Probabilistic Programming in Scala

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What is a PPL?

- Probabilistic graphical models (Bayesian networks) important in machine learning, statistics, robotics, vision, biology, neuroscience, artificial intelligence (AI) and cognitive science.

- **Probabilistic Programming Languages** unify general purpose programming with probabilistic modeling

- **Examples**
  - Functional, extending Scheme (Church) or Scala (Figaro, FACTORIE, ScalaPPL)
  - Logical, extending Prolog (ProbLog, PRISM, BLOG, Dyna)
  - Extending C# (Infer.NET)

- **Tasks**
  - Compute probabilities, most likely assignments given observations
  - Learn parameters and programs

- Applications in natural language processing, computer vision, machine learning, bioinformatics, probabilistic planning, seismic monitoring, ...
Idea behind functional PPLs
Any function \((A, B, \ldots) \Rightarrow R\) can also operate on probability distributions \((Distr[A], Distr[B], \ldots) \Rightarrow Distr[R]\)

Starting Point
Many library functions operate on Booleans:

- \&\&, \|\|, !, ==, !=
- exists, forall

Idea

1. Make a version of those functions that operates on distributions and returns a \text{Distr[Boolean]}.
2. Extend with probabilistic data structures.
3. Use them to model complex probability distributions.
PPLs: The Easy Case

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Examples
Boolean formulae

- Random Variables are objects (each object independent)

```scala
val a = Flip(0.3)
val b = Flip(0.6)
```

- BooleanDistr has member functions that build Formula objects.

```scala
val xor = a && !b || !a && b
```

- Run inference on BooleanDistr

```scala
println("Probability = " + xor.probability())
```

Probability = 0.54
Boolean formulae

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  ```
  Probability = 0.54
abstract class Timestep {
    def rainy: BooleanDistr
    def umbrella = If(rainy, Flip(0.9), Flip(0.1))
}

object StartState extends Timestep {
    val rainy = Flip(0.2)
}

class SuccessorState(predecessor: Timestep) extends Timestep {
    val rainy = If(predecessor.rainy, Flip(0.5), Flip(0.1))
}

var timestep: Timestep = StartState
for(i <- 1 until 2000) timestep = new SuccessorState(timestep)
println("Probability = " + timestep.umbrella.probability())
2-state weather HMM

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Probabilistic Data Structures

- Probabilistic List: objects are in the list with a certain probability, given by a BooleanDistr

```scala
trait ListDistr[T] extends Distribution[List[T]]{
  def forall(f: T => BooleanDistr) : BooleanDistr
  def exists(f: T => BooleanDistr) : BooleanDistr
}
```

- Replace all Boolean by BooleanDistr in member functions
- Many possibilities (Set, Tree, ... )
Probabilistic Graphs

- Viral marketing
- Learning biological pathways
- Spread of influence in social networks

```scala
class Person {
  val influencedFriends = new ListDistr[Person]
  def influences(target: Person): BooleanDistr = {
    if(target == this) True
    else friends.exists(_.influences(target))
  }
}

val p1, p2, p3, p4, p5, p6 = new Person

val influence1to2 = Flip(0.9)
n1.influencedFriends += (influence1to2, p2)
n2.influencedFriends += (influence1to2, p1)
...

println("Probability = " + p1.influences(p4).probability())
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```
Probabilistic Values

▶ Model any discrete distribution

```scala
class ValDistr[T] extends Distribution[T]{
  def ==(v: T): BooleanDistr
  def map[R](f: T => ValDistr[R]): ValDistr[R]
}
```

▶ Apply any deterministic function to ValDistr arguments

```scala
def Apply[A,R](f: (A) => R)(a: ValDistr[A]): ValDistr[R]
def Apply[A,B,R](f: (A,B) => R) ...
... 
```

Example

▶ Two dice: probability that their sum is 8?

```scala
val die1 = Uniform(1 to 6)
val die2 = Uniform(1 to 6)

val sum = Apply(_+_)(die1,die2)
println("Probability = " + (sum == 8).probability())
```
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```
3-state weather HMM

object Sunny extends Weather
object Foggy extends Weather
object Rainy extends Weather

abstract class Timestep {
  def weather: ValDistr[Weather]
  def umbrella = weather.map{
    case Sunny => Flip(0.1)
    case Foggy => Flip(0.3)
    case Rainy => Flip(0.8)
  }
}

object StartState extends Timestep {
  val weather = ValDistr((0.3,Sunny), (0.3,Foggy), (0.4,Rainy))
}

class SuccessorState(predecessor: Timestep) extends Timestep {
  val weather = predecessor.weather.map{
    case Sunny => ValDistr((0.8,Sunny), (0.15,Foggy), (0.05,Rainy))
    case Foggy => ValDistr((0.05,Sunny), (0.3,Foggy), (0.65,Rainy))
    case Rainy => ValDistr((0.15,Sunny), (0.35,Foggy), (0.5,Rainy))
  }
}
Why Scala for Probabilistic Programming?

- Probabilistic programming as a library
  - No separate compiler or VM
- mixin DSL
- Higher-order functions and generics
  - Pass existing deterministic code to probabilistic model
  - Memoization for efficient inference
- Object-oriented easy to model probabilistic databases
- Other advantages carry over to the probabilistic case
Thanks