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Utility Proportional Optimization Flow Control for Overlay Multicast

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***Abstract*—There was a lot of interest in multicast communications within this decade as it is an essential part of many network applications, e.g. video-on-demand, etc. In this paper, we model flow rate allocation for application overlay as a utility based optimization problem constrained by capacity limitations of physical links and overlay constraints. The optimization flow control presented here addresses not only concave utility functions which are suitable for applications with elastic traffics, but also especial forms of non-concave utilities that are used to model applications with inelastic traffics, which might demand for hard delay and rate requirements. We then propose an iterative algorithm as the solution to the optimization flow control problem and investigate especial forms of non-concave utilities that are supported by this model. Simulation results show that the iterative algorithm can be used to deal with sigmoidal-like utilities which are useful for modeling real-time applications such as live streaming.**

***Keywords*-Overlay Multicast; Resource Allocation; Optimization Flow Control; Utility Proportional Flow Control; Duality Theorem; Iterative Algorithm.**

I. INTRODUCTION

The use of Multicast communications is a part of many network applications, such as video conferencing, video-on-demand and peer-to-peer file sharing. The lack of a widely available IP multicast service at the network layer in backbone networks, has led to an interest in application-level multicast, e.g., [1], [2], [3]. In application-level multicast, end hosts are self organized in an overlay network. This approach has the following advantages over traditional IP multicast. First, multicast support is not needed in network layer. By organizing end hosts into an overlay network, multicast data can be transmitted via unicast. Second, peers in a session can receive multicast data at different rates, thus network heterogeneity can be achieved [15].

The lack of IP multicast causes a large recent research on how to implement multicast service at the network layer, e.g., [1], [2], [3]. Some overlay multicast systems has been built using structured peer-to-peer networks that are claimed to be largely scalable [4], [5], [6], each of which implements application-level multicast using either flooding or tree construction. Flooding approach utilizes routing information already maintained by nodes, to broadcast messages. This method is sufficient to networks in which most of peers are interested in receiving the messages. If a subset of peers are interested in messages, overlay networks including those nodes have to be constructed and used. The alternative approach constructs a tree for each group instead of building separate overlay network. The application manages the tree and uses it to propagate multicast messages.

In an overlay multicast, different sources may transmit data through the same physical link, causing excessive traffic on a link. Furthermore, unicast data traverses different end hosts in a parent-child mode and needs to be considered that a child can not relay data at a rate higher than its parent. The last constraint has been introduced by Cui et al. [15], for the first time. It is critical to assign rate of each source in a way that constraints are maintained. Cui et al. [15] modeled the resource allocation problem in an overlay multicast as a utility based optimization problem in which utility functions are assumed to be strictly concave.

In order to provide flow control mechanism in a data network, we can use utility based optimal flow control methods that has been proposed by Kelly [14] and Low [16]. In these solutions, a price is associated to each link that is updated based on the aggregate rates of sources passing it. A utility function is associated to each source that can be considered as a measurement of application's QoS over the allocated bandwidth. The resource allocation problem is formulated as an optimization problem which maximizes the aggregate utility of all users under the link capacity constraint. Solving the problem distributively, results in a link algorithm that measures the link price and a source algorithm that adapts the source rate according to congestion feedbacks form the network. Choosing appropriate utility functions, different fairness objectives, such as proportional, max-min and etc. can be achieved.

There are some works following the work presented by Kelly et al. [14]. Authors in [7], [8] applied price-based resource allocation in multirate multicast sessions. In multirate

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multicast session, different receivers in the same multicast session can receive data at different rates. These works considered the heterogeneity of user requirements. Also, some work has been done for multiple-path networks [9], [10]. In these works, flow control and routing are optimized. Some research has been carried out to study optimal flow control based on optimization and game theory, e.g., [11], [12], [13].

In this paper, we model flow rate allocation for application overlay as a utility based optimization problem with capacity limitations of physical links and overlay constraints. Our model is influenced by the work of [15], however in our work, we address not only concave utility functions which are suitable for applications with elastic traffics, but also especial forms of non-concave utilities which are used to model applications with inelastic traffics, which might demand for hard delay and rate requirements.

