novel reparameterization algorithm, creates a simple but powerful mechanism for multiresolution deformable curve, surface, and solid models to segment objects with complex geometries and topologies, adapting their shape to conform to the object boundaries. The ACD framework enables the models to maintain the traditional properties associated with classical deformable models, such as user interaction and the incorporation of constraints through energy functions or force functions, while overcoming many of their limitations. The framework also provides a convenient mechanism for the incorporation of “hard” geometric, topological and global shape constraints. Fig. 1 provides examples using topologically adaptable surface models (T-surfaces) [5] to segment anatomic structures from medical image volumes.

## 2 Realistic Human Facial Modeling from Scanned Data

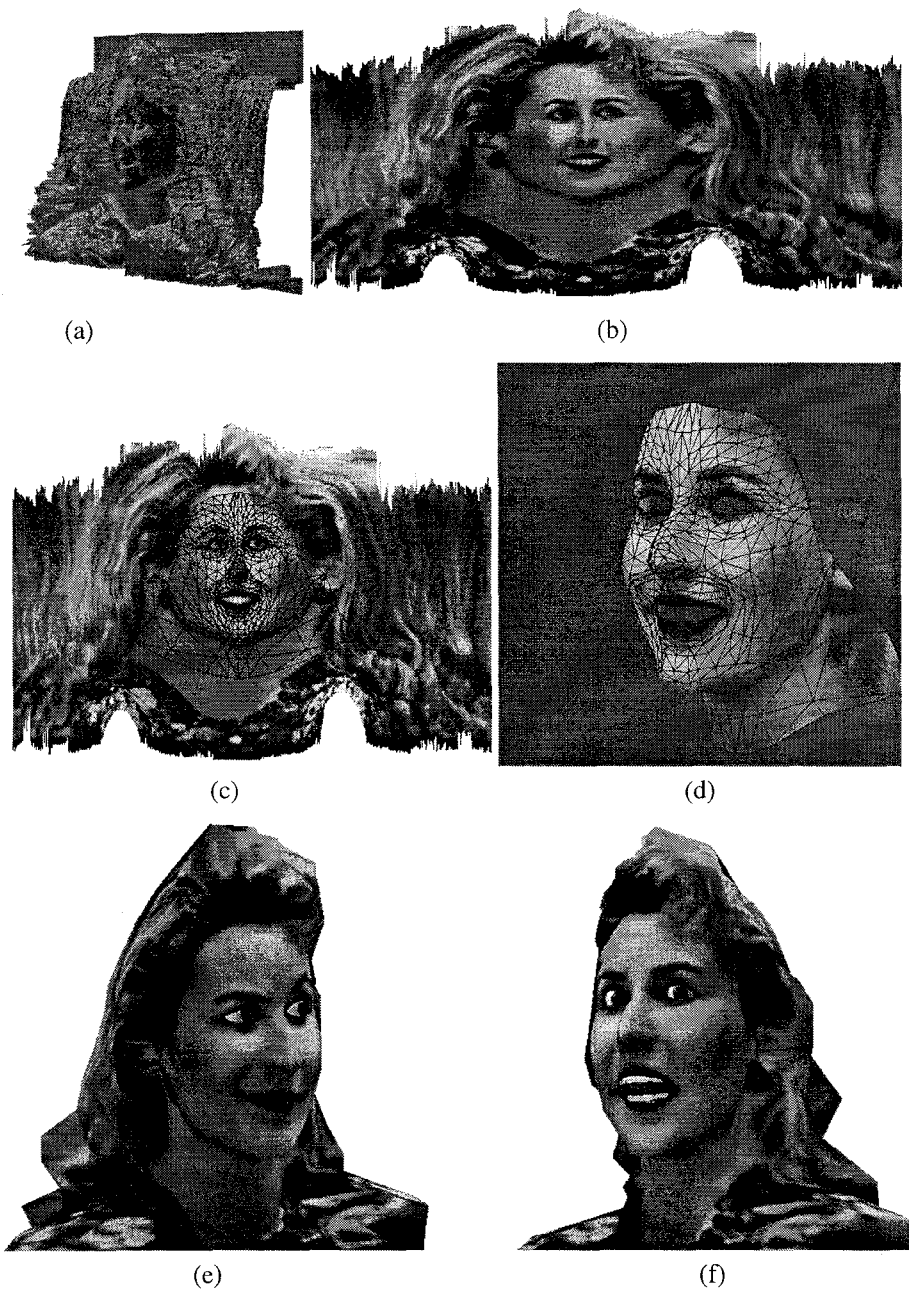
Facial image analysis and synthesis is useful for numerous applications. A primary application is for computer animation of human faces for entertainment and education. Another application is low bandwidth teleconferencing which may involve the real-time extraction of facial control parameters from live video at the transmission site and the reconstruction of a dynamic facsimile of the subject’s face at a remote receiver. Teleconferencing and other applications require facial models that are computationally efficient and also realistic enough to accurately synthesize the various nuances of facial structure and motion. We have argued that the anatomy and physics of the human face, especially the arrangement and actions of the primary facial muscles, provide a good basis for facial image analysis and synthesis [12].

