INTRODUCTION TO DIGITAL SYSTEMS

- DESCRIPTION AND DESIGN OF DIGITAL SYSTEMS
- FORMAL BASIS: SWITCHING ALGEBRA
- IMPLEMENTATION: MODULES (ICs) AND NETWORKS
- IMPLEMENTATION OF ALGORITHMS IN “HARDWARE”
- COURSE EMPHASIS: CONCEPTS, ANALYSIS AND DESIGN

- Follow-on courses:
  - Digital Lab
  - Computer Architecture
  - Computer Architecture Lab
  - Digital Design - Advanced Topics
OVERVIEW

- WHAT IS A DIGITAL SYSTEM?
- HOW IT DIFFERS FROM AN ANALOG SYSTEM?
- WHY ARE DIGITAL SYSTEMS IMPORTANT?
- BASIC TYPES OF DIGITAL SYSTEMS: COMBINATIONAL AND SEQUENTIAL
- SPECIFICATION AND IMPLEMENTATION OF DIGITAL SYSTEMS
- ANALYSIS AND DESIGN OF DIGITAL SYSTEMS
- DESIGN PROCESS AND CAD TOOLS
WHAT IS DIGITAL

DIGITAL SYSTEMS
   – inputs and outputs:
     finite number of discrete values

ANALOG SYSTEMS
   – inputs and output values
     from a continuous (infinite) set

Example: digital vs. analog scale for measuring weights

MAIN USE OF DIGITAL SYSTEMS:
• INFORMATION PROCESSING (text, audio, visual, video)
• TRANSMISSION (communication)
• STORAGE
Figure 1.1: System S: a) Block diagram. b) Analog I/O signals. c) Digital I/O signals. d) I/O sequence pair.
1. FOR BOTH NUMERICAL AND NONNUMERICAL INFORMATION PROCESSING

2. INFORMATION PROCESSING CAN USE A GENERAL-PURPOSE SYSTEM (a computer)

3. DIGITAL REPRESENTATION:
   - vector of signals with just two values (binary signals)

Example:

<table>
<thead>
<tr>
<th>digit</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>vector</td>
<td>0000</td>
<td>0001</td>
<td>0010</td>
<td>0011</td>
<td>0100</td>
<td>0101</td>
<td>0110</td>
<td>0111</td>
<td>1000</td>
<td>1001</td>
</tr>
</tbody>
</table>

- All signals binary
- Simple devices to process binary signals:
  (SWITCHES with two STATES: open and closed).
4. DIGITAL SIGNALS INSENSITIVE TO VARIATIONS OF COMPONENT PARAMETER VALUES

![Figure 1.2: Separation of digital signal values.](image-url)
5. Numerical digital systems can be made MORE ACCURATE by simply increasing the number of digits used in the representation.

6. PHENOMENAL ADVANCES OF MICROELECTRONICS TECHNOLOGY:

   - Possible to fabricate extremely complex digital systems, which are small, fast, and cheap

   - Digital systems built as *integrated circuits* composed of a large number of very simple devices
7. DIFFERENT IMPLEMENTATIONS OF SYSTEMS WHICH TRADE-OFF SPEED AND AMOUNT OF HARDWARE (COST)

Example:
– add two integers represented by six decimal digits

Figure 1.3: Six-digit adder: a) Parallel implementation. b) Serial implementation.
SUMMARY

• DIGITAL REPRESENTATION AND PROCESSING METHODS WIDELY USED

• EXTRAORDINARY PROGRESS IN DIGITAL TECHNOLOGY AND USE

• INDISPENSABLE IN MODERN SOCIETY

• NEW APPLICATIONS FUELED BY THE DEVELOPMENT OF COMPUTER TECHNOLOGY

• KNOWLEDGE ABOUT THE DESIGN AND USE OF DIGITAL SYSTEMS REQUIRED IN A LARGE VARIETY OF HUMAN ACTIVITIES
Figure 1.4: a) A system with analog and digital signals. b) Analog-to-digital conversion.
• DIGITAL SYSTEMS - TWO CLASSES:

• **COMBINATIONAL SYSTEMS**

\[ z(t) = F(x(t)) \]

– no memory, the output does not depend on previous inputs

• **SEQUENTIAL SYSTEMS**

\[ z(t) = F(x(0, t)) \]

\( x(0, t) \): input sequence from time 0 to time \( t \)

– \( z(t) \) depends also on previous inputs - the system has MEMORY
Figure 1.5: Input-output functions for: a) Combinational system; b) Sequential system.
EXAMPLE 1.1: SEQUENTIAL SYSTEM