The remainder of this paper is organized as follows. The system model and problem formulation is given in Sections II and III, respectively. In Section IV, we solve the problem and present the optimal solution. Section V investigates the non-concave choices of utility functions which can be supported using this framework. Simulation results are presented in Section VI. Finally, Section VII concludes the paper.

II. System Model

We consider an overlay network consisting of a set of N peers, denoted by $\mathcal{N} = \{1, 2, ..., N\}$. In the multicast session, one peer acts as the streaming source, and other peers are the receivers. Receivers relay the streaming data through unicast in a peer-to-peer mode. The multicast session includes F unicast peer-to-peer flows. This set of flows can be represented by $\mathcal{F} = \{1, 2, ..., F\}$, where each flow $f \in \mathcal{F}$ has a rate x_f bps. For notational convenience, we use the matrix notation and define the flow rate vector $\mathbf{x} = (x_f, f \in \mathcal{F})$. We denote by $m_f \geq 0$ and $M_f < \infty$ the minimum and maximum transmission rate of flow f, respectively.

If flow f arrives a peer and flow f' departs from that peer, then f' is the child of flow f, represented by $f \to f'$. If the source of f and the destination of f^p is the same peer, then flow f^p is the parent of f which is represented by $f^p \to f$.

Let us suppose that our overlay network has a set of L physical links denoted by $\mathcal{L} = \{1, 2, ..., L\}$. Each physical link $l \in \mathcal{L}$ has capacity c_l bps. We define the link capacity vector as $\mathbf{c} = (c_l, l \in \mathcal{L})$. Each flow $f \in \mathcal{F}$ corresponds to multiple physical links, represented by $\mathcal{L}(f) \subseteq \mathcal{L}$. Each link is associated with a set of flows that pass through it, denoted by $\mathcal{F}(l) \subseteq \mathcal{F}$.

For the sake of convenience, we define a $L \times F$ routing matrix $\mathbf{A}$ where $A_{lf} = 1$, if flow f passes through link l, i.e., $f \in \mathcal{F}(l)$, and $A_{lf} = 0$ otherwise. Such a routing matrix gives a precise insight of the resource usage pattern. The first requirement of this model to be realistic is the capacity constraint: the total load on each physical link $l \in \mathcal{L}$ should not exceed the capacity c_l. The capacity constraint for a particular link l can be expressed as follows:

$$\sum_f A_{lf} x_f \leq c_l \tag{1}$$

Considering this for all physical links, we obtain the matrix form as below

$$\mathbf{Ax} \leq \mathbf{c} \tag{2}$$

While the capacity constraint restricts physical link usage, application constraint expresses the relationship between the rate of different flows. In the Peer-to-peer multicast session, a peer can not relay the data to its downstream peer at a rate higher than receiving rate from its parent. Formally, if $f \to f'$, then $x_{f'} \leq x_f$. We define a $F \times F$ matrix B as follows:

$$B_{f'f} = \begin{cases} -1 & \text{if} \quad f \to f' \\ 1 & \text{if} \quad f' = f \text{and } f \text{ has a parent flow} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

$\mathbf{B}$ indicates the data relaying relationship between different flows in application level. This data dependency is determined by the topology of the overlay multicast tree [2]. Hence, the data constraint is formulated as follows:

$$\mathbf{Bx} \leq \mathbf{0} \tag{4}$$

The notations used to capture the network model as above, are summarized in Tab. I.

Table I
Notations used in system model

Notations	*Definitions*
$n \in \mathcal{N} = \{1, 2, ..., N\}$	End Host
$f \in \mathcal{F} = \{1, 2, ..., F\}$	Unicast Flow in Overlay Multicast
$\mathbf{x} = (x_f, f \in \mathcal{F})$	Flow Rate of $f \in \mathcal{F}$
$l \in \mathcal{L} = \{1, 2, ..., L\}$	Physical Network Link
$\mathbf{c} = (c_l, l \in \mathcal{L})$	Link Capacity of $l \in \mathcal{L}$
$f \to f'$	f' is the Child Flow of f
$\mathcal{L}(f) \subseteq \mathcal{L}$	Set of Links that f passes through
$\mathcal{F}(l) \subseteq \mathcal{F}$	Set of Flows that go through l
$\mathbf{A} = (A_{lf})_{L \times F}$	Link Capacity Constraint Matrix
$\mathbf{B} = (B_{f'f})_{F \times F}$	Data Constraint Matrix

