

UCLA

**Computer
Science**



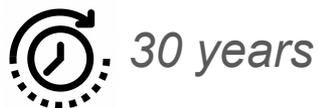
Symbolic Reasoning in the Age of Large Language Models

Guy Van den Broeck



Reasoning with Symbolic AI

- Logic and probabilistic
- **Deductive** reasoning algorithms
- Correct on *all* problems
- Limited scope
- Intractable

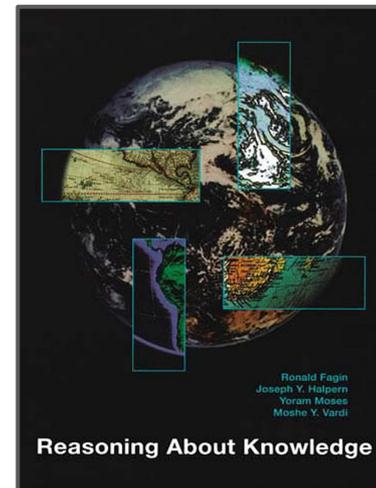


30 years



Reasoning with Transformers

- Build chains of thought
- **Inductive & Transductive** reasoning from data
- Correct on *many* problems
- Unlimited scope
- Tractable



Use the digits [0-9] to make three numbers: x,y,z so that $x+y=z$

Thought for 270 seconds ^

Okay, so I need to use each digit from 0 to 9 exactly once to create three numbers x , y , and z such that $x + y = z$. Hmm, let me think about how to approach this.

...

Verification:

- The sum $752 + 346 = 1098$ is correct.
- All digits from 0 to 9 are used exactly once.

Thus, the final answer is:

$752 + 346 = 1098$

<https://www.datacamp.com/blog/deepseek-r1-vs-v3>



Questions for this talk:

1. Do *deductive reasoning algorithms* still have a purpose in the age of LLMs?
2. Do *neurosymbolic AI researchers* still have a purpose in the age of LLMs?

What did I do this past week?

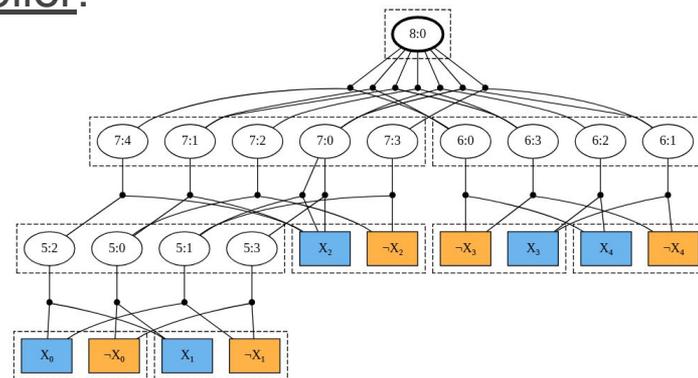
Wednesday: Submitted 30 pages of pure theory to SAT conference, defining a new logical circuit language (alternative to OBDD) with nicer properties.

Thursday: Gave the latex source to Claude Code, got a working compiler for this new circuit language by EOD. Passes all tests, gives correct model counts, ...

Friday: Asked Claude Code to optimize performance
Tool became faster than the SotA SDD compiler.

Weekend: Tool keeps getting better and better;
Existential crisis

Monday: Panic about this talk



Why did this happen?

- **Not** because of **me**: *I don't know Rust and have written 0 LOC*
- **Not** because of **the LLM**: *it makes too many mistakes to write this code*
- **Not** because of **me and the LLM** working together:
I don't have enough time to understand diffs for 143,920 LOC

Why?

- Called 10,000 **symbolic** tools (rust compiler, profiler, sat solvers, ...)
- Strong **symbolic** guardrails: 300 unit tests and ground-truth model counts
- Strong **symbolic** objective: runtime on benchmarks

It happened because of neurosymbolic AI!

Robotics has become **Neurosymbolic** (EAI Benchmark)

Task: use the rag to clean the refrigerator

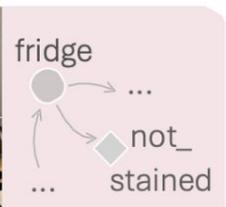
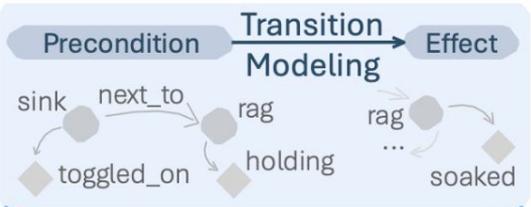
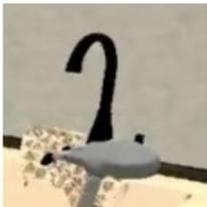
<https://arxiv.org/pdf/2410.07166>

stained(fridge) *next_to(rag, sink)* *toggled_on(sink)*

Subgoal Decomposition

soaked(rag) ...

not_stained(fridge)



GRASP(rag) PLACE_NEXTTO(sink) TOGGLE_ON(sink)

SOAK(rag)

Action Sequencing

Goal Interpretation

[TASK] Transform initial → target environment state.

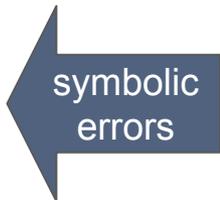
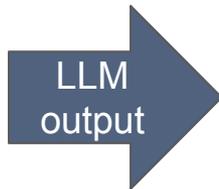
[OUTPUT] JSON list (execution order): {"action": ACTION_NAME, "object": OBJ}; use "OBJ1, OBJ2" only for *_NEXTTO_ONTOP.

[ACTIONS] From catalog: GRASP (LEFT/RIGHT), PLACE_(INSIDE/ONTOP/NEXTTO), OPEN/CLOSE, TOGGLE_ON/OFF, CLEAN, SOAK, DRY, SLICE. One object per hand; PLACE releases. OPEN/CLOSE/... require ≥1 empty hand & valid properties. Act only if target not inside a closed container.

[STATE] ['predicate', obj1, (obj2?)].

[INPUT] initial states..., target states..., interactable objects...

[TARGET (this instance)] rags NEXTTO sink_82; soap_0 INSIDE sink_82; trays (all) INSIDE fridge_97 & NOT stained; bowl_0 NEXTTO sink_82 & NOT dusty; fridge_97 NOT stained.



```
[{"action": "RIGHT_GRASP", "object": "rag_0"},
{"action": "RIGHT_PLACE_INSIDE", "object": "sink_82"},
{"action": "TOGGLE_ON", "object": "sink_82"},
{"action": "SOAK", "object": "rag_0"},
{"action": "RIGHT_GRASP", "object": "soap_0"},
{"action": "RIGHT_PLACE_INSIDE", "object": "sink_82"},
{"action": "LEFT_GRASP", "object": "rag_0"},
{"action": "TOGGLE_OFF", "object": "sink_82"},
```

```
{"action": "OPEN", "object": "fridge_97"},
{"action": "CLEAN", "object": "tray_0"},
{"action": "CLEAN", "object": "tray_1"},
{"action": "CLEAN", "object": "fridge_97"},
```

```
{"action": "RIGHT_GRASP", "object": "bowl_0"},
{"action": "RIGHT_PLACE_NEXTTO", "object": "sink_82"},
{"action": "CLEAN", "object": "bowl_0"},
```

```
{"action": "LEFT_PLACE_NEXTTO", "object": "sink_82"},
...
```

Math and CS Theory have become **Neurosymbolic**

```
def Equation1689 (M : Type _) (Magma M) := ∀ x y z : M, x * (y * z) = (x * z) * y
def Equation2 (M : Type _) (Magma M) := ∀ x y : M, x * y = y * x
variable (M : Type _) (Magma M)
/-
Human-readable proof that  $\forall x \cdot (y \cdot (x \cdot z)) \text{ (equation 1689)}$ 
implies the 'singleton law' (equation 2). -/
lemma main_eq (x y z : M) : x * (y * z) = (x * z) * y := by
  rw [f_eq]
  -- unfold f
  rw [f_S_eq]
  -- unfold S
  rw [S_eq]
```

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Summary

Mathematics is the source of significant technological advances. However, progress in math is slow for two primary reasons.