We have developed a highly automated approach to con-

structing realistic, functional models of human heads [2]. These physics-based models are anatomically accurate and may be made to conform closely to specific individuals. Currently, we begin by scanning a subject with a laser sensor which circles around the head to acquire detailed range and reflectance information. Next, an automatic conformation algorithm adapts a triangulated face mesh of predetermined topological structure to these data. The generic mesh, which is reusable with different individuals, reduces the range data to an efficient, polygonal approximation of the facial geometry and supports a high-resolution texture mapping of the skin reflectivity.

The conformed polygonal mesh forms the epidermal layer of a physics-based model of facial tissue. An automatic algorithm constructs the multilayer synthetic skin and estimates an underlying skull substructure with a jointed jaw. Finally, the algorithm inserts synthetic muscles into the deepest layer of the facial tissue. These contractile actuators, which emulate the primary muscles of facial expression, generate forces that deform the synthetic tissue into meaningful expressions. To increase realism, we include constraints to emulate tissue incompressibility and to enable the tissue to slide over the skull without penetrating into it.

Fig. 2 illustrates the aforementioned steps. The figure shows a 360° head-to-shoulder scan of a woman, “Heidi,” acquired by Cyberware, Inc., using their Color 3D Digitizer. The data set consists of a radial range map (Fig. 2(a)) and a registered RGB photometric map (Fig. 2(b)). The range and RGB maps are high-resolution  $512 \times 256$  arrays in cylindrical coordinates, where the  $x$  axis is the latitudinal angle around the head and the  $y$  axis is vertical distance. Fig. 2(c) shows the generic mesh projected into the 2D cylindrical domain and overlaid on the RGB map. The triangle edges in the mesh are elastic springs, and the mesh has been conformed semi-interactively to the woman’s face using both the



**Figure 2. Facial modeling using scanned data. (a) Radial range map. (b) RGB photometric map. (c) RGB map with conformed epidermal mesh overlaid. (d) 3D mesh and texture mapped triangles. (e-f) Animate face model.**