In the sequel, we give an example to make data constraint and capacity constraint more clear. In the topology shown in Fig. 2, we have 5 overlay multicast flows and 7 physical links in the underlying network. From example, inequality (2) becomes

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \leq \begin{pmatrix} 6 \\ 3 \\ 8 \\ 15 \\ 10 \\ 2 \\ 12 \end{pmatrix}$$

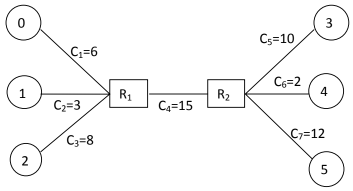

Figure 1. Physical Network Topology

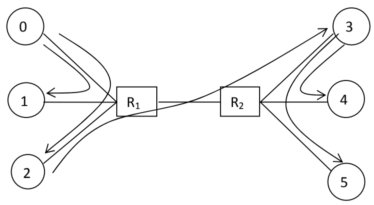

Figure 2. Overlay Multicast on the Physical Network

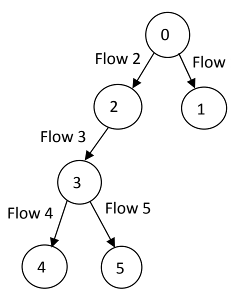

Figure 3. overlay multicast

and equality (4) becomes

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \leq \mathbf{0}$$

III. Problem Formulation

In this section, we model the flow rate allocation optimization problem. We focus on a utility-based optimization to allocate flow rates. Our model is influenced by the model of Low [16] and Cui [15]. [15] studied the resource allocation problem using a standard optimization flow control. In the optimization flow control framework each flow $f \in \mathcal{F}$, when submitting at rate x_f attains a utility $U_f(x_f)$.

We consider an optimization flow control problem as in [15]. The authors have focused on best effort traffic which admit well-behaved optimization problems due to concave utility functions. However, in our problem we have included the possibility of some non-concave utility functions which provides the possibility of streaming traffic as well as best effort.

The main objective of optimization flow control is to allocate flow rates so that the total utility over the network is maximized. [15] have addressed maximizing such an optimization flow control for overlay multicast problem.

$$\text{maximize} \quad \sum_{f \in \mathcal{F}} U_f(x_f) \tag{5}$$

$$\text{subject to} \quad \mathbf{Ax} \leq \mathbf{c} \tag{6}$$

$$\mathbf{Bx} \leq \mathbf{0} \tag{7}$$

$$\text{over} \quad \mathbf{x} \in I_f \tag{8}$$

The main requirement for the optimization flow control mentioned above is the strict concavity of utility functions. For many applications which demand for elastic traffic, strictly concave utility functions will suffice to capture the traffic characteristics. However, many other applications exist which demand for inelastic traffic, whose utility functions are shown to be non-convex [18]. Such applications are mainly relying on data streams with relatively hard delay and rate constraints for the traffic. For these applications, the optimization flow control problem is no longer concave and the iterative solution may lead to local optimal point of the underlying non-convex problem.

As stated in previous sections, here we adopt a recently proposed framework for optimization flow control, called utility proportional flow control, to deal with well-behaved non-concave utility functions. Utility proportional flow control is originally proposed as a remedy to unfair rate allocation and to extend the idea of utility max-min rate allocation proposed by [19]. In utility proportional flow control, instead of maximizing the aggregate utility functions $U_f(z)$, the aggregate of $\int \frac{dz}{U_f(z)}$ will be maximized. Andrew et. al. [17] have shown that Such a strange modification in utility function will improve the fairness properties of the flow rate allocation, especially when there are competing sources (peers) in the network. However, we are not assuming any selfish behavior for peers and will use this paradigm only to address utility functions with non-concave utility functions. Later on, we discuss how non-convex utility functions of especial forms will be dealt with in this manner.

Utility proportional flow control for our network model is defined as

$$\text{maximize} \quad \sum_{f \in \mathcal{F}} \int_{m_f}^{x_f} \frac{dz}{U_f(z)} \qquad m_f \leq x_f \leq M_f \tag{9}$$