1. Decomposing problems into useful lemmas is a laborious and manual process. To advance the field of mathematics, mathematicians use their knowledge and experience to explore candidate lemmas, which, when composed together, prove theorems. Ideally, these lemmas are generalizable beyond the specifics of the current problem so they can be easily understood and ported to new contexts.

How AI is changing the nature of mathematical research

What machine learning theorists learned using AI agents to generate proofs — and what comes next.

By Michael Kearns, Aaron Roth

Share

March 9, 2026 | 10 min read

Modern AI coding tools have revolutionized software engineering, with developers now using AI assistants to write a [substantial fraction of their code](#) across a range of applications. As scientists studying the theory of machine learning, we're already seeing a similar transformation in basic scientific methodology, especially for research of a mathematical nature.

More precisely, AI tools are now able to develop and write rigorous mathematical proofs only from prompts providing high-level proof sketches. These proofs are written in longstanding "languages" for detailing mathematical arguments, in the same way that code is written in formal programming languages like Python. AI seems to have become proficient in both kinds of languages and their underlying logics.

We came to this realization during a three-week period last summer, when we used agentic AI tools to write a mathematical paper that normally would have taken months. The [50-page paper](#) describes and solves an optimization problem based on concepts from graph theory and machine learning. A typical prompt we would give the AI to set up the general framework for our paper looked like this: "Imagine a directed acyclic network of linear least-squares learning agents, each of which shares a common

"Working with proof-based AI tools is akin to collaborating with a smart, broadly educated but occasionally error-prone colleague."

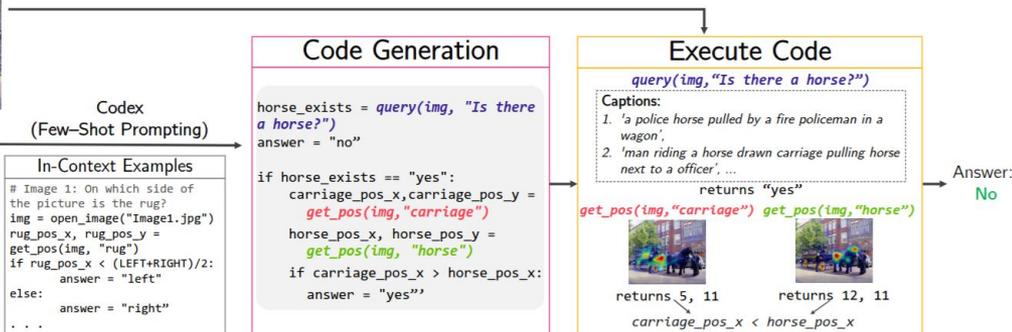
The world has become Neurosymbolic

Victory!

- Software engineering has become Neurosymbolic
- Robotics has become Neurosymbolic
- Math and CS Theory have become Neurosymbolic
- Algorithms, Architecture has become Neurosymbolic (AlphaEvolve)
- AI4Science has become Neurosymbolic (cf. Mathias Niepert)
- Computer Vision has become Neurosymbolic



Question:
Is the carriage to the right of a horse?



<https://arxiv.org/pdf/2306.05392>

What does the world think?

Will our community win a Turing Award for neurosymbolic AI?

- Warning: ***offensive, unfair, facetious***

The world thinks we mostly care about fuzzy logic, MNIST addition, philosophy about hallucinations & guarantees, proving that doing the right thing is impossible, repeating Kahneman's System 1 and System 2, and fighting culture wars from the Imagenet era of AI (†2020)

- **We don't even know that we've "won",**
so how do we expect others to know?

If no thanks to us, then why did neurosymbolic AI “win”?

The philosophy was always right.

Neurosymbolic won because of the overwhelming gravitational pull of **tools** and **languages** that provide

{
Compositionality
Abstraction
Soundness
}

Neurosymbolic data, training, and inference is vastly more powerful because of these properties!

Or what do you think?

How could this have happened without us?

Success stories **despite** not understanding the symbolic side well.

- Ask a Computer Vision researcher about the VQA code they generate. Answer: I don't know, it's just Python.
- AlphaEvolve generated *hundreds of millions of programs*.
No attempt to be smart about it.
- A \$200-per-month Claude Code subscription uses \$5,000 in compute, subsidized by Anthropic [forbes]

Scale and brute force compensate for this lack of understanding.

What is the role of the neurosymbolic AI researchers?

- We have a unique invaluable skill!
- Can we do better than prompting until it works (prompt and pray)
 - World needs tight integration of deductive reasoning and LLM
 - We know how to bridge using probabilistic logic!
- Can we do better than treating every problem as costly RL?
 - World needs deductive reasoning about LLM alignment goals.
- Can we have deductive reasoning that is tractable at scale?
- ...?

Discussion?

Or some examples of how my lab approaches:

1. World needs tight integration of deductive reasoning and LLM

Put symbolic probability into the LLM

2. World needs deductive reasoning about LLM alignment goals.
3. World needs tight integration of deductive reasoning and LLM

Put LLM probability into the symbolic reasoning

4. Can we have deductive reasoning that is tractable at scale?

Discussion?

Or some examples of how my lab approaches:

- 1. World needs tight integration of deductive reasoning and LLM**

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Put LLM probability into the symbolic reasoning

4. Can we have deductive reasoning that is tractable at scale?



Generate a sentence using "frisbee", "caught", and "dog" in that order



The frisbee flew through the air and the dog caught it with excitement.

GPT 4.1-mini (apr 2025)



The energetic dog leapt into the air, caught the frisbee in mid-flight, and proudly trotted back to its owner.

Llama-3.2-3B (sep 2024)



You got the order wrong, please respect the given order



Here is a new sentence with the words in that order: The dog's owner was about to catch the frisbee that had been flying through the air.



A frisbee is caught by a dog.

A pair of frisbee players are caught in a dog fight.

Ctrl-G (jun 2024)

Reasoning about all Future Tokens: *Constraints*

$p(\text{next-token} \mid \alpha, \text{prefix})$

Constrained Generation: $\Pr(x_{t+1} \mid \alpha, x_{1:t} = \text{"the weather is"})$

Lexical Constraint α : sentence contains keyword "winter"

Reasoning about all Future Tokens: *Constraints*

$p(\text{next-token} \mid \alpha, \text{prefix})$

Constrained Generation: $\Pr(x_{t+1} \mid \alpha, x_{1:t} = \text{"the weather is"})$

Lexical Constraint α : sentence contains keyword "winter"

$\propto p(\text{next-token} \mid \text{prefix}) \cdot p(\alpha \mid \text{next-token}, \text{prefix})$



Bayes' rule lets us reason backwards in time!