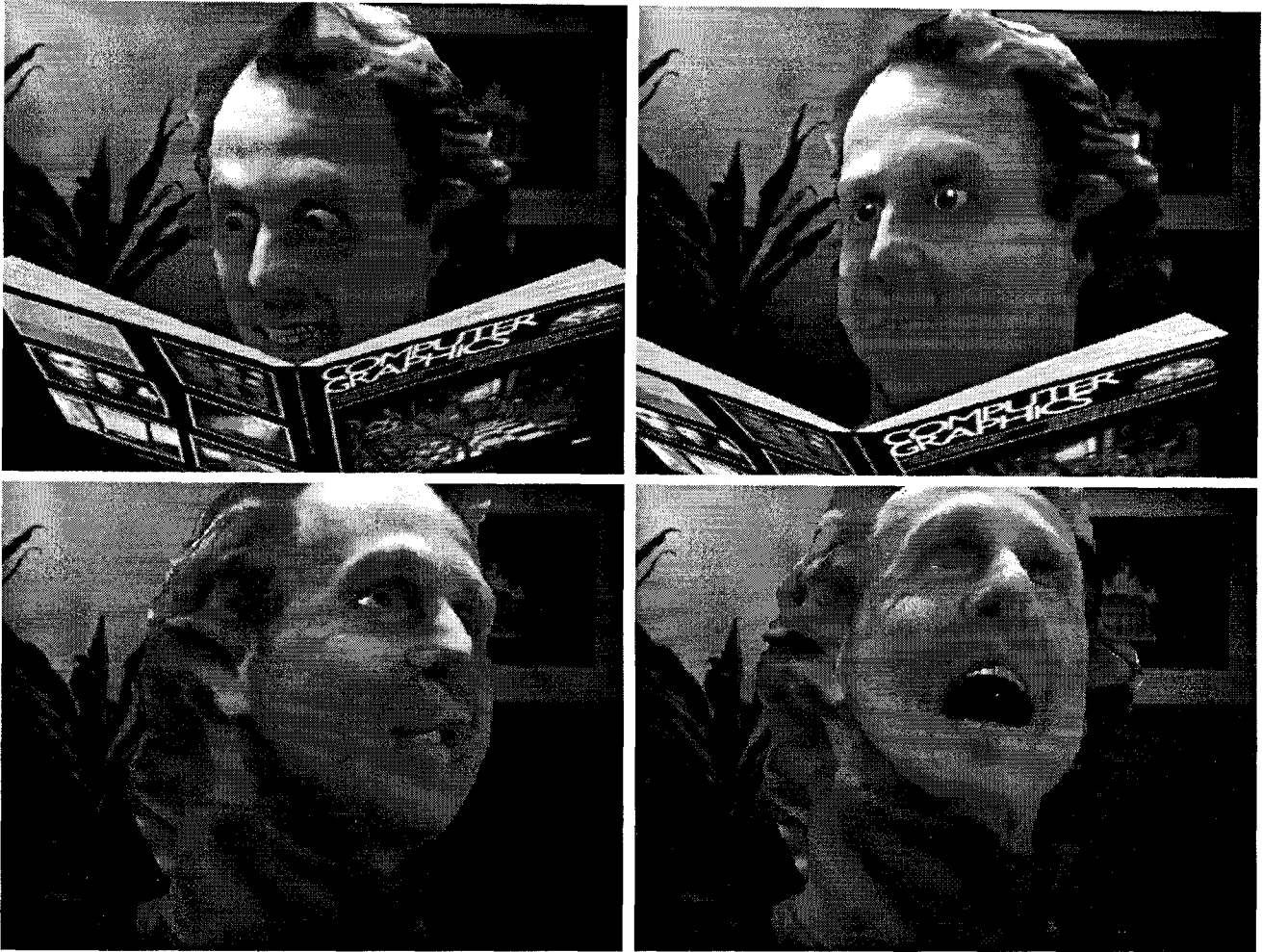


Figure 3. Synthetic George in four scenes from “*Bureaucrat Too*”.

range and RGB maps. The nodes of the conformed mesh serve as sample points in the range map. Their cylindrical coordinates and the sampled range values are employed to compute 3D Euclidean space coordinates for the polygon vertices. In addition, the nodal coordinates serve as polygon vertex texture map coordinates into the RGB map. Fig. 2(d) shows the 3D facial mesh with the texture mapped photometric data. Once we have reduced the scanned data to the 3D epidermal mesh of Fig. 2(d), we can assemble a physics-based face model of Heidi. Fig. 2(e,f) demonstrates that we can animate the resulting face model by activating muscles. Fig. 3 shows frames from the expressive animation of a different synthetic face.