- INPUT $x$ with VALUES 0, 1, or 2
- OUTPUT $z$ with VALUES 0 or 1
- FUNCTION:
  
  $z(t) = \begin{cases} 
  1 & \text{if} \ (x(0), x(1), \ldots, x(t)) \text{ has even 2's and odd 1's} \\
  0 & \text{otherwise} 
  \end{cases}$

- AN INPUT-OUTPUT PAIR:

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$z$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
EXAMPLE 1.2: COMBINATIONAL SYSTEM

- INPUT $x(t)$ with values from the set of letters (upper and lower case)
- INPUT $y(t)$ with values 0 and 1
- FUNCTION:
  - change $x(t)$ to opposite case when $y(t) = 1$
  - leave it unchanged when $y(t) = 0$
- AN INPUT-OUTPUT PAIR:

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>E</td>
<td>X</td>
<td>A</td>
<td>M</td>
<td>P</td>
<td>L</td>
<td>E</td>
</tr>
<tr>
<td>$y$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$z$</td>
<td>E</td>
<td>x</td>
<td>A</td>
<td>M</td>
<td>P</td>
<td>I</td>
<td>E</td>
</tr>
</tbody>
</table>
Figure 1.6: Relationship among system specification and implementation.
SPECIFICATION of a system describes its function.

Objective:

- to use the system as a component in more complex systems; and

- to serve as the basis for the implementation of the system by a network of simpler components.
SPECIFICATION LEVELS

- HIGH-LEVEL
- BINARY-LEVEL
- ALGORITHMIC-LEVEL

- Spec. of combinational systems: Chapter 2
- Spec. of sequential systems: Chapter 7
- Spec. of algorithmic systems: Chapter 13
IMPLEMENTATION

As a *DIGITAL NETWORK* – interconnection of modules

- SEVERAL LEVELS depending on the complexity of the primitive modules
  - from very simple *gates* to complex *processors*
- Need for *HIERARCHICAL IMPLEMENTATION*

- PHYSICAL LEVEL: interconnection of electronic elements such as transistors, resistors, and so on (Chapter 3).

- IMPLEMENTATION OF COMBINATIONAL SYSTEMS
  - at the gate level: Chapters 5 and 6
  - at the module level: Chapters 9, 10, and 12

- IMPLEMENTATION OF SEQUENTIAL SYSTEMS
  - elementary: Chapter 8
  - more complex: Chapters 11 and 12

- IMPLEMENTATION OF ALGORITHMIC SYSTEMS: Chapters 13-15
Figure 1.7: Hierarchical implementation: a) Top-down approach. b) Bottom-up approach.
● **ANALYSIS:**
  * get specification from an implementation

● **DESIGN:**
  * obtain an implementation that satisfies the specification

● Use of MULTILEVEL APPROACH necessary

● The TOP-DOWN and BOTTOM-UP approaches

● A combination of the two approaches
Example: \( Z(t) = \sum_{i=0}^{t} X(i) \)

Figure 1.8: Digital system: a) module level; b) logical level; and c) physical level.
Design of digital systems an involved and laborious process

Various computer-aided design (CAD) tools available

Main types of CAD tools support the main phases of digital design:

(i) description (specification),

(ii) design (synthesis) including various optimizations to reduce cost and improve performance, and

(iii) checking of the design with respect to its specification.

The design phases typically require several passes
DESCRIPTION of digital systems for design purposes

- At a high-level, use hardware-description language (HDL)
- At the binary level, use HDLs to describe the system structure
- Editors used to produce HDL programs
- Graphical forms - *logic diagrams* also used for structure
USE WORK.ALL;
ENTITY sample_system IS
  PORT (xin: IN BIT_VECTOR;
        z : OUT BIT_VECTOR;
        clk: IN BIT);
END sample_system;

ARCHITECTURE structural OF sample_system IS
  SIGNAL xreg, yreg, addout: BIT_VECTOR(7 DOWNTO 0);
BEGIN
  RX: ENTITY BitReg8 PORT MAP(xin,xreg,clk);
  RY: ENTITY BitReg8 PORT MAP(addout,yreg,clk);
  ADD: ENTITY Adder PORT MAP(xreg,yreg,addout);
      z <= addout;
END structural;

Figure 1.9: $\mu$VHDL-based description of a system.
SYNTHESIS AND OPTIMIZATION
   – Semi-automated

SIMULATION tools generate behavior of a system for given input
   – Logic simulation
   – Timing simulation