Reasoning about all Future Tokens: *Constraints*

$$p(\text{next-token} \mid \alpha, \text{prefix})$$

cold	0.025
warm	0.001

$$\propto p(\text{next-token} \mid \text{prefix})$$

cold	0.05
warm	0.10

Constrained Generation: $\Pr(x_{t+1} \mid \alpha, x_{1:t} = \text{"the weather is"})$

Lexical Constraint α : sentence contains keyword "winter"

$$p(\alpha \mid \text{next-token}, \text{prefix})$$

cold	0.50
warm	0.01



Reasoning about all Future Tokens

$$p_{lm}(\text{next-token} \mid \alpha, \text{prefix})$$

Using Bayes rule,

$$\propto p_{lm}(\text{next-token} \mid \text{prefix}) \cdot \cancel{p_{lm}(\alpha \mid \text{next-token}, \text{prefix})}$$



Intractable



Reasoning about all Future Tokens

$$p_{lm}(\text{next-token} \mid \alpha, \text{prefix})$$

Abusing Bayes rule,

$$\propto p_{lm}(\text{next-token} \mid \text{prefix}) \cdot p_{circuit}(\alpha \mid \text{next-token}, \text{prefix})$$



Use a tractable circuit model distilled from the transformer LLM...

A 'tractable digital twin'

Reasoning about all Future Tokens: Constraints

$$p_{lm}(\text{next-token} \mid \alpha, \text{prefix})$$

Abusing Bayes rule,

$$\propto p_{lm}(\text{next-token} \mid \text{prefix}) \cdot p_{circuit}(\alpha \mid \text{next-token}, \text{prefix})$$



Theorem. Given

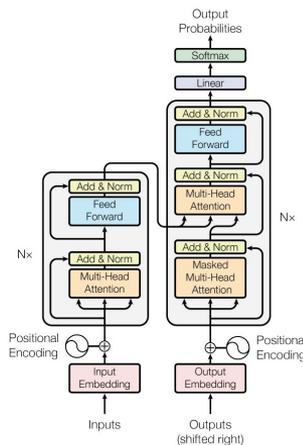
1. a deterministic finite automata constraint α with m edges and
 2. a probabilistic circuit $p(\cdot)$ with h hidden states (representing a Hidden Markov Model),
- computing $p(\alpha \mid x_{1:t})$ over a sequence of n future tokens takes $O(nmh^2)$ time.

Reasoning about all Future Tokens: Constraints

$$p_{lm}(\text{next-token} \mid \alpha, \text{prefix})$$

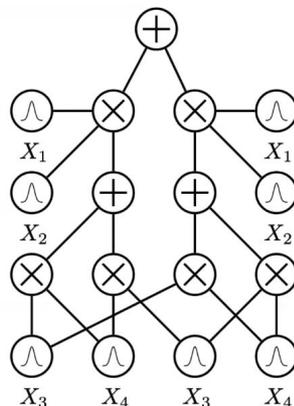
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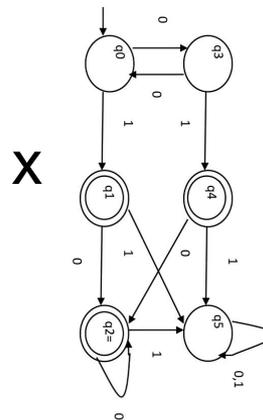


transformer

$\cdot \sum_{\text{future}}$



weights/circuit



models/dfa

Interactive Text Editing

“First they've defeated a small squad [BLANK] are few humans left, and despite their magical power, their numbers are getting fewer.”

Interactive Text Editing

User: given the following context, generate infilling text for [BLANK] using key phrases "alien mothership", "far from over"; generated text must contain 25 - 30 words.

"First they've defeated a small squad [BLANK] are few humans left, and despite their magical power, their numbers are getting fewer."

Ctrl-G



"First they've defeated a small squad of aliens, then a larger fleet of their ships. Eventually they've even managed to take down the alien mothership. But their problems are far from over. There are few humans left, and despite their magical power, their numbers are getting fewer."

Interactive Text Editing with key phrase (K) or length (L) constraints



	K	L	K&L
<i>Quality</i>			
TULU2	2.64	2.78	2.74
GPT3.5	2.22	2.27	2.31
GPT4	3.33	3.53	3.10
Ctrl-G	3.56	3.73	3.59
<i>Success</i>			
TULU2	12%	20%	3%
GPT3.5	22%	54%	10%
GPT4	60%	20%	27%
Ctrl-G	100%	100%	100%
<i>Overall</i>			
TULU2	7%	10%	1%
GPT3.5	0%	5%	2%
GPT4	41%	17%	14%
Ctrl-G	76%	78%	82%

→ *How many stars by humans?*

→ *Follows instructions?*

→ ★★☆☆☆ & Up + *Follows instructions?*

→ **Ctrl-G based on Llama2-7B wipes the floor with GPT4, which is a >100x bigger LLM**

Probabilistic Circuit Language Model

How did we train a probabilistic circuit to solve Ctrl-G?

Keep it simple... just a classic **Hidden Markov Model (HMM)** with 32,768 hidden states and 2 billion parameters... on the GPU



Grade School Math Benchmark

Question: Kylar went to the store to buy glasses for his new apartment. One glass costs \$5, but every second glass costs only 60% of the price. Kylar wants to buy 16 glasses. How much does he need to pay for them?

Vanilla LLM Answer: The price of the 2nd glass is $(16 / 2) * 60\% = 8$ dollars. So one pair of glasses costs $16 + 8 = 24$ dollars. **So the answer is 24.**

Grade School Math Benchmark

Question: Kylar went to the store to buy glasses for his new apartment. One glass costs \$5, but every second glass costs only 60% of the price. Kylar wants to buy 16 glasses. How much does he need to pay for them?

Vanilla LLM Answer: The price of the 2nd glass is $(16 / 2) * 60\% = 8$ dollars. So one pair of glasses costs $16 + 8 = 24$ dollars. **So the answer is 24.**

Ctrl-G Answer: The second glass costs $5 * .6 = \$3$. So each set of two glasses actually costs $5 + 3 = \$8$. He wants $16 / 2 = 8$ sets of two. That means he needs to pay $8 * 8 = \$64$. **So the answer is 64.**

Which constraint improves accuracy?

Grade School Math Benchmark

Question: *Kylar went to the store to buy glasses for his new apartment. One glass costs \$5, but every second glass costs only 60% of the price. Kylar wants to buy 16 glasses. How much does he need to pay for them?*

Vanilla LLM Answer: The price of the 2nd glass is $(16 / 2) * 60\% = 8$ dollars. So one pair of glasses costs $16 + 8 = 24$ dollars. So the answer is 24.

Ctrl-G Answer: The second glass costs $5 * .6 = \$3$. So each set of two glasses actually costs $5 + 3 = \$8$. He wants $16 / 2 = 8$ sets of two. That means he needs to pay $8 * 8 = \$64$. So the answer is 64.

Use all the numbers in the problem statement!

Robotics has become **Neurosymbolic** (EAI Benchmark)

Task: use the rag to clean the refrigerator

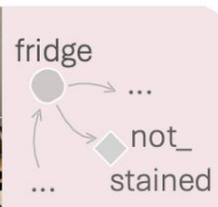
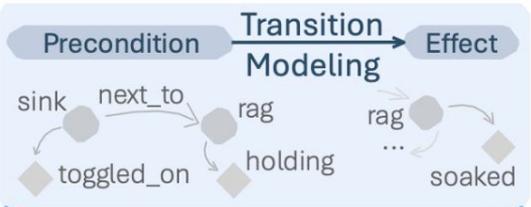
<https://arxiv.org/pdf/2410.07166>

stained(fridge) *next_to(rag, sink)* *toggled_on(sink)*

Subgoal Decomposition

soaked(rag) ...

not_stained(fridge)



GRASP(rag) PLACE_NEXTTO(sink) TOGGLE_ON(sink)

SOAK(rag)

Action Sequencing

Goal Interpretation

[TASK] Transform initial → target environment state.