This physics-based anatomically motivated facial model has allowed us to develop a new approach to the analysis of dynamic facial images for the purposes of estimating and resynthesizing dynamic facial expressions [12]. Part of the difficulty of facial image analysis is that the face is

highly deformable, particularly around the forehead, eyes, and mouth, and these deformations convey a great deal of meaningful information. Techniques for tracking the deformation of facial features include snakes [1]. Motivated by the anatomically consistent musculature in our model, we have considered the estimation of dynamic facial muscle contractions from video sequences of expressive faces. We have developed an analysis technique that uses snakes to track the nonrigid motions of facial features in video. Features of interest include the eyebrows, nasal furrows, mouth, and jaw in the image plane. We are able to estimate dynamic facial muscle contractions directly from the snake state variables. These estimates make appropriate control parameters for resynthesizing facial expressions through our face model. The model resynthesizes facial images at real time rates.

### 3 Modeling with Dynamic NURBS

In 1975 Versprille proposed the Non-Uniform Rational B-Splines or NURBS for geometric design. NURBS quickly gained popularity and were incorporated into several commercial modeling systems. The NURBS representation has several attractive properties. It offers a unified mathematical formulation for representing not only free-form curves and surfaces, but also standard analytic shapes such as conics, quadrics, and surfaces of revolution. The most frequently used NURBS design techniques are the specification of a control polygon, and interpolation or approximation of data points to generate the initial shape. For surfaces or solids, cross-sectional design including skinning, sweeping, and swinging operations is also popular. By adjusting the positions of control points, associated weights, and knots of the initial shape, one can design a large variety of shapes using NURBS. Despite modern interactive devices, however, this conventional refinement process can be clumsy and laborious when it comes to designing complex, real-world objects.

To address these problems, we have developed *Dynamic NURBS*, or D-NURBS [11]. D-NURBS are physics-based models that incorporate mass distributions, internal deformation energies, and other physical quantities into the NURBS geometric substrate. The models are governed by dynamic differential equations which, when integrated numerically through time, continuously evolve the control points and weights in response to applied forces. The D-NURBS formulation supports interactive direct manipulation of NURBS objects, which results in physically meaningful hence intuitively predictable motion and shape variation.

Using D-NURBS, a modeler can interactively sculpt complex shapes not merely by kinematic adjustment of control points and weights, but dynamically as well—by applying simulated forces. Additional control over dynamic sculpting stems from the modification of physical parameters such as mass, damping, and elastic properties. Elastic functionals allow the imposition of qualitative “fairness” criteria through quantitative means. Linear or nonlinear constraints may be imposed either as hard constraints that must not to be violated, or as soft constraints to be satisfied approximately. The latter may be interpreted intuitively as simple forces. Optimal shape design results when D-NURBS are allowed to achieve static equilibrium subject to shape constraints. All of these capabilities are subsumed under an elegant formulation grounded in physics.

Physics-based design augments (rather than supersedes) standard geometry and geometric design, offering attractive new advantages. We have used Lagrangian mechanics to formulate D-NURBS curves, tensor-product D-NURBS surfaces [11], swung D-NURBS surfaces [6], and triangular D-NURBS surfaces [7]. We apply finite element analysis

to reduce these models to efficient numerical algorithms computable at interactive rates on common graphics workstations. We have implemented a prototype modeling environment based on D-NURBS, and demonstrated that D-NURBS can be effective tools in a wide range of CAGD applications such as shape blending, scattered data fitting, and interactive sculpting. Fig. 4 illustrates the results of four interactive sculpting sessions using swung D-NURBS surfaces and simple forces.

