Iterative Optimization in the Polyhedral Model

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January 18th, 2010

Ph.D Defense

A Brief History...

- A Quick look backward:
  - 20 years ago: 80486 (1.2 M trans., 25 MHz, 8 kB cache)
  - **10 years ago:** Pentium 4 (42 M trans., **1.4 GHz**, 256 kB cache, SSE)
  - **7 years ago:** Pentium 4EE (169 M trans., **3.8 GHz**, 2 Mo cache, SSE2)
  - **4 years ago:** Core 2 Duo (**291 M** trans., **3.2 GHz**, 4 Mo cache, SSE3)
  - **1 years ago:** Core i7 Quad (**781 M** trans., **3.2 GHz**, 8 Mo cache, SSE4)

- Memory Wall: 400 MHz FSB speed vs 3+ GHz processor speed
- Power Wall: going multi-core, "slowing" processor speed
- Heterogeneous: CPU(s) + accelerators (GPUs, FPGA, etc.)

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- Heterogeneous: CPU(s) + accelerators (GPUs, FPGA, etc.)

Compilers are facing a much harder challenge

Important Issues

- New architecture $\longrightarrow$ New high-performance libraries needed
- **New architecture $\longrightarrow$ New optimization flow needed**
- Architecture complexity/diversity increases faster than optimization progress
- **Traditional approaches are not oriented towards performance portability...**

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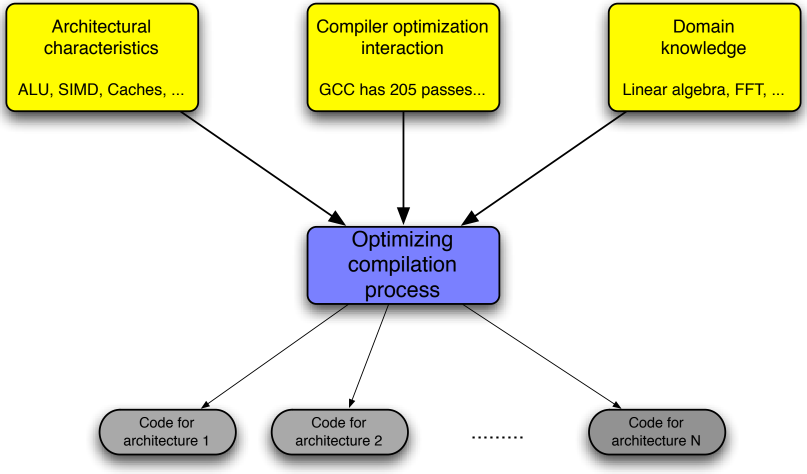
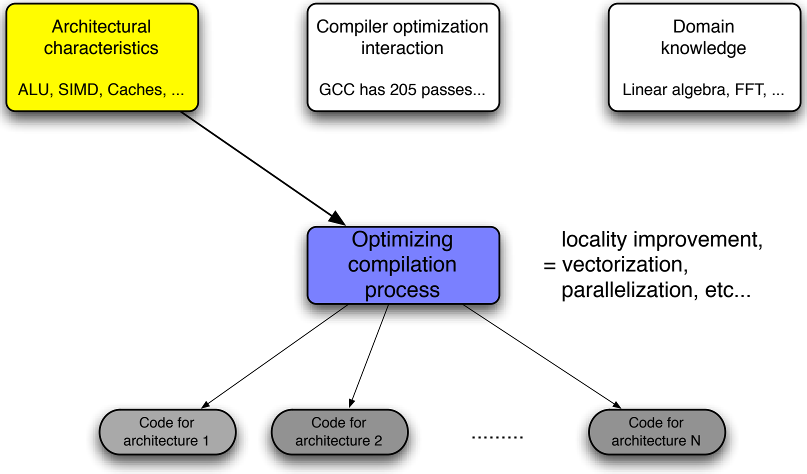
We need a portable optimization process

The Optimization Problem



The Optimization Problem



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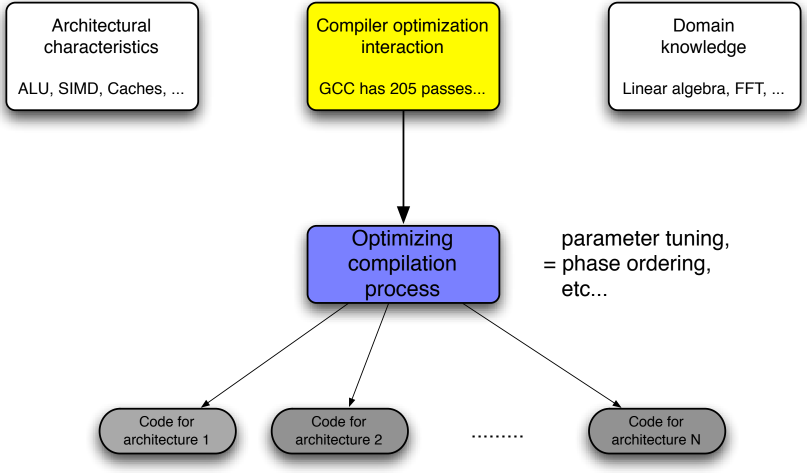

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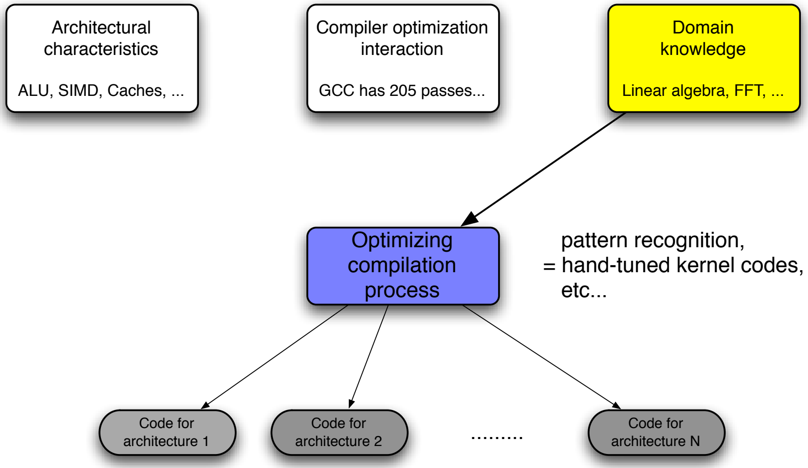

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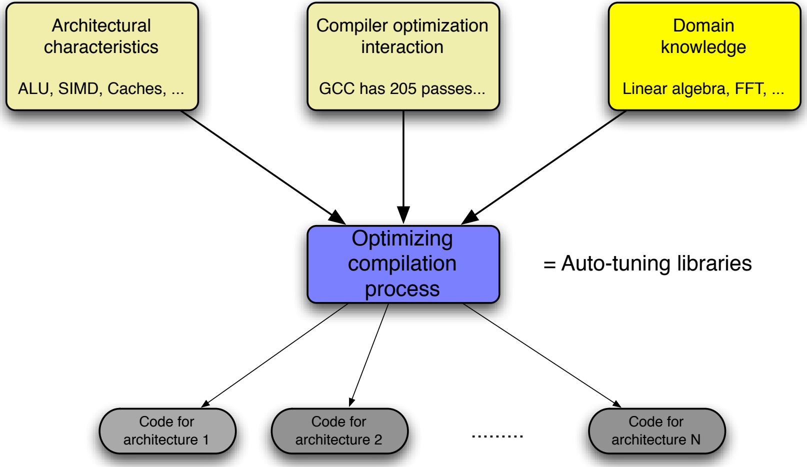
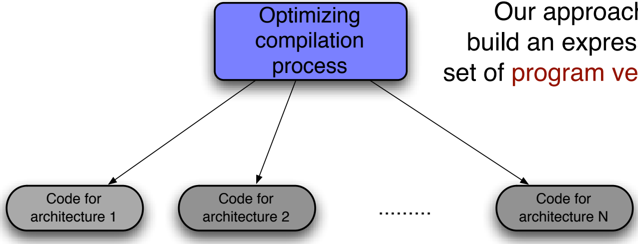