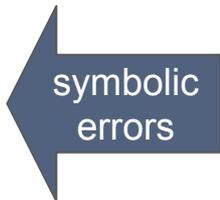
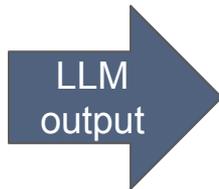
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[STATE] ['predicate', obj1, (obj2?)].

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[TARGET (this instance)] rags NEXTTO sink_82; soap_0 INSIDE sink_82; trays (all) INSIDE fridge_97 & NOT stained; bowl_0 NEXTTO sink_82 & NOT dusty; fridge_97 NOT stained.



```
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{"action": "RIGHT_PLACE_INSIDE", "object": "sink_82"},
{"action": "TOGGLE_ON", "object": "sink_82"},
{"action": "SOAK", "object": "rag_0"},
{"action": "RIGHT_GRASP", "object": "soap_0"},
{"action": "RIGHT_PLACE_INSIDE", "object": "sink_82"},
{"action": "LEFT_GRASP", "object": "rag_0"},
{"action": "TOGGLE_OFF", "object": "sink_82"},
```

```
{"action": "OPEN", "object": "fridge_97"},
{"action": "CLEAN", "object": "tray_0"},
{"action": "CLEAN", "object": "tray_1"},
{"action": "CLEAN", "object": "fridge_97"},
```

```
{"action": "RIGHT_GRASP", "object": "bowl_0"},
{"action": "RIGHT_PLACE_NEXTTO", "object": "sink_82"},
{"action": "CLEAN", "object": "bowl_0"},
```

```
{"action": "LEFT_PLACE_NEXTTO", "object": "sink_82"},
...
```

Go Beyond Token-Level Control (out soon)

To generate high-quality code, we need complex DFAs that capture long-range dependencies.

Decompose problem:

1. Token-level Syntax DFA
2. Meta-DFA capturing high-level Semantics

Now Llama 3 8B **success rate goes from 24% to 88%!**

On par with proprietary LLMs.

Model	Action Sequencing		Subgoal Decomposition	
	Task SR	Execution SR	Task SR	Execution SR
Claude-3 Haiku	54.8	60.7	78.4	82.8
Claude-3 Sonnet	58.0	63.3	83.1	86.4
Claude-3 Opus	64.6	69.5	86.7	89.9
Claude-3.5 Sonnet	76.1	81.3	89.1	92.0
Cohere Command R	44.9	44.3	71.3	78.1
Cohere Command R+	54.1	65.2	77.8	83.7
Gemini 1.0 Pro	45.6	56.7	70.4	84.6
Gemini 1.5 Flash	69.5	75.4	89.1	94.1
Gemini 1.5 Pro	76.7	83.6	87.0	91.1
GPT-3.5-turbo	24.9	40.7	69.2	81.4
GPT-4-turbo	60.0	65.2	85.5	94.1
GPT-4o	71.5	81.3	87.6	91.1
o1-mini	71.5	76.4	79.3	84.6
o1-preview	65.2	72.5	89.4	93.2
Llama 3 8B Instruct	21.3	23.6	48.8	58.0
Llama 3 70B Instruct	59.0	66.6	78.4	87.3
Mistral Large	78.4	84.6	84.3	92.0
Mixtral 8x22B MoE	63.3	67.9	80.5	90.2
ctrlg				
Llama 3 8B Instruct	21.3	23.6	48.8	58.0
+Syntax Ctrlg	48.7	51.5	70.8	78.1
+Semantic Ctrlg	81.3	88.0	–	–

Advantages of Ctrl-G:

1. Constraint α is guaranteed to be satisfied:

if next-token makes α unsatisfiable, $p_{lm}(\text{next-token} \mid \alpha, \text{prefix}) = 0$.

$$p_{lm}(\text{next-token} \mid \text{prefix}) \cdot p_{circuit}(\alpha \mid \text{next-token}, \text{prefix}) = 0$$

2. Generalizes well to unseen reasoning tasks, because all tasks are unseen :-)
(training on a distribution over tasks is slow and brittle!)
3. Bayesian = goal-oriented (\leftrightarrow structured generation tools)

You can control an intractable generative model using a generative model that is *tractable for symbolic reasoning*.

Discussion?

Or some examples of how my lab approaches:

1. World needs tight integration of deductive reasoning and LLM

Put symbolic probability into the LLM

2. **World needs deductive reasoning about LLM alignment goals.**

3. World needs tight integration of deductive reasoning and LLM

Put LLM probability into the symbolic reasoning

4. Can we have deductive reasoning that is tractable at scale?

Reasoning about all Future Tokens: *Alignment*

$p(\text{next-token} \mid \alpha, \text{prefix})$

Prefix: It's a pain ...

Constraint α : non-toxic

Reasoning about all Future Tokens: *Alignment*

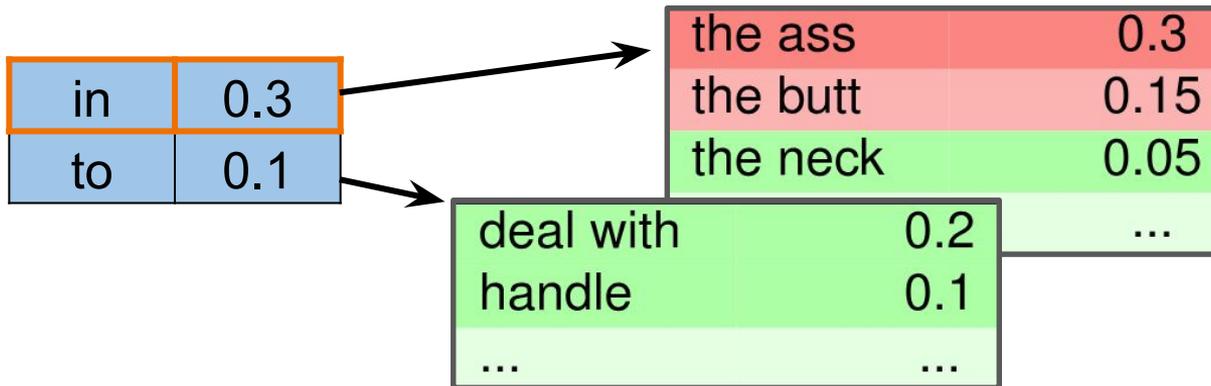
$p(\text{next-token} \mid \alpha, \text{prefix})$

Prefix: It's a pain ...

Constraint α : non-toxic

$\propto p(\text{next-token} \mid \text{prefix})$

$p(\alpha \mid \text{next-token}, \text{prefix})$



Reasoning about all Future Tokens: *Alignment*

$$p(\text{next-token} \mid \alpha, \text{prefix})$$

in	0.03
to	0.08

Prefix: It's a pain ...