### References

- [1] M. Kass, A. Witkin, and D. Terzopoulos. Snakes: Active contour models. *International Journal of Computer Vision*, 1(4):321–331, 1988.
- [2] Y. Lee, D. Terzopoulos, and K. Waters. Realistic facial modeling for animation. In *Computer Graphics Proceedings, Annual Conference Series, Proc. SIGGRAPH '95* (Los Angeles, CA), pages 55–62. ACM SIGGRAPH, August 1995.
- [3] T. McInerney and D. Terzopoulos. Medical image analysis with topologically adaptive snakes. In *First International Conference on Computer Vision, Virtual Reality, and Robotics in Medicine (CVRMed'95)*, Nice, France, April 1995. In press.
- [4] T. McInerney and D. Terzopoulos. Deformable models in medical image analysis: A survey. *Medical Image Analysis*, 1(2):91–108, June 1996.
- [5] T. McInerney and D. Terzopoulos. Medical image segmentation using topologically adaptable deformable surface models. In *First Joint Conf. of CVRMed II and MRCAS III, Grenoble, France, March, 1997*, 1997.
- [6] H. Qin and D. Terzopoulos. Dynamic NURBS swung surfaces for physics-based shape design. *Computer-Aided Design*, 27(2):111–127, 1995.
- [7] H. Qin and D. Terzopoulos. Triangular NURBS and dynamic generalizations. *Computer-Aided Geometric Design*, 1997. in press.
- [8] D. Terzopoulos. Regularization of inverse visual problems involving discontinuities. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, PAMI-8(4):413–424, 1986.
- [9] D. Terzopoulos. The computation of visible-surface representations. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, PAMI-10(4):417–438, 1988.
- [10] D. Terzopoulos and K. Fleischer. Deformable models. *The Visual Computer*, 4(6):306–331, 1988.
- [11] D. Terzopoulos and H. Qin. Dynamic NURBS with geometric constraints for interactive sculpting. *ACM Transactions on Graphics*, 13(2):103–136, 1994.
- [12] D. Terzopoulos and K. Waters. Analysis and synthesis of facial image sequences using physical and anatomical models. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 15(6):569–579, 1993.
- [13] D. Terzopoulos, A. Witkin, and M. Kass. Constraints on deformable models: Recovering 3D shape and nonrigid motion. *Artificial Intelligence*, 36(1):91–123, 1988.



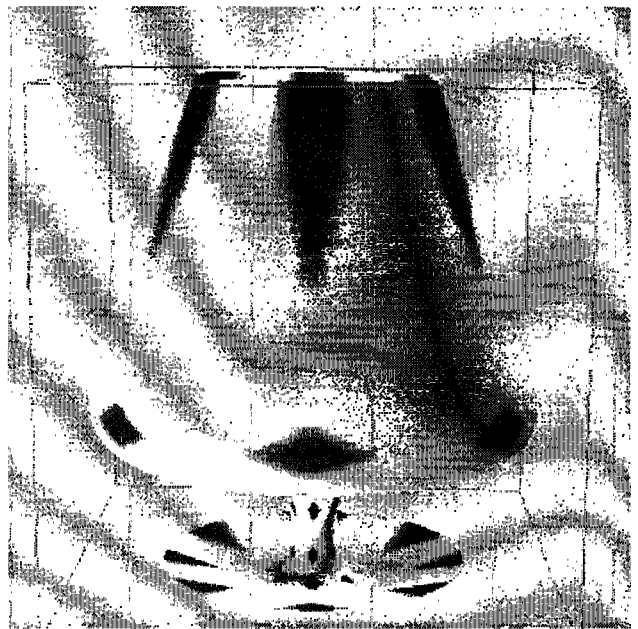
(a)



(b)



(c)



(d)

**Figure 4. Interactive sculpting of swung D-NURBS surfaces. Open and closed surfaces shown were sculpted interactively from prototype shapes noted in parentheses (a) Egg shape (sphere). (b) Deformed toroid (torus). (c) Hat (open surface). (d) Wine glass (cylinder).**