The Optimization Problem

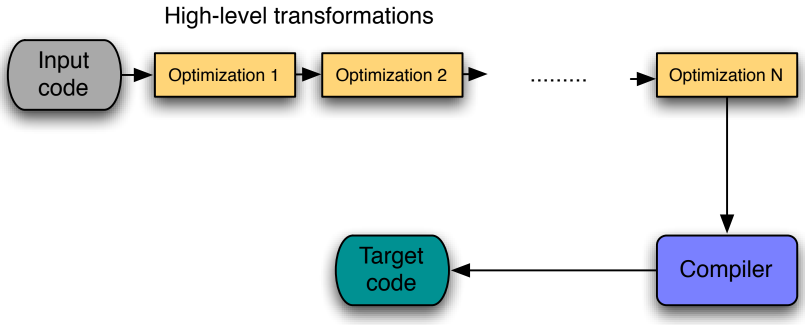
Architectural characteristics	Compiler optimization interaction	Domain knowledge
ALU, SIMD, Caches, ...	GCC has 205 passes...	Linear algebra, FFT, ...

In reality, there is a complex interplay between all components



Our approach:
build an expressive
set of program versions

Iterative Optimization Flow



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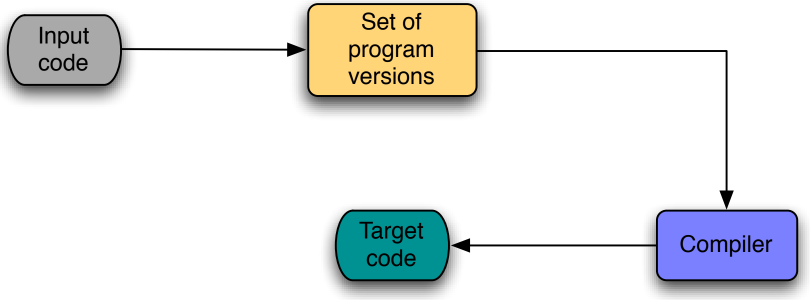
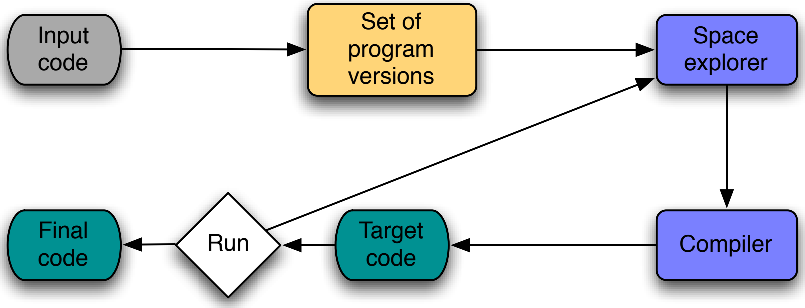


Program version = result of a sequence of loop transformation

Iterative Optimization Flow



Program version = result of a sequence of loop transformation

Other Iterative Frameworks

- Focus usually on composing existing compiler flags/passes
  - Optimization flags [Bodin et al.,PFDC98] [Fursin et al.,CGO06]
  - Phase ordering [Kulkarni et al.,TACO05]
  - Auto-tuning libraries (ATLAS, FFTW, ...)

- Others attempt to select a transformation sequence
  - SPIRAL [Püschel et al.,HPEC00]
  - Within UTF [Long and Fursin,ICPPW05], GAPS [Nisbet,HPCN98]
  - CHiLL [Hall et al.,USCRR08], POET [Yi et al.,LCPC07], etc.
  - URUK [Girbal et al.,IJPP06]

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  - URUK [Girbal et al.,IJPP06]

- Capability proven for efficient optimization
- Limited in applicability (legality)
- Limited in expressiveness (mostly simple sequences)
- Traversal efficiency compromised (uniqueness)

Our Approach: Set of Polyhedral Optimizations

What matters is the **result of the application of optimizations**, not the optimization sequence

All-in-one approach: [Pouchet et al.,CGO07/PLDI08]

- **Legality:** semantics is always preserved
- **Uniqueness:** all versions of the set are distinct
- **Expressiveness:** a version is the result of an arbitrarily complex sequence of loop transformation

- **Completion algorithm** to instantiate a legal version from a partially specified one
- **Dedicated traversal heuristics** to focus the search

The Polyhedral Model

The Polyhedral Model vs Syntactic Frameworks

Limitations of standard syntactic frameworks:

- Composition of transformations may be tedious
- Approximate dependence analysis
  - Miss optimization opportunities
  - Scalable optimization algorithms

The polyhedral model:

- Works on executed statement instances, finest granularity
- Model arbitrary compositions of transformations
- Requires computationally expensive algorithms

A Three-Stage Process

1 Analysis: from code to model

$\longrightarrow$ Existing prototype tools (some developed during this thesis)

- PoCC (Clan-Candl-LetSee-Pluto-Cloog-Polylib-PIPLib-ISL-FM)
- URUK, Omega, Loopo, . . .

$\longrightarrow$ GCC GRAPHITE (now in mainstream)

$\longrightarrow$ Reservoir Labs R-Stream, IBM XL/Poly

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2 Transformation in the model

→ Build and select a program transformation

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2 Transformation in the model

$\longrightarrow$ Build and select a program transformation

3 Code generation: from model to code

$\longrightarrow$ "Apply" the transformation in the model

$\longrightarrow$ Regenerate syntactic (AST-based) code

Polyhedral Representation of Programs

Static Control Parts

- Loops have affine control only (over-approximation otherwise)

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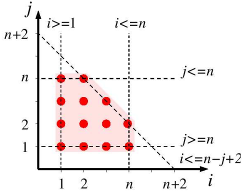
- Iteration domain: represented as integer polyhedra

```
for (i=1; i<=n; ++i)
. for (j=1; j<=n; ++j)
. . if (i<=n-j+2)
. . . s[i] = ...
```

$$\mathcal{D}_{S1} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ -1 & -1 & 1 & 2 \end{bmatrix} . \begin{pmatrix} i \\ j \\ n \\ 1 \end{pmatrix} \geq \vec{0}$$



Iteration domain of S_1

Polyhedral Representation of Programs

Static Control Parts

- Loops have affine control only (over-approximation otherwise)
- Iteration domain: represented as integer polyhedra
- Memory accesses: static references, represented as affine functions of $\vec{x_S}$ and $\vec{p}$

```
for (i=0; i<n; ++i) {
. s[i] = 0;
. for (j=0; j<n; ++j)
. . s[i] = s[i]+a[i][j]*x[j];
}
```

$$f_{\mathbf{s}}(\vec{x_{S2}}) = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \end{array}\right] . \begin{pmatrix} \vec{x_{S2}} \\ n \\ 1 \end{pmatrix}$$

$$f_{\mathbf{a}}(\vec{x_{S2}}) = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right] . \begin{pmatrix} \vec{x_{S2}} \\ n \\ 1 \end{pmatrix}$$

$$f_{\mathbf{x}}(\vec{x_{S2}}) = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \end{array}\right] . \begin{pmatrix} \vec{x_{S2}} \\ n \\ 1 \end{pmatrix}$$