Constraint α : non-toxic

$$\propto p(\text{next-token} \mid \text{prefix})$$

in	0.3
to	0.1

$$\cdot p(\alpha \mid \text{next-token}, \text{prefix})$$

in	0.1
to	0.8



RL Hammer: Ad-Hoc Probabilistic Reasoning

Toxicity

Base LLM	0.385
PPO ⁽⁷⁾	0.218
Quark ⁽⁸⁾	0.196
DPO ⁽⁹⁾	0.180

lower = less toxic



Perplexity

Base LLM	25.57
PPO ⁽⁷⁾	14.27
Quark ⁽⁸⁾	12.47
DPO ⁽⁹⁾	21.59

lower = higher LLM probability



Base LLM	52.06	Lower entropy means
DPO	39.52	lower diversity

DPO “thinks” that 99.9996% of all internet text is toxic.

Attribute Probability



0 (toxic)

1 (nontoxic)

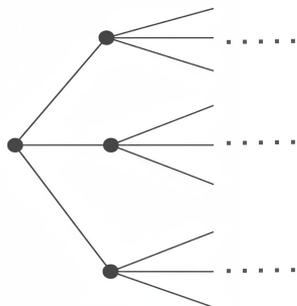
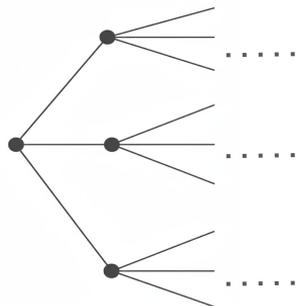
It's a pain

in

$p_{LM} = 0.3$

to

$p_{LM} = 0.1$



future text

the ass

the butt

the neck

...

...

future text

deal with

handle

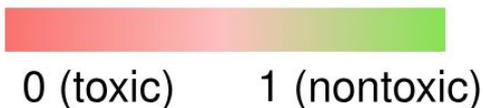
...

...

Intractable to know
expected future toxicity



Attribute Probability



It's a pain

in

$$p_{LM} = 0.3$$

to

$$p_{LM} = 0.1$$

future text

the ass
the butt
the neck
...
...

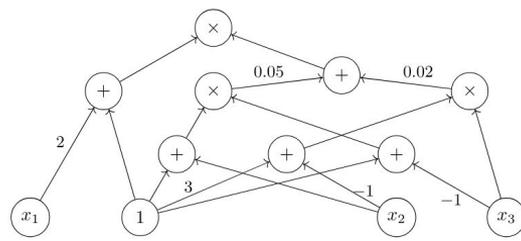
$EAP = 0.1$

future text

deal with
handle
...
...

$EAP = 0.8$

LLM continuations with *tractable probabilistic circuit*



+
Goal attribute with
log-linear classifier



=
**Efficient Expected
Attribute Probability!**



Attribute Probability



0 (toxic)

1 (nontoxic)

It's a pain

in

$$p_{LM} = 0.3$$

to

$$p_{LM} = 0.1$$

future text

the ass

the butt

the neck

...

...

$$EAP = 0.1$$

$$= p_{TRACE} \propto 0.03$$

future text

deal with

handle

...

...

$$EAP = 0.8$$

$$= p_{TRACE} \propto 0.08$$



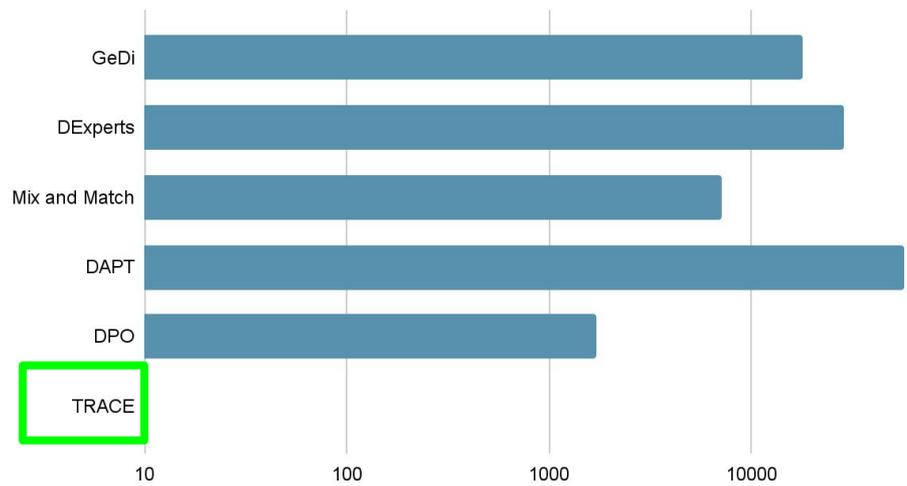
State-of-the-art LLM Detoxification

Model	Toxicity (↓)		Diversity (↑)		Fluency (↓)	Approach Type
	avg. max.	prob.	dist-2	dist-3		
GPT-2 Large Results						
GPT2	0.385	0.254	0.87	0.86	25.57	Baseline
DAPT ⁽¹⁾	0.428	0.360	0.84	0.84	31.21	Finetuning
GeDi ⁽²⁾	0.363	0.217	0.84	0.83	60.03	Decoding (Trained Guide)
FUDGE ⁽³⁾	0.302	0.371	0.78	0.82	12.97 *	Decoding (Trained Guide)
DExperts ⁽⁴⁾	0.314	0.128	0.84	0.84	32.41	Decoding (Trained Guide)
PPLM ⁽⁵⁾	0.520	0.518	0.86	0.86	32.58	Decoding (Logit Control)
MuCoLa ⁽⁶⁾	0.308	0.088	0.82	0.83	29.92	Decoding (Sampling)
PPO ⁽⁷⁾	0.218	0.044	0.80	0.84	14.27 *	RL
Quark ⁽⁸⁾	0.196	0.035	0.80	0.84	12.47 *	RL
DPO ⁽⁹⁾	0.180	0.026	0.76	0.78	21.59 *	RL
TRACE	0.163	0.016	0.85	0.85	29.83	Decoding (HMM Reasoning)
Gemma-2B Results						
Gemma-2B	0.359	0.23	0.86	0.85	15.75	Baseline
DPO ⁽⁹⁾	0.222	0.06	0.74	0.77	14.39 *	RL
TRACE	0.189	0.02	0.86	0.85	17.68	Decoding (HMM Reasoning)

TRACE is Blazingly Fast

Given a language model, and its tractable circuit twin,
train log-linear attribute classifier

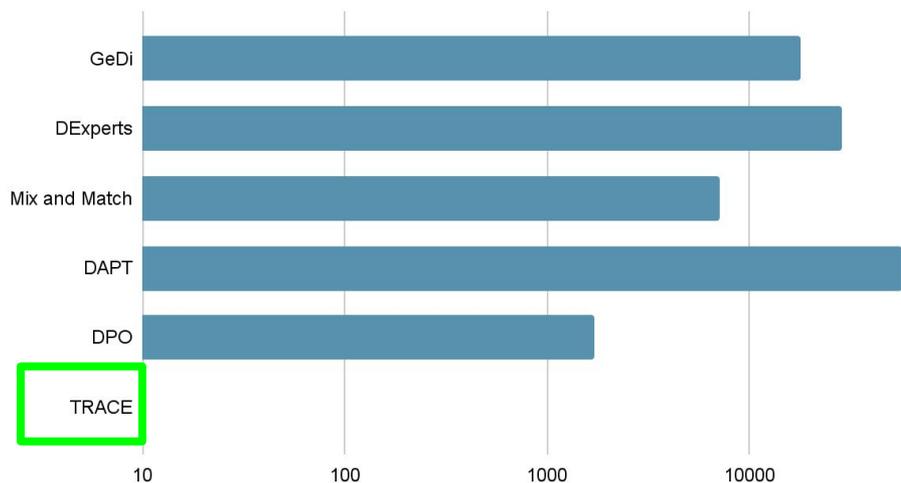
Training Time per Attribute (seconds)



