implementation may be by linking together fixed size buckets using overflow chains. Deletion is difficult with open hashing as *all* the buckets may have to inspected before we can ascertain that a key value has been deleted, whereas in closed hashing only that bucket whose address is obtained by hashing the key value need be inspected. Deletions are more common in databases and hence closed hashing is more appropriate for them. For a small, static set of data lookups may be more efficient using open hashing. The symbol table of a compiler would be a good example.

12.11 What are the causes of bucket overflow in a hash file organization? What can be done to reduce the occurrence of bucket overflows?
Answer: The causes of bucket overflow are :-

- **a.** Our estimate of the number of records that the relation will have was too low, and hence the number of buckets allotted was not sufficient.
- **b.** Skew in the distribution of records to buckets. This may happen either because there are many records with the same search key value, or because the the hash function chosen did not have the desirable properties of uniformity and randomness.

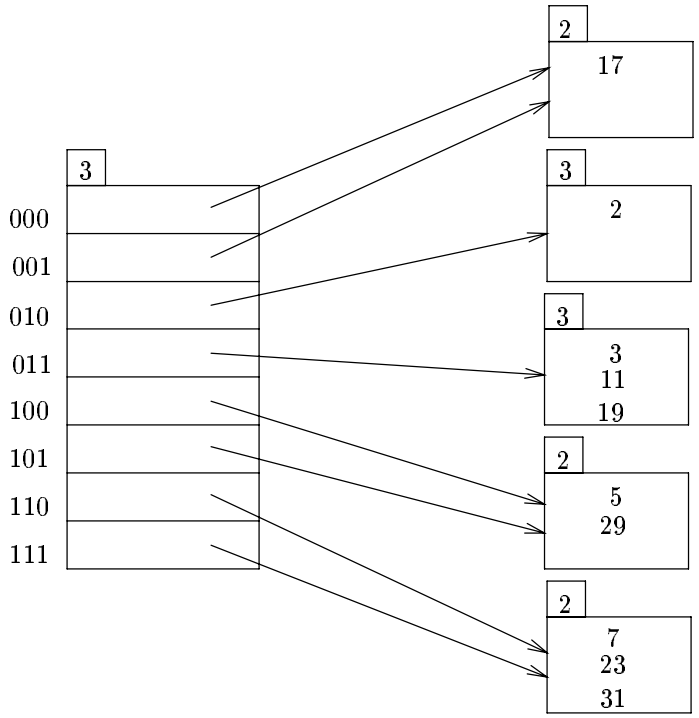
To reduce the occurrence of overflows, we can :-

- **a.** Choose the hash function more carefully, and make better estimates of the relation size.
- **b.** If the estimated size of the relation is n_r and number of records per block is f_r, allocate $(n_r/f_r) * (1 + d)$ buckets instead of (n_r/f_r) buckets. Here d is a fudge factor, typically around 0.2. Some space is wasted: About 20 percent of the space in the buckets will be empty. But the benefit is that some of the skew is handled and the probability of overflow is reduced.

12.12 Suppose that we are using extendable hashing on a file that contains records with the following search-key values:

2, 3, 5, 7, 11, 17, 19, 23, 29, 31

Show the extendable hash structure for this file if the hash function is $h(x) = x$ mod 8 and buckets can hold three records.
Answer:

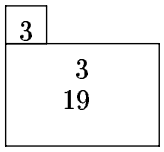


12.13 Show how the extendable hash structure of Exercise 12.12 changes as the result of each of the following steps:

a. Delete 11.
b. Delete 31.
c. Insert 1.
d. Insert 15.

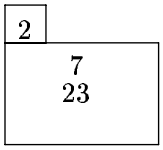
Answer:

a. Delete 11: From the answer to Exercise 12.12, change the third bucket to:

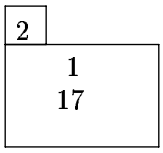


At this stage, it is possible to coalesce the second and third buckets. Then it is enough if the bucket address table has just four entries instead of eight. For the purpose of this answer, we do not do the coalescing.