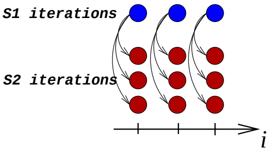
Polyhedral Representation of Programs

Static Control Parts

- Loops have affine control only (over-approximation otherwise)
- Iteration domain: represented as integer polyhedra
- Memory accesses: static references, represented as affine functions of $\vec{x}_S$ and $\vec{p}$
- Data dependence between S1 and S2: a subset of the Cartesian product of $\mathcal{D}_{S1}$ and $\mathcal{D}_{S2}$ (**exact analysis**)

```
for (i=1; i<=3; ++i) {
. s[i] = 0;
. for (j=1; j<=3; ++j)
. . s[i] = s[i] + 1;
}
```

$$\mathcal{D}_{S1\delta S2}:\left[\begin{array}{cccc} 1 & -1 & 0 & 0 \\ \hline 1 & 0 & 0 & -1 \\ -1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 3 \end{array}\right] \cdot \begin{pmatrix} i_{S1} \\ i_{S2} \\ j_{S2} \\ 1 \end{pmatrix} \begin{array}{c} =0 \\ \hline \geq \vec{0} \end{array}$$



Program Transformations

Original Schedule

```
 for (i = 0; i < n; ++i)
  for (j = 0; j < n; ++j){
S1: C[i][j] = 0;
    for (k = 0; k < n; ++k)
S2:   C[i][j] += A[i][k]*
                 B[k][j];

  }
```

$$\Theta^{S1}.\vec{x}_{S1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} . \begin{pmatrix} \mathbf{i} \\ \mathbf{j} \\ \mathbf{n} \\ \mathbf{1} \end{pmatrix}$$

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- Represent Static Control Parts (control flow and dependences must be statically computable)
- Use code generator (e.g. CLooG) to generate C code from polyhedral representation (provided iteration domains + schedules)

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$$\Theta^{S2}.\vec{x}_{S2} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} . \begin{pmatrix} \mathbf{i} \\ \mathbf{j} \\ \mathbf{k} \\ \mathbf{n} \\ \mathbf{1} \end{pmatrix}$$

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Program Transformations

Distribute loops

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```
for (i = 0; i < n; ++i)
 for (j = 0; j < n; ++j)
  C[i][j] = 0;
for (i = n; i < 2*n; ++i)
 for (j = 0; j < n; ++j)
  for (k = 0; k < n; ++k)
   C[i-n][j] += A[i-n][k]*
                B[k][j];
```

- **All instances of S1 are executed before the first S2 instance**

Program Transformations

Distribute loops + Interchange loops for S2

```
 for (i = 0; i < n; ++i)
  for (j = 0; j < n; ++j){
S1: C[i][j] = 0;
   for (k = 0; k < n; ++k)
S2:  C[i][j] += A[i][k]*
                B[k][j];
  }
```

$$\Theta^{S1}.\vec{x}_{S1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} . \begin{pmatrix} \mathbf{i} \\ \mathbf{j} \\ \mathbf{n} \\ \mathbf{1} \end{pmatrix}$$

$$\Theta^{S2}.\vec{x}_{S2} = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} . \begin{pmatrix} \mathbf{i} \\ \mathbf{j} \\ \mathbf{k} \\ \mathbf{n} \\ \mathbf{1} \end{pmatrix}$$

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for (i = 0; i < n; ++i)
 for (j = 0; j < n; ++j)
  C[i][j] = 0;
for (k = n; k < 2*n; ++k)
 for (j = 0; j < n; ++j)
  for (i = 0; i < n; ++i)
   C[i][j] += A[i][k-n]*
              B[k-n][j];
```

- The outer-most loop for S2 becomes k

Program Transformations

Illegal schedule

```
 for (i = 0; i < n; ++i)
  for (j = 0; j < n; ++j){
S1: C[i][j] = 0;
   for (k = 0; k < n; ++k)
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```
for (k = 0; k < n; ++k)
 for (j = 0; j < n; ++j)
  for (i = 0; i < n; ++i)
   C[i][j] += A[i][k]*
              B[k][j];
for (i = n; i < 2*n; ++i)
 for (j = 0; j < n; ++j)
  C[i-n][j] = 0;
```

- **All instances of S1 are executed <u>after</u> the last S2 instance**

Program Transformations

A legal schedule

```
 for (i = 0; i < n; ++i)
  for (j = 0; j < n; ++j){
S1: C[i][j] = 0;
   for (k = 0; k < n; ++k)
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$$\Theta^{S2}.\vec{x}_{S2} = \begin{pmatrix} 0 & 0 & 1 & \mathbf{1} & \mathbf{1} \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} . \begin{pmatrix} \mathbf{i} \\ \mathbf{j} \\ \mathbf{k} \\ \mathbf{n} \\ \mathbf{1} \end{pmatrix}$$

```
for (i = n; i < 2*n; ++i)
 for (j = 0; j < n; ++j)
  C[i][j] = 0;
for (k= n+1; k<= 2*n; ++k)
 for (j = 0; j < n; ++j)
  for (i = 0; i < n; ++i)
   C[i][j] += A[i][k-n-1]*
              B[k-n-1][j];
```

- **Delay the S2 instances**
- Constraints must be expressed between Θ^{S1} and Θ^{S2}

Program Transformations

Implicit fine-grain parallelism

```
 for (i = 0; i < n; ++i)
  for (j = 0; j < n; ++j){
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   C[i][j] += A[i][k-n]*
              B[k-n][j];
```

- **Number of rows of $\Theta \leftrightarrow$ number of outer-most <u>sequential</u> loops**

Program Transformations

Representing a schedule

```
 for (i = 0; i < n; ++i)
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```

$$\Theta.\vec{x} = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} . \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{i} & \mathbf{j} & \mathbf{k} & \mathbf{n} & \mathbf{n} & \mathbf{1} & \mathbf{1} \end{pmatrix}^T$$

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  C[i][j] = 0;
for (k= n+1; k<= 2*n; ++k)
 for (j = 0; j < n; ++j)
  for (i = 0; i < n; ++i)
   C[i][j] += A[i][k-n-1]*
              B[k-n-1][j];
```

$$\Theta.\vec{x} = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} . (\underbrace{\mathbf{i}\ \mathbf{j}\ \mathbf{i}\ \mathbf{j}\ \mathbf{k}}_{\vec{\imath}}\ \underbrace{\mathbf{n}\ \mathbf{n}}_{\vec{p}}\ \underbrace{\mathbf{1}\ \mathbf{1}}_{\mathbf{c}})^T$$

Program Transformations

Representing a schedule

```
 for (i = 0; i < n; ++i)
  for (j = 0; j < n; ++j){
S1: C[i][j] = 0;
    for (k = 0; k < n; ++k)
S2:   C[i][j] += A[i][k]*
                 B[k][j];

  }
```

$$\Theta^{S1}.\vec{x}_{S1} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} . \begin{pmatrix} i \\ j \\ n \\ 1 \end{pmatrix}$$

$$\Theta^{S2}.\vec{x}_{S2} = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} . \begin{pmatrix} i \\ j \\ k \\ n \\ 1 \end{pmatrix}$$