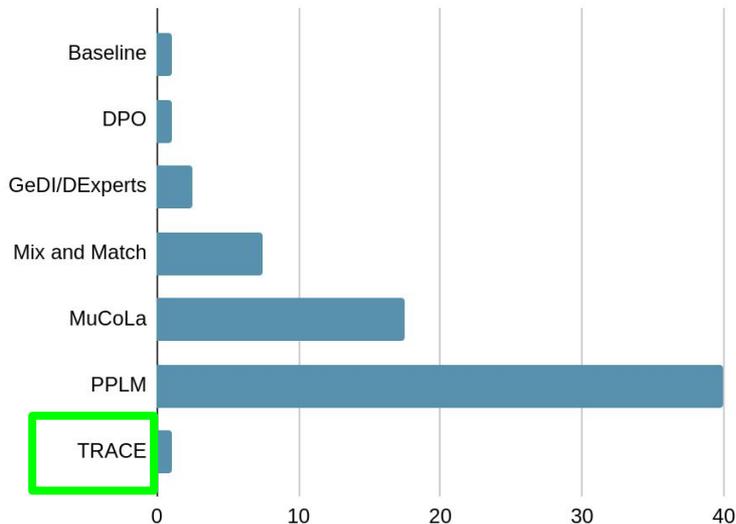
TRACE is Blazingly Fast

Given a language model, and its tractable circuit twin,
train log-linear attribute classifier,
then use Bayesian logits at decoding time (**weighted model counting**)

Training Time per Attribute (seconds)



Inference Time



Discussion?

Or some examples of how my lab approaches:

1. World needs tight integration of deductive reasoning and LLM

Put symbolic probability into the LLM

2. World needs deductive reasoning about LLM alignment goals.

3. **World needs tight integration of deductive reasoning and LLM**

Put LLM probability into the symbolic reasoning

4. Can we have deductive reasoning that is tractable at scale?

Programs-of-thought for grade school math

Question: A class of 200 students is split into 3 groups such that 2 of them are equal in number and the last one (which is the smallest) is 10 less than each of the other groups. How many students are in this (smallest) group?

🤔 Let's ask an LLM.

```
def compute_answer():  
    total_students = 200  
    smallest_group = (total_students - 10) // 3  
    return smallest_group
```

Not exactly correct! Let's ask again.

Programs-of-thought for grade school math

Question: A class of 200 students is split into 3 groups such that 2 of them are equal in number and the last one (which is the smallest) is 10 less than each of the other groups. How many students are in this (smallest) group?

🤔 Let's ask an LLM a second time.

```
def compute_answer():  
    total_students = 200  
    smallest_group = (total_students + 10) // 3  
    return smallest_group
```

Not exactly correct! Let's ask again.

Programs-of-thought for grade school math

Question: A class of 200 students is split into 3 groups such that 2 of them are equal in number and the last one (which is the smallest) is 10 less than each of the other groups. How many students are in this (smallest) group?

🤔 Let's ask an LLM a third time.

```
def compute_answer():  
    total_students = 200  
    smallest_group = (total_students + 10) // 3  
    return smallest_group
```

Same program
Ask again.

Programs-of-thought for grade school math

Question: A class of 200 students is split into 3 groups such that 2 of them are equal in number and the last one (which is the smallest) is 10 less than each of the other groups. How many students are in this (smallest) group?

🤔 Let's ask an LLM a fourth time.

```
def compute_answer():  
    total_students = 200  
    smallest_group = (total_students // 3) - 10  
    return smallest_group
```

Not exactly correct! Let's ask again.

Programs-of-thought for grade school math

Question: A class of 200 students is split into 3 groups such that 2 of them are equal in number and the last one (which is the smallest) is 10 less than each of the other groups. How many students are in this (smallest) group?

🤔 Let's ask an LLM a fifth time.

```
def compute_answer():  
    total_students = 200  
    difference = 10  
    group1 = group2 = (total_students + difference) // 3  
    group3 = group1 - difference  
    return group3
```

Phew!
finally

Programs-of-thought for grade school math

Question: A class of 200 students is split into 3 groups such that 2 of them are equal in number and the last one (which is the smallest) is 10 less than each of the other groups. How many students are in this (smallest) group?

What's happening here? Why do we need so many samples from the language model?

Deterministic Programs-of-thought for grade school math

Question: A class of 200 students is split into 3 groups such that 2 of them are equal in number and the last one (which is the smallest) is 10 less than each of the other groups. How many students are in this (smallest) group?

What's happening here? Why do we need so many samples from the language model?

Because language model is a distribution over token sequences.

And each time, we sample a deterministic program and throw away the uncertainty!!!

Probabilistic Programs-of-thought for grade school math

Question: A class of 200 students is split into 3 groups such that 2 of them are equal in number and the last one (which is the smallest) is 10 less than each of the other groups. How many students are in this (smallest) group?

What's happening here? Why do we need so many samples from the language model?

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Probabilistic Programs-of-thought for grade school math

Question: A class of 200 students is split into 3 groups such that 2 of them are equal in number and the last one (which is the smallest) is 10 less than each of the other groups. How many students are in this (smallest) group?

🤔 Let's ask an LLM but let's not throw away the uncertainty

```
def compute_answer():  
    total_students = 200  
    smallest_group = (total_students + 10) // 3  
    return smallest_group
```

Probabilistic Programs-of-thought for grade school math

Question: A class of 200 students is split into 3 groups such that 2 of them are equal in number and the last one (which is the smallest) is 10 less than each of the other groups. How many students are in this (smallest) group?

🤔 Let's ask an LLM but let's not throw away the uncertainty

```
def compute_answer():  
    total_students = [x12, x13, x14]  
    smallest_group = (total_students + [x25, x26]) // x27  
    return smallest_group
```

- Interpret integers as random variables

Probabilistic Programs-of-thought for grade school math

Question: A class of 200 students is split into 3 groups such that 2 of them are equal in number and the last one (which is the smallest) is 10 less than each of the other groups. How many students are in this (smallest) group?

🤔 Let's ask an LLM but let's not throw away the uncertainty

```
def compute_answer():  
    total_students = [X12, X13, X14]  
    smallest_group = (total_students Z23 [X25, X26]) Z28 X27  
    return smallest_group
```

- Interpret integers as random variables.
- Interpret operators as random variables.



A Probabilistic Program!!!

Probabilistic Programs-of-thought for grade school math

Question: A class of 200 students is split into 3 groups such that 2 of them are equal in number and the last one (which is the smallest) is 10 less than each of the other groups. How many students are in this (smallest) group?

```
def compute_answer():  
    total_students = [X12, X13, X14]  
    smallest_group = (total_students Z23 [X25, X26]) Z28 X27  
    return smallest_group
```

Sample from probabilistic program

- Much cheaper than sampling from LLM

```
def compute_answer():
```

...