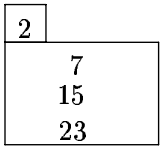
b. Delete 31: From the answer to 12.12, change the last bucket to:



c. Insert 1: From the answer to 12.12, change the first bucket to:



d. Insert 15: From the answer to 12.12, change the last bucket to:



12.14 Give pseudocode for deletion of entries from an extendable hash structure, including details of when and how to coalesce buckets. Do not bother about reducing the size of the bucket address table.
Answer: Let i denote the number of bits of the hash value used in the hash table. Let **BSIZE** denote the maximum capacity of each bucket.

```
delete(value K_l)
begin
    j = first i high-order bits of h(K_l);
    delete value K_l from bucket j;
    coalesce(bucket j);
end

coalesce(bucket j)
begin
    i_j = bits used in bucket j;
    k = any bucket with first (i_j − 1) bits same as that
        of bucket j while the bit i_j is reversed;
    i_k = bits used in bucket k;
    if(i_j ≠ i_k)
        return; /* buckets cannot be merged */
    if(entries in j + entries in k > BSIZE)
        return; /* buckets cannot be merged */
    move entries of bucket k into bucket j;

    decrease the value of i_j by 1;
    make all the bucket-address-table entries,
    which pointed to bucket k, point to j;

    coalesce(bucket j);
end
```

Note that we can only merge two buckets at a time. The common hash prefix of the resultant bucket will have length one less than the two buckets merged. Hence we look at the buddy bucket of bucket j differing from it only at the last bit. If the common hash prefix of this bucket is not i_j, then this implies that the buddy bucket has been further split and merge is not possible.

When merge is successful, further merging may be possible, which is handled by a recursive call to *coalesce* at the end of the function.

12.15 Suggest an efficient way to test if the bucket address table in extendable hashing can be reduced in size, by storing an extra count with the bucket address table. Give details of how the count should be maintained when buckets are split, coalesced or deleted.

(Note: Reducing the size of the bucket address table is an expensive operation, and subsequent inserts may cause the table to grow again. Therefore, it is best not to reduce the size as soon as it is possible to do so, but instead do it only if the number of index entries becomes small compared to the bucket address table size.)

Answer: If the hash table is currently using i bits of the hash value, then maintain a count of buckets for which the length of common hash prefix is exactly i.

Consider a bucket j with length of common hash prefix i_j. If the bucket is being split, and i_j is equal to i, then reset the count to 1. If the bucket is being split and i_j is one less that i, then increase the count by 1. It the bucket if being coalesced, and i_j is equal to i then decrease the count by 1. If the count becomes 0, then the bucket address table can be reduced in size at that point.

However, note that if the bucket address table is not reduced at that point, then the count has no significance afterwards. If we want to postpone the reduction, we have to keep an array of counts, i.e. a count for each value of common hash prefix. The array has to be updated in a similar fashion. The bucket address table can be reduced if the i^{th} entry of the array is 0, where i is the number of bits the table is using. Since bucket table reduction is an expensive operation, it is not always advisable to reduce the table. It should be reduced only when sufficient number of entries at the end of count array become 0.

12.16 Why is a hash structure not the best choice for a search key on which range queries are likely?

Answer: A range query cannot be answered efficiently using a hash index, we will have to read all the buckets. This is because key values in the range do not occupy consecutive locations in the buckets, they are distributed uniformly and randomly throughout all the buckets.

12.17 Consider a grid file in which we wish to avoid overflow buckets for performance reasons. In cases where an overflow bucket would be needed, we instead reorganize the grid file. Present an algorithm for such a reorganization.

Answer: Let us consider a two-dimensional grid array. When a bucket overflows, we can split the ranges corresponding to that row and column into two, in both the linear scales. Thus the linear scales will get one additional entry each, and the bucket is split into four buckets. The ranges should be split in such a way as to ensure that the four resultant buckets have nearly the same number of values.

There can be several other heuristics for deciding how to reorganize the ranges, and hence the linear scales and grid array.

12.18 Consider the *account* relation shown in Figure 12.25.

- **a.** Construct a bitmap index on the attributes *branch-name* and *balance*, dividing *balance* values into 4 ranges: below 250, 250 to below 500, 500 to below 750, and 750 and above.
- **b.** Consider a query that requests all accounts in Downtown with a balance of 500 or more. Outline the steps in answering the query, and show the final and intermediate bitmaps constructed to answer the query.

Answer: We reproduce the account relation of Figure 12.25 below.