```
for (i = n; i < 2*n; ++i)
 for (j = 0; j < n; ++j)
  C[i][j] = 0;
for (k= n+1; k<= 2*n; ++k)
 for (j = 0; j < n; ++j)
  for (i = 0; i < n; ++i)
   C[i][j] += A[i][k-n-1]*
              B[k-n-1][j];
```

	Transformation	Description
$\vec{i}$	reversal	Changes the direction in which a loop traverses its iteration range
	skewing	Makes the bounds of a given loop depend on an outer loop counter
	interchange	Exchanges two loops in a perfectly nested loop, a.k.a. permutation
$\vec{p}$	fusion	Fuses two loops, a.k.a. jamming
	distribution	Splits a single loop nest into many, a.k.a. fission or splitting
c	peeling	Extracts one iteration of a given loop
	shifting	Allows to reorder loops

Example: Semantics Preservation (1-D)

Example: Semantics Preservation (1-D)

Property (Causality condition for schedules)

Given $R\delta S$, θ_R *and* θ_S *are legal iff for each pair of instances in dependence:*

$$\theta_R(\vec{x_R}) < \theta_S(\vec{x_S})$$

Equivalently: $\Delta_{R,S} = \theta_S(\vec{x_S}) - \theta_R(\vec{x_R}) - 1 \geq 0$

Example: Semantics Preservation (1-D)

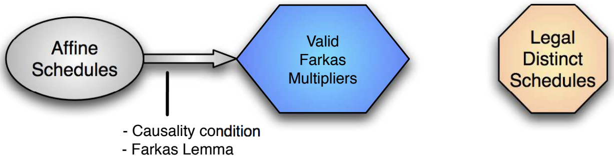
Lemma (Affine form of Farkas lemma)

Let $\mathcal{D}$ be a nonempty polyhedron defined by $A\vec{x}+\vec{b} \geq \vec{0}$. Then any affine function $f(\vec{x})$ is non-negative everywhere in $\mathcal{D}$ iff it is a positive affine combination:

$$f(\vec{x}) = \lambda_0 + \vec{\lambda}^T(A\vec{x}+\vec{b}), \text{ with } \lambda_0 \geq 0 \text{ and } \vec{\lambda} \geq \vec{0}.$$

λ_0 and $\vec{\lambda}^T$ are called the Farkas multipliers.

Example: Semantics Preservation (1-D)



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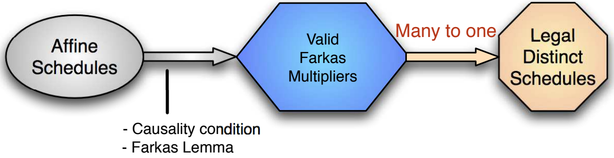
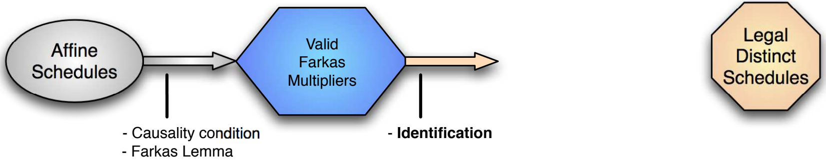


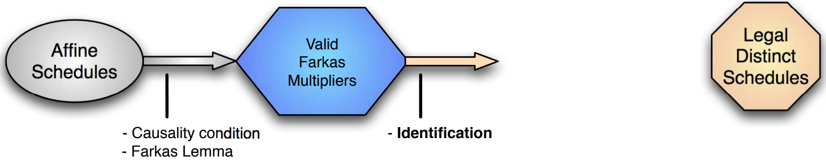
Example: Semantics Preservation (1-D)



$$\theta_S(\vec{x_S}) - \theta_R(\vec{x_R}) - 1 = \lambda_0 + \vec{\lambda}^T \left(D_{R,S} \begin{pmatrix} \vec{x_R} \\ \vec{x_S} \end{pmatrix} + \vec{d}_{R,S} \right) \geq 0$$

$$D_{R\delta S} \left\{ \begin{array}{lcl} \mathbf{i_R} & : & \lambda_{D_{1,1}} - \lambda_{D_{1,2}} + \lambda_{D_{1,3}} - \lambda_{D_{1,4}} \\ \mathbf{i_S} & : & -\lambda_{D_{1,1}} + \lambda_{D_{1,2}} + \lambda_{D_{1,5}} - \lambda_{D_{1,6}} \\ \mathbf{j_S} & : & \lambda_{D_{1,7}} - \lambda_{D_{1,8}} \\ \mathbf{n} & : & \lambda_{D_{1,4}} + \lambda_{D_{1,6}} + \lambda_{D_{1,8}} \\ \mathbf{1} & : & \lambda_{D_{1,0}} \end{array} \right.$$

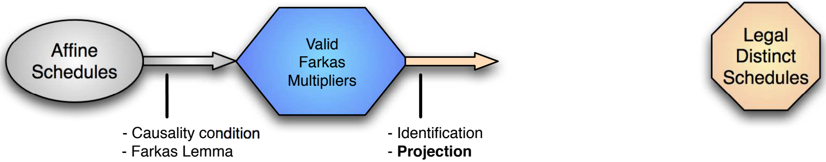
Example: Semantics Preservation (1-D)



$$\theta_S(\vec{\mathbf{x_S}}) - \theta_R(\vec{\mathbf{x_R}}) - 1 = \lambda_0 + \vec{\lambda}^T \left(D_{R,S} \begin{pmatrix} \vec{\mathbf{x_R}} \\ \vec{\mathbf{x_S}} \end{pmatrix} + \vec{d}_{R,S} \right) \geq 0$$

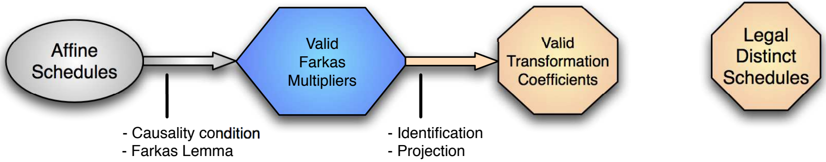
$$\left\{ \begin{array}{llcrcl} D_{R\delta S} & \mathbf{i_R} & : & -t_{1_R} & = & \lambda_{D_{1,1}} - \lambda_{D_{1,2}} + \lambda_{D_{1,3}} - \lambda_{D_{1,4}} \\ & \mathbf{i_S} & : & t_{1_S} & = & -\lambda_{D_{1,1}} + \lambda_{D_{1,2}} + \lambda_{D_{1,5}} - \lambda_{D_{1,6}} \\ & \mathbf{j_S} & : & t_{2_S} & = & \lambda_{D_{1,7}} - \lambda_{D_{1,8}} \\ & \mathbf{n} & : & t_{3_S} - t_{2_R} & = & \lambda_{D_{1,4}} + \lambda_{D_{1,6}} + \lambda_{D_{1,8}} \\ & \mathbf{1} & : & t_{4_S} - t_{3_R} - 1 & = & \lambda_{D_{1,0}} \end{array} \right.$$

Example: Semantics Preservation (1-D)

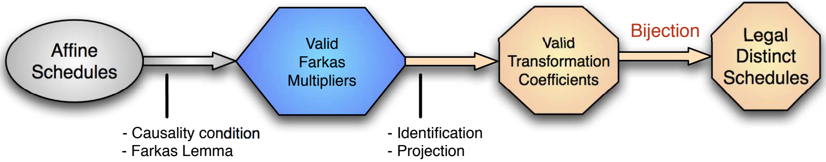


- Solve the constraint system
- Use (purpose-optimized) Fourier-Motzkin projection algorithm
  - Reduce redundancy
  - Detect implicit equalities

Example: Semantics Preservation (1-D)



Example: Semantics Preservation (1-D)



- One point in the space $\Leftrightarrow$ one set of legal schedules w.r.t. the dependences
- These conditions for semantics preservation are not new! [Feautrier,92]
- But never coupled with iterative search before

Generalization to Multidimensional Schedules

p-dimensional schedule **is not** $p \times 1$-dimensional schedule:

- Once a dependence is strongly satisfied ("loop"-carried), must be discarded in subsequent dimensions
- Until it is strongly satisfied, must be respected ("non-negative")

$\longrightarrow$ Combinatorial problem: lexicopositivity of dependence satisfaction

A solution:

- Encode dependence satisfaction with decision variables [Feautrier,92]

$$\Theta_k^S(\vec{x}_S) - \Theta_k^R(\vec{x}_R) \geq \delta, \quad \delta \in \{0,1\}$$

- Bound schedule coefficients, and nullify the precedence constraint when needed [Vasilache,07]

Legality as an Affine Constraint

Lemma (Convex form of semantics-preserving affine schedules)