Probabilistic Programs-of-thought for grade school math

Question: A class of 200 students is split into 3 groups such that 2 of them are equal in number and the last one (which is the smallest) is 10 less than each of the other groups. How many students are in this (smallest) group?

```
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    total_students = [X12, X13, X14]  
    smallest_group = (total_students Z23 [X25, X26]) Z28 X27  
    return smallest_group
```

Sample from probabilistic program

- Much cheaper than sampling from LLM

```
def compute_answer():  
    ...
```

```
def compute_answer():  
    total_students = 200  
    smallest_group = (total_students + 20) // 3  
    return smallest_group  
    ...
```



```
def compute_answer():  
    ...
```

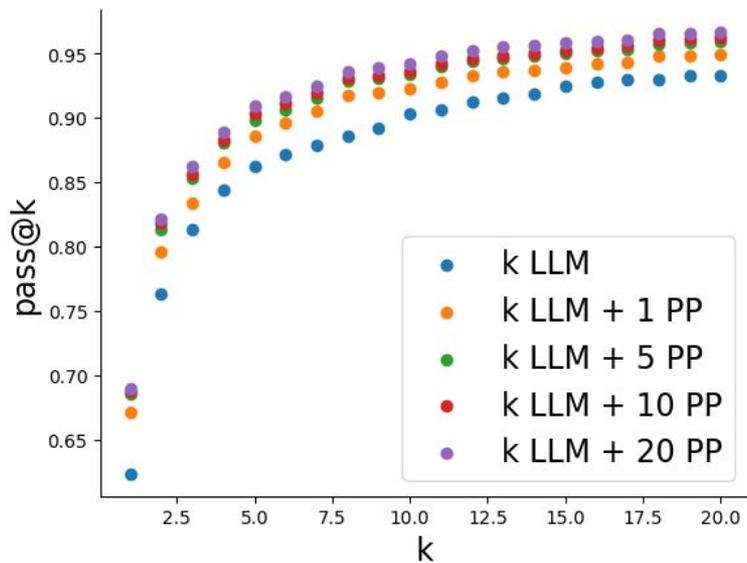
Probabilistic Programs-of-thought: Results

Pass@k accuracy

1.5-6.5% boost
for the same amount of GPU compute

Method	GSM8k			Qwen2.5-Coder-Instruct CruxEval		
	0.5B	3B	7B	0.5B	3B	7B
1 LLM + 0 PP	24.94	72.63	82.71	30.87	40.5	57.12
1 LLM + 5 PP	28.51	76.19	84.76	37.37	45.87	60.62
5 LLM + 0 PP	42.53	86.2	91.2	46.13	59.13	73.37
5 LLM + 25 PP	49.13	89.08	93.02	53.13	63.87	76.87

10 LLM samples
with probabilistic \approx 20 LLM samples
program



Discussion?

Or some examples of how my lab approaches:

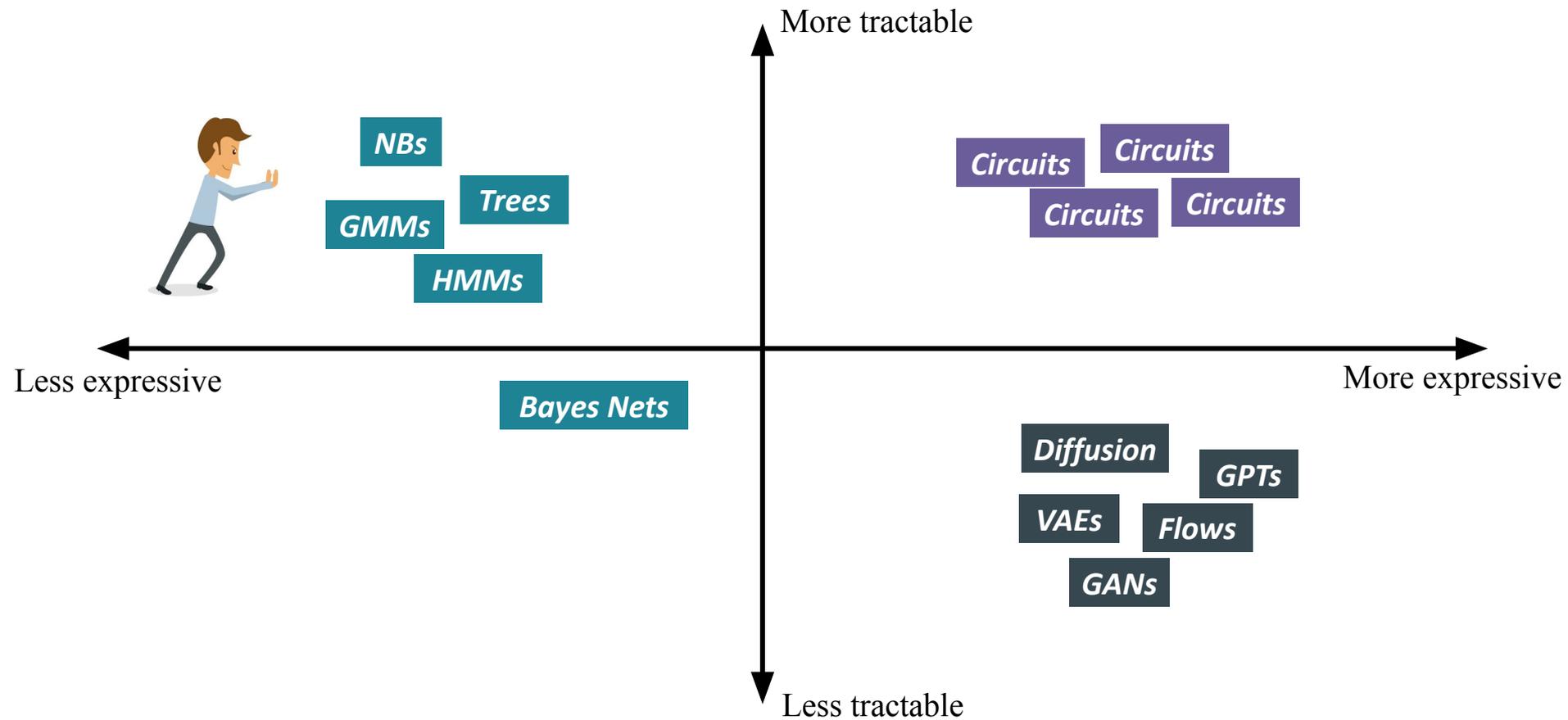
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Put symbolic probability into the LLM

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3. World needs tight integration of deductive reasoning and LLM

Put LLM probability into the symbolic reasoning

4. **Can we have deductive reasoning that is tractable at scale?**



Generative Models

polynomials model **joint distributions**

$$p(x_1, x_2, x_3) = .1x_1 + .05x_2 + .1x_1x_2 + .01x_3 - .07x_2x_3 + .02x_1x_3 - .14x_1x_2x_3 + .05$$

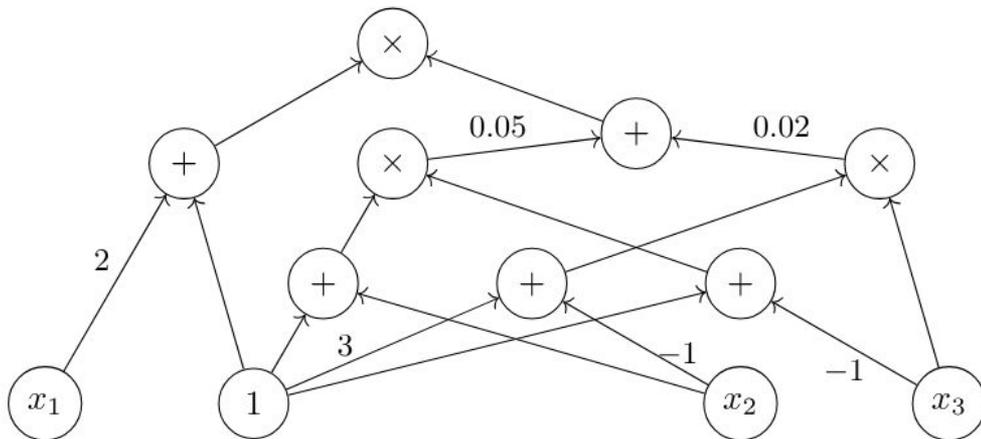
X_1	X_2	X_3	p
0	0	0	0.05
1	0	0	0.15
0	1	0	0.1
1	1	0	0.3
0	0	1	0.06
1	0	1	0.18
0	1	1	0.04
1	1	1	0.12