Given a set of affine schedules $\Theta^R, \Theta^S \dots$ of dimension m, the program semantics is preserved if the three following conditions hold:

$$\begin{aligned}
&(i) && \forall \mathcal{D}_{R,S},\ \delta_p^{\mathcal{D}_{R,S}} \in \{0,1\} \\
&(ii) && \forall \mathcal{D}_{R,S},\ \sum_{p=1}^{m} \delta_p^{\mathcal{D}_{R,S}} = 1 && (1) \\
&(iii) && \forall \mathcal{D}_{R,S},\ \forall p \in \{1,\dots,m\},\ \forall \langle \vec{x}_R, \vec{x}_S \rangle \in \mathcal{D}_{R,S}, && (2) \\
& && \Theta_p^S(\vec{x}_S) - \Theta_p^R(\vec{x}_R) \geq -\sum_{k=1}^{p-1} \delta_k^{\mathcal{D}_{R,S}}.(K.\vec{n}+K) + \delta_p^{\mathcal{D}_{R,S}}
\end{aligned}$$

$\longrightarrow$ Note: schedule coefficients must be bounded for Lemma to hold

$\longrightarrow$ Severe scalability challenge for large programs

Search Space Construction and Evaluation

Objectives for the Search Space Construction

- Provide **scalable** techniques to construct the search space
- **Adapt** the space construction to the machine specifics (esp. parallelism)
- Search space is infinite: requires appropriate **bounding**
- **Expressiveness:** allow for a rich set of transformations sequences
- Compiler optimization heuristics are fragile, manage it!

Overview of the Proposed Approach

1. Build a convex set of candidate program versions
   - Affine set of schedule coefficients
   - Enforce legality and uniqueness as affine constraints
2. Shape this set to a form which allows an efficient traversal
   - Redundancy-less Fourier-Motzkin elimination algorithm
   - Force FM-property by applying Fourier-Motzkin elim. on the set
3. Traverse the set
   - Exhaustively, for performance analysis
   - Heuristically, for scalability

Search Space Construction

Principle: Feautrier's + coefficient bounding

Output: 1 **independent polytope per schedule dimension**

Algorithm

Init: Set all dependencies as unresolved

1. $k = 1$
2. *Set $\mathcal{T}_k$ as the* ***polytope*** *of valid schedules with all unresolved dependencies weakly satisfied (i.e., set $\delta = 0$)*
3. *For each unresolved dependence $\mathcal{D}_{R,S}$:*
   1. *build $\mathcal{S}_{\mathcal{D}_{R,S}}$ the set of schedules strongly satisfying $\mathcal{D}_{R,S}$ (i.e., set $\delta = 1$)*
   2. $\mathcal{T}_k^{'} = \mathcal{T}_k \bigcap \mathcal{S}_{\mathcal{D}_{R,S}}$
   3. *if $\mathcal{T}_k^{'} \neq \emptyset$, $\mathcal{T}_k = \mathcal{T}_k^{'}$. Mark $\mathcal{D}_{R,S}$ as resolved*
4. *If unresolved dependence remains, increment k and go to 1*

Some Properties of the Algorithm

- Without bounding, equivalent to Feautrier's genuine scheduling algorithm
- With bounding, **sensitive to the dependence traversal order**
  - Heuristics to select the dependence order: pairwise interference, traffic ranking, etc.
  - May also search for different orders
- May not minimize the schedule dimensionality
- **Outer dimensions** (i.e., outer loops) **are more constrained**
- Inner dimensions tend to be parallel, if possible (SIMD friendly)

Search Space Size

- Bound each coefficient between $[-1, 1]$ to avoid complex control overhead and drive the search

Benchmark	#Inst.	#Dep.	#Dim.	dim 1	dim 2	dim 3	dim 4	Total
compress	6	56	3	20	136	10857025	n/a	2.9×10^{10}
edge	3	30	4	27	54	90534	43046721	5.6×10^{15}
iir	8	66	3	18	6984	$> 10^{15}$	n/a	$> 10^{19}$
fir	4	36	2	18	52953	n/a	n/a	9.5×10^{7}
lmsfir	9	112	2	27	10534223	n/a	n/a	2.8×10^{8}
mult	3	27	3	9	27	3295	n/a	8.0×10^{5}
latnrm	11	75	3	9	1896502	$> 10^{15}$	n/a	$> 10^{22}$
lpc-LPC_analysis	12	85	2	63594	$> 10^{20}$	n/a	n/a	$> 10^{25}$
ludcmp	14	187	3	36	$> 10^{20}$	$> 10^{25}$	n/a	$> 10^{46}$
radar	17	153	3	400	$> 10^{20}$	$> 10^{25}$	n/a	$> 10^{48}$

Figure: Search Space Statistics

Performance Distribution for 1-D Schedules [1/2]

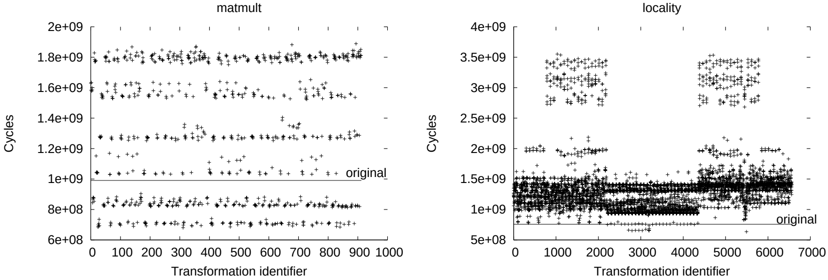


Figure: Performance distribution for `matmult` and `locality`

Performance Distribution for 1-D Schedules [2/2]

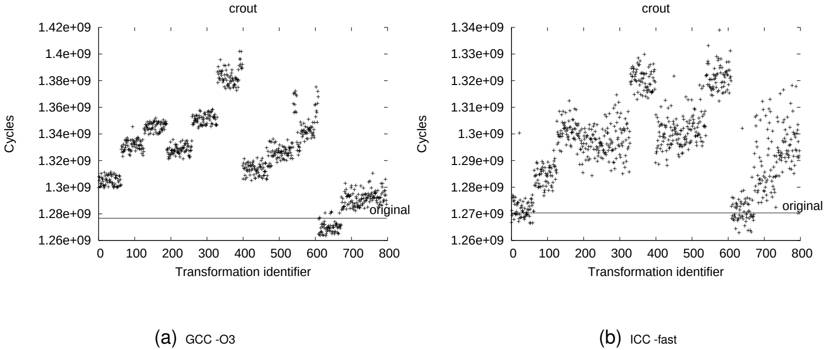


Figure: The effect of the compiler

Quantitative Analysis: The Hypothesis

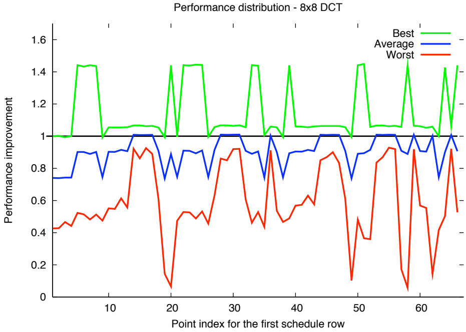
Extremely large generated spaces: $> 10^{50}$ points

$\longrightarrow$ we must leverage static and dynamic characteristics to build traversal mechanisms

Hypothesis: [Pouchet et al,SMART08]

- **It is possible to statically order the impact on performance of transformation coefficients, that is, decompose the search space in subspaces where the performance variation is maximal or reduced**
- **First rows of Θ are more performance impacting than the last ones**

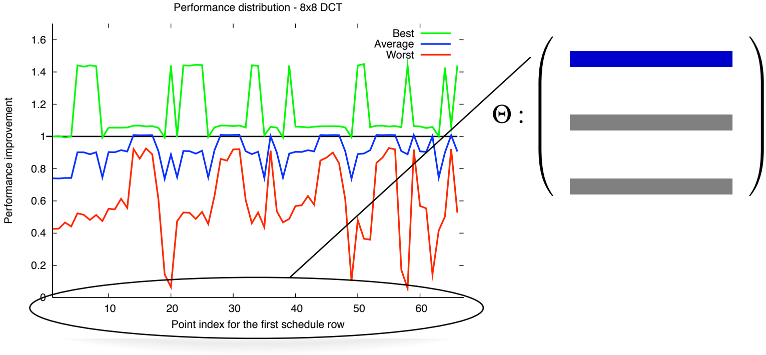
Observations on the Performance Distribution