Deep Generative Models

circuit polynomials model **joint distributions** compactly
(and can have billions of trainable parameters)

$$p(x_1, x_2, x_3) = .1x_1 + .05x_2 + .1x_1x_2 + .01x_3 - .07x_2x_3 + .02x_1x_3 - .14x_1x_2x_3 + .05$$

X_1	X_2	X_3	p
0	0	0	0.05
1	0	0	0.15
0	1	0	0.1
1	1	0	0.3
0	0	1	0.06
1	0	1	0.18
0	1	1	0.04
1	1	1	0.12

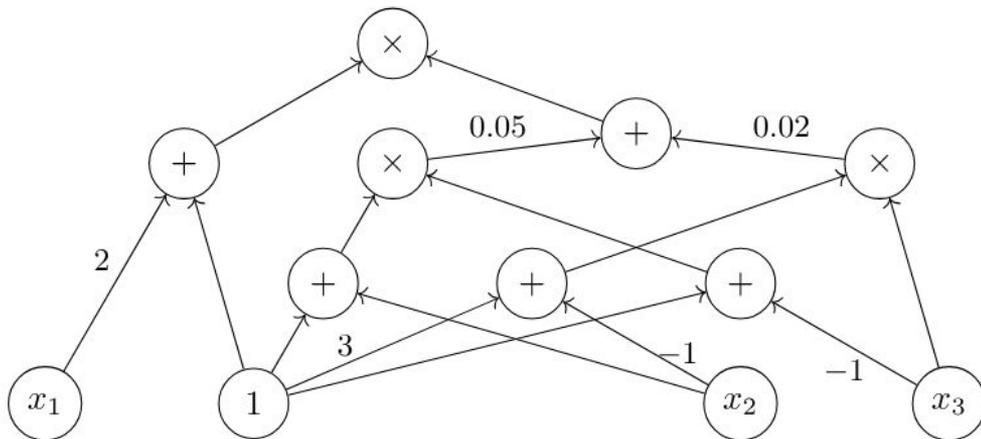


Tractable Deep Generative Models

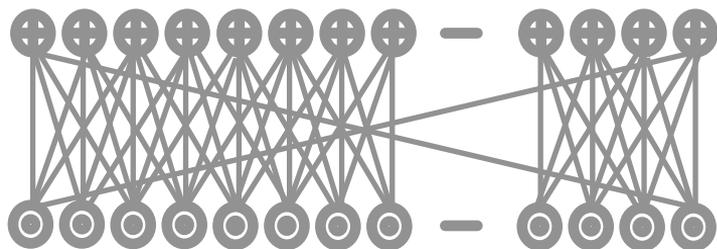
Multilinear circuit polynomials model **joint distributions** compactly *and* allow **efficient** probabilistic reasoning (marginalization)

$$p(x_1, x_2, x_3) = .1x_1 + .05x_2 + .1x_1x_2 + .01x_3 - .07x_2x_3 + .02x_1x_3 - .14x_1x_2x_3 + .05$$

X_1	X_2	X_3	p
0	0	0	0.05
1	0	0	0.15
0	1	0	0.1
1	1	0	0.3
0	0	1	0.06
1	0	1	0.18
0	1	1	0.04
1	1	1	0.12



Scaling Up Probabilistic Circuits



d nodes

$O(d^2)$ edges

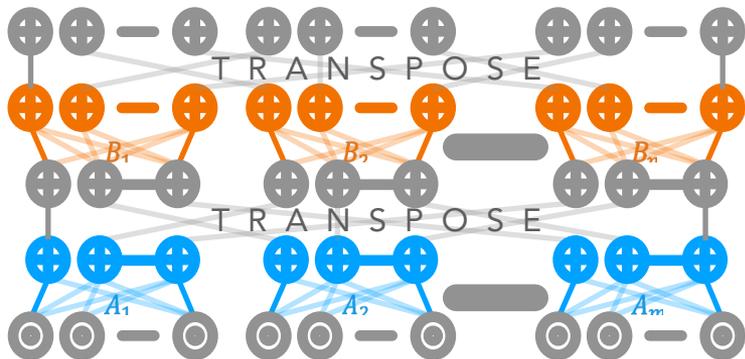
$$y_{ij} = \sum_{kl} A_{ijkl} x_{kl}$$

Linear Layers

Dense Matrices



e.g. a model w/ just 250K nodes requires 69B parameters (memory + time)...



d nodes

$O(d^{3/2})$ edges

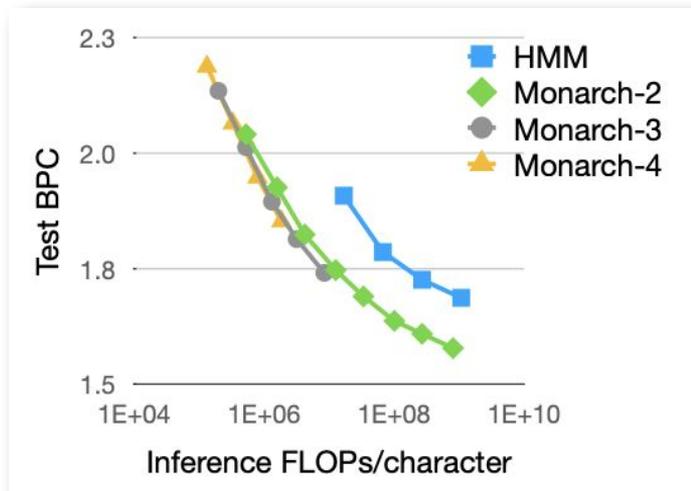
$$y_{ij} = \sum_{kl} B_{ijk} A_{jkl} x_{kl}$$

Monarch Matrices



... now just 134M parameters required!

Scaling Up Probabilistic Circuits



Type	Model	BPC (↓)	Time (s) (↓)
Flow	IAF/SCF	1.88	0.04
Flow	Argmax Coup Flow	1.80	0.40
Diffusion	D3PM Uniform	≤ 1.61	3.60
Diffusion	SEDD Uniform	≤ 1.47	-
PC	SparsePC	2.60	-
PC	NPC ²	3.17	-
PC	HMM	1.69	0.006
PC	Monarch-HMM	1.57	0.017

Text8 Character-Level Language Modelling
Roughly on par with Flow and Diffusion models

You Tricked Us



You promised us reasoning algorithms...

... and all we got was another lousy feedforward neural network!

***Theorem.** If there exists a polynomial time (real RAM) algorithm that computes (virtual evidence) marginal probabilities for a class of distributions, then there exist **poly-size circuits** for their **multilinear polynomials**.*



What is the role of the neurosymbolic AI researchers?

- We have a unique invaluable skill!
- Can we do better than prompting until it works (prompt and pray)
 - World needs tight integration of deductive reasoning and LLM
 - We know how to bridge using probabilistic logic!
- Can we do better than treating every problem as costly RL?
 - World needs deductive reasoning about LLM alignment goals.
- Can we have deductive reasoning that is tractable at scale?
- ...?

Thanks

This was the work of many wonderful students/postdocs/collaborators!



References: <http://starai.cs.ucla.edu>