```
for (i = 0; i < M; i++)
  for (j = 0; j < M; j++) {
    tmp[i][j] = 0.0;
    for (k = 0; k < M; k++)
      tmp[i][j] += block[i][k] *
                   cos1[j][k];
 }
for (i = 0; i < M; i++)
  for (j = 0; j < M; j++) {
    sum2 = 0.0;
    for (k = 0; k < M; k++)
      sum2 += cos1[i][k] * tmp[k][j];
      block[i][j] = ROUND(sum2);
 }
```

- Extensive study of 8x8 Discrete Cosine Transform (UTDSP)
- Search space analyzed: $66 \times 19683 = 1.29 \times 10^6$ different legal program versions

Observations on the Performance Distribution



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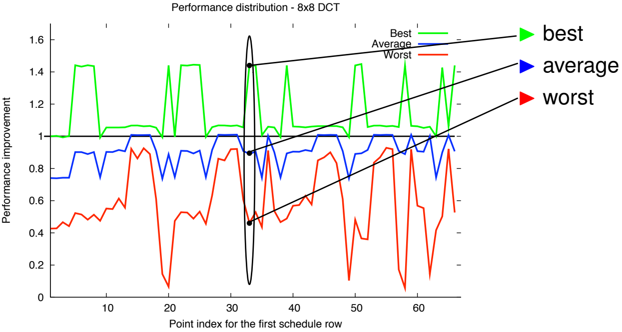
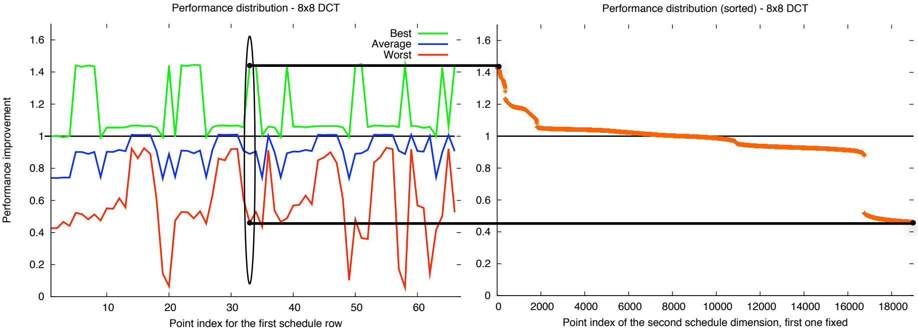


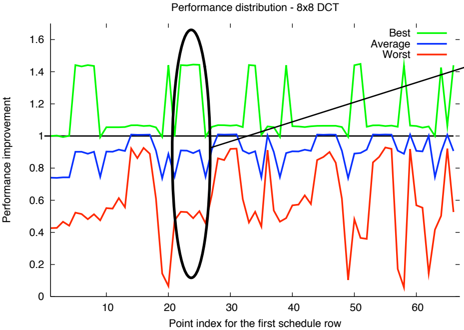
- Take one specific value for the first row
- Try the 19863 possible values for the second row

Observations on the Performance Distribution



- Take one specific value for the first row
- Try the 19863 possible values for the second row
- Very low proportion of best points: $< 0.02\%$

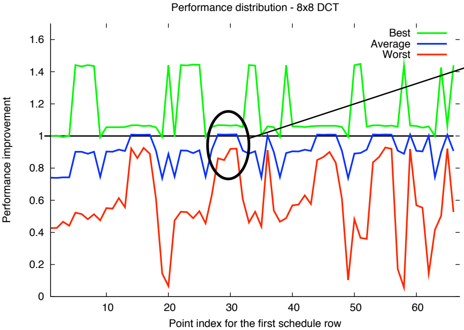
Observations on the Performance Distribution



Large **performance variation**

- Performance variation is large for good values of the first row

Observations on the Performance Distribution



Small **performance variation**

- Performance variation is large for good values of the first row
- It is usually reduced for bad values of the first row

Scanning The Space of Program Versions

The search space:

- Performance **variation** indicates to partition the space: $\vec{\imath} > \vec{p} > c$
- **Non-uniform distribution of performance**
- No clear analytical property of the optimization function

$\longrightarrow$ Build dedicated **heuristic** and **genetic operators** aware of these **static and dynamic characteristics**

Search Space Traversal

Objectives for Efficient Traversal

Main goals:

- Enable feedback-directed search
- **Focus the search on interesting subspaces**

Provide mechanisms to decouple the traversal:

- Leverage our knowledge on the performance distribution
- Leverage static properties of the search space
- Completion mechanism, to instantiate a full schedule from a partial one
- Traversal heuristics adapted to the problem complexity
  - **Decoupling heuristic:** explore first iterator coefficients (deterministic)
  - **Genetic algorithm:** improve further scalability (non-deterministic)

Some Results for 1-D Schedules

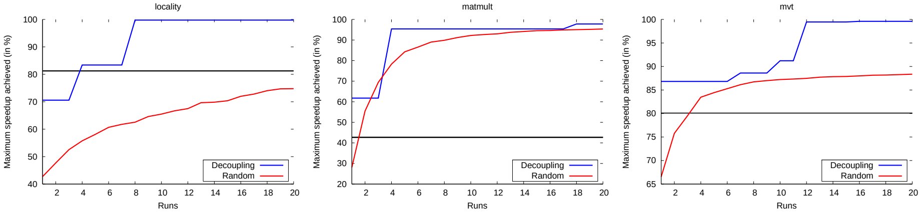
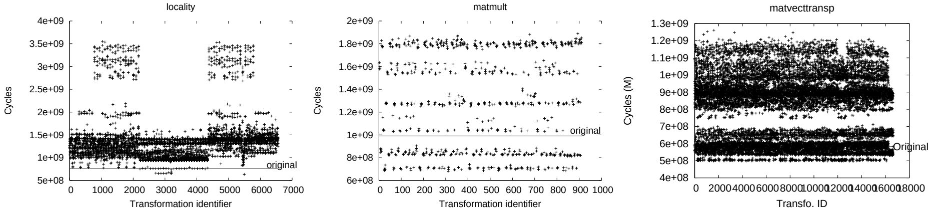


Figure: Comparison between random and decoupling heuristics



Inserting Randomness in the Search

About the performance distribution:

- The performance distribution is not uniform
- Wild jump in the space: tune $\vec{\imath}$ coefficients of upper dimensions
- Refinement: tune $\vec{p}$ and $\vec{c}$ coefficients

About the space of schedules:

- Highly constrained: **small change in $\vec{\imath}$ may alter many other coefficients**
- Rows are independent: no inter-dimension constraint
- Some transformations (e.g., interchange) must **operate between rows**

Genetic Operators

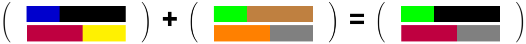
Mutation

- Probability varies along with evolution
- Tailored to focus on the most promising subspaces
- Preserves legality (closed under affine constraints)

Cross-over

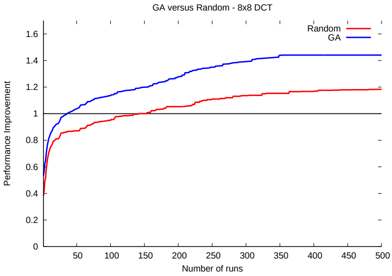
- Row cross-over

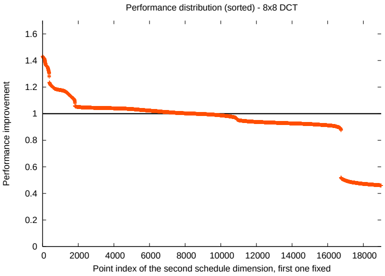
- Column cross-over



- Both preserve legality

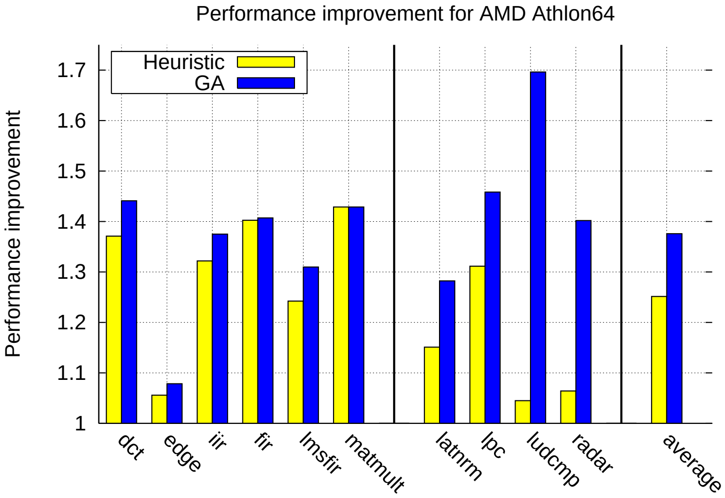
Dedicated GA Results





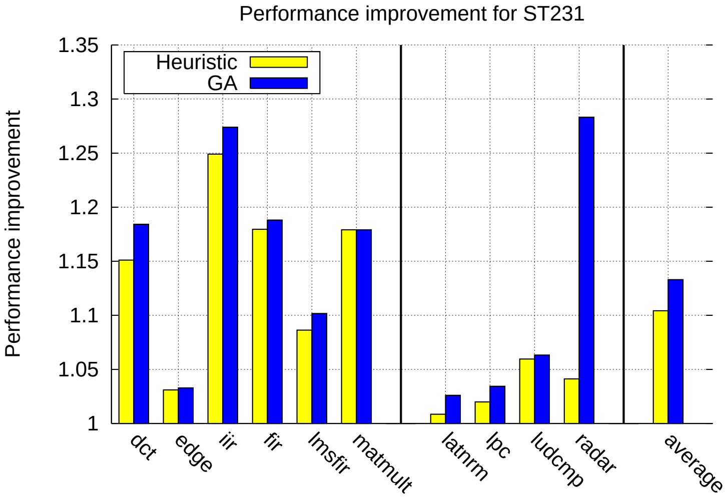
- GA converges towards the maximal space speedup

Experimental Results [1/2]



baseline: gcc -O3 -ftree-vectorize -msse2

Experimental Results [2/2]



baseline: st200cc -O3 -OPT:alias=restrict -mauto-prefetch

Assessments from Experimental Results

Looking into details (hardware counters+compilation trace):

- **Better activity** of the processing units
- Best version may **vary significantly for different architectures**
- Different source code may **trigger different compiler optimizations**

$\longrightarrow$ **Portability of the optimization process validated w.r.t. architecture/compiler**

Assessments from Experimental Results

Looking into details (hardware counters+compilation trace):

- **Better activity** of the processing units
- Best version may **vary significantly for different architectures**
- Different source code may **trigger different compiler optimizations**

$\longrightarrow$ **Portability of the optimization process validated w.r.t. architecture/compiler**

- Limitation: poor compatibility with coarse-grain parallelism

  Can we reconcile tiling, parallelization, SIMD and iterative search?

Multidimensional Interleaving Selection

Overview of the Problem

Objectives:

- Achieve efficient coarse-grain parallelization
- Combine iterative search of profitable transformations for tiling
  - $\longrightarrow$ loop fusion and loop distribution

Existing framework: tiling hyperplane [Bondhugula,08]

- Model-driven approach for automatic parallelization + locality improvement
- Tiling-oriented
- Poor model-driven heuristic for the selection of loop fusion (<u>not portable</u>)
- Overly relaxed definition of fused statements

Our Strategy in a Nutshell...

1. Introduce the concept of **fusability**
2. Introduce a modeling for arbitrary loop fusion/distribution combinations
   1. Equivalence 1-d interleaving with total preorders
   2. **Affine encoding of total preorders**
   3. Generalization to multidimensional interleavings
   4. Pruning technique to keep only semantics-preserving ones
3. Design a **mixed iterative and model-driven algorithm** to build optimizing transformations

Fusability of Statements

- Fusion $\Leftrightarrow$ interleaving of statement **instances**
- Two statements are fused if their timestamp overlap

$$\Theta_k^R(\vec{x_R}) \leq \Theta_k^S(\vec{x_S}) \wedge \Theta_k^S(\vec{x_S}') \leq \Theta_k^R(\vec{x_R}')$$

- Better approach: at most c instances are not fused (approximation)

Definition (Fusability restricted to non-negative schedule coefficients)

Given two statements R, S such that R is surrounded by d^R loops, and S by d^S loops. They are fusable at level p if, $\forall k \in \{1 \ldots p\}$, there exists two semantics-preserving schedules Θ_k^R and Θ_k^S such that:

$$\begin{array}{ll} (i) & \forall k \in \{1, \ldots, p\}, \quad -c < \Theta_k^R(\vec{0}) - \Theta_k^S(\vec{0}) < c \\ (ii) & \displaystyle\sum_{i=1}^{d^R} \theta_{k,i}^R > 0, \ \sum_{i=1}^{d^S} \theta_{k,i}^S > 0 \end{array}$$

Exact solution is hard: may require Ehrart polynomials for general case

Affine Encoding of Total Preorders

Principle: [Pouchet,PhD10]

- Model a total preorder with 3 binary variables

  $p_{i,j} : i < j \qquad s_{i,j} : i > j \qquad e_{i,j} : i = j$
- Enforce totality and mutual exclusion
- Enforce all cases of transitivity through affine inequalities connecting some variables. Ex: $e_{i,j} = 1 \wedge e_{j,k} = 1 \Rightarrow e_{i,k} = 1$

$$O = \left\{ \begin{array}{c} 0 \le p_{i,j} \le 1 \\ 0 \le e_{i,j} \le 1 \\ 0 \le s_{i,j} \le 1 \end{array} \right\} \quad \text{constrained to:} \quad O = \left\{ \begin{array}{lrl} & \left.\begin{array}{r} 0 \le p_{i,j} \le 1 \\ 0 \le e_{i,j} \le 1 \end{array}\right\} & \text{Variables are binary} \\ & \left.\begin{array}{r} p_{i,j} + e_{i,j} \le 1 \end{array}\right\} & \text{Relaxed mutual exclusion} \\ \forall k \in]j,n] & \left.\begin{array}{r} e_{i,j} + e_{i,k} \le 1 + e_{j,k} \\ e_{i,j} + e_{j,k} \le 1 + e_{i,k} \end{array}\right\} & \text{Basic transitivity on } e \\ \forall k \in]i,j[& \left.\begin{array}{r} p_{i,k} + p_{k,j} \le 1 + p_{i,j} \end{array}\right\} & \text{Basic transitivity on } p \\ \begin{array}{l} \forall k \in]j,n] \\ \\ \forall k \in]i,j[\end{array} & \left.\begin{array}{r} e_{i,j} + p_{i,k} \le 1 + p_{j,k} \\ e_{i,j} + p_{j,k} \le 1 + p_{i,k} \\ e_{k,j} + p_{i,k} \le 1 + p_{i,j} \end{array}\right\} & \text{Complex transitivity on } p \text{ and } e \\ \forall k \in]j,n] & \left.\begin{array}{r} e_{i,j} + p_{i,j} + p_{j,k} \le 1 + p_{i,k} + e_{i,k} \end{array}\right\} & \text{Complex transitivity on } s \text{ and } p \end{array} \right.$$

Search Space Statistics

Pruning for semantics preservation ($\mathcal{F}$):

- Start from all total preorders ($\mathcal{O}$)
- Prove when fusability is a transitive relation: equivalent to checking the existence of **pairwise compatible loop permutations**
- Check graph of compatible permutations to determine fusable sets, prune $\mathcal{O}$ from non-fusable ones

			$\mathcal{O}$			$\mathcal{F}^1$				
Benchmark	#loops	#refs	#dim	#cst	#points	#dim	#cst	#points	#Tested	Time
advect3d	12	32	12	58	75	9	43	26	52	0.82s
atax	4	10	12	58	75	6	25	16	32	0.06s
bicg	3	10	12	58	75	10	52	26	52	0.05s
gemver	7	19	12	58	75	6	28	8	16	0.06s
ludcmp	9	**35**	**182**	**3003**	$\approx 10^{12}$	**40**	**443**	**8**	16	**0.54s**
doitgen	5	7	6	22	13	3	10	4	8	0.08s
varcovar	7	26	42	350	**47293**	22	193	**96**	192	**0.09s**
correl	5	12	30	215	4683	21	162	176	352	0.09s

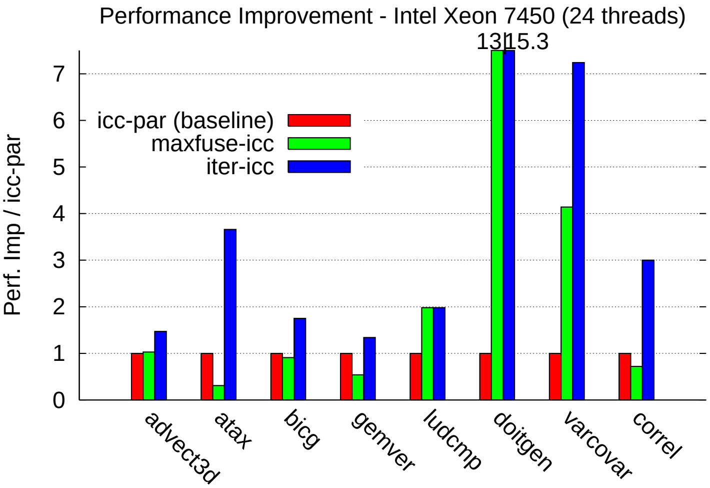
Figure: Search space statistics

Optimization Algorithm

- Proceeds **level-by-level**
- Starting from the outer-most level, **iteratively select an interleaving**
- For this interleaving, compute an optimization which respects it
  - Compound of skewing, shifting, fusion, distribution, interchange, tiling and parallelization (OpenMP)
  - **Maximize locality** for each partition of statements

- **Automatically adapt to the target architecture**
- Solid improvement over existing model-driven approach
- Up to 150$\times$ speedup on 24 cores, 15$\times$ speedup over autopll compiler

Performance Results for Intel Xeon 24-cores



baseline: ICC 11.0 -fast -parallel -fopenmp

Conclusions and Future Work

Summary of Contributions

We have designed, built and experimented **all required blocks to perform an efficient iterative selection of fine-grain loop transformations** in the polyhedral model.

- Theoretically sound and practical iterative optimization algorithms
  - Significant increase in expressiveness of iterative techniques
  - Well-designed (but complex) problems
  - Extensive experimental analysis of the performance distribution
  - Subspace-driven traversal techniques for polytopes
- Theoretical framework for generalized fusion
- Practical solution for machine-dependent parallelization + vectorization + locality
- Implementation in publicly available tools: PoCC, LetSee, FM, etc.

Future Work: Machine Learning

Machine Learning could improve the scalability:

- Currently, no reuse from previous compilation / space traversal
- Efficiency proved on (simpler) compilation problems

Main issues:

- Fine-grain vs. coarse-grain optimization
- **Knowledge representation**
- Features for similarity computation

Take-Home Message

Iterative Optimization: the last hope, or a new hope?

- **Efficient, more expressive and portable mechanisms can be built**
- **The polyhedral representation is adaptable to iterative compilation**
- Performance-demanding programmers can afford long compilation time
- Still require to execute different codes: not always possible
- Downside of polyhedral expressiveness: algorithmic complexity

Questions:

- Can we increase the accuracy of static models, given the complexity of modern compilers and chips?
- Can we systematically reach the performance of hand-tuned code with an automatic approach?

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Thank you!

Supplementary Slides

Yet Another Completion Algorithm

Principle: [Pouchet et al,PLDI08]

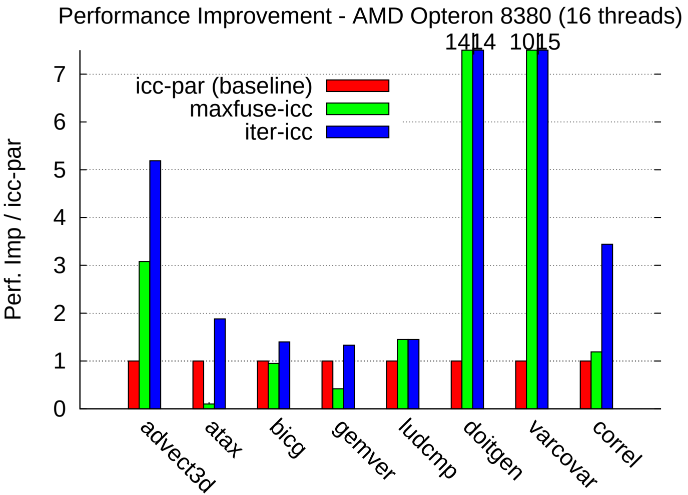
- Rely on a pre-pass to normalize the space (improved full polytope projection)
- Works in polynomial time w.r.t. the number of constraints in the normalized space

See also [Li et al,IJPP94] [Griebl,PACT98] [Vasilache,PACT07]...

Three fundamental properties:

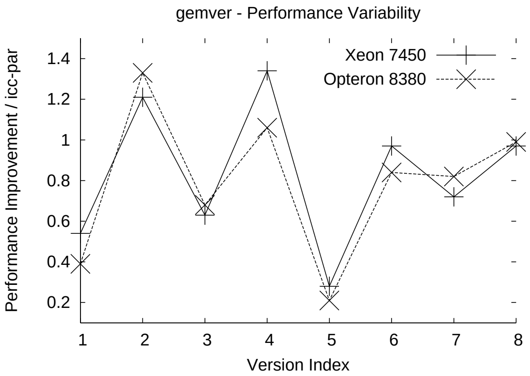
1. If $v_1, \ldots, v_k$ is a prefix of a legal point v, a completion is always found
2. This completion will only update $v_{k+1}, \ldots, v_{d_{\text{max}}}$, if needed;
3. When $v_1, \ldots, v_k$ are the $\vec{\imath}$ coefficients, the heuristic looks for the smallest absolute value for the $\vec{p}$ and c coefficients

Performance Results for AMD Opteron 16-cores



baseline: ICC 11.0 -fast -parallel -fopenmp

Variability for GEMVER



Future Work: Knowledge Transfer

Current approach:

- Training: 1 program $\longrightarrow$ 1 effective transformation
- On-line: Compute similarities with existing program, apply the same transformation

$\longrightarrow$ Does not work well for fine-grain optimization

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Proposed approach:

- Don't care about the sequence, only about properties of the schedule (parallelism degree, locality, etc.)
- Learn how to prioritize **performance anomaly** solving instead
- Rely on the polyhedral model to compute a matching optimization
- Some open problems:
  - How to compute (polyhedral) features? They are parametric
  - How to compute the optimization (combinatorial decision problem)?