Recent advances in lifted inference
@ Leuven

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SML, April 19, 2012
Outline

- Introduction to lifted inference
- Four contributions
  - Arbitrary constraints
  - Completeness results
  - Conditioning
  - An approximate method
Lifted inference

Exact

Variable Elimination (2003)

Knowledge compilation (2011)

…and many more!

Approximate

Belief propagation (2008)

…and many more!
MLN

1.5  Attends(person) → Series

1.2  Topic → Attends(person)
MLN

1.5 \text{Attends(person)} \rightarrow \text{Series}

1.2 \text{Topic} \rightarrow \text{Attends(person)}

\begin{align*}
\text{Attends}(p_1) & \quad \phi_1 \quad \phi_1 \quad \phi_1 \\
\text{Attends}(p_2) & \quad \phi_2 \quad \phi_2 \\
\ldots & \\
\text{Attends}(p_N) & \quad \phi_2 \quad \phi_2
\end{align*}

\text{Series} \quad \text{Topic}
MLN

1.5 \text{Attends(person)} \rightarrow \text{Series}

1.2 \text{Topic} \rightarrow \text{Attends(person)}

\begin{tabular}{|c|c|c|}
\hline
A_1 & T & \phi_2(A_1, T) \\
\hline
true & true & 3.3 \\
true & false & 3.3 \\
false & true & 1.0 \\
false & false & 3.3 \\
\hline
\end{tabular}
MLN

1.5 Attends(person) → Series

1.2 Topic → Attends(person)

Series

Attends(p₁)

Attends(p₂)

...

Attends(pₙ)

Topic

<table>
<thead>
<tr>
<th>A₁</th>
<th>T</th>
<th>φ₂(A₁,T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>true</td>
<td>true</td>
<td>3.3</td>
</tr>
<tr>
<td>true</td>
<td>false</td>
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</tr>
<tr>
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<td>true</td>
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</tr>
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<td>false</td>
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</tr>
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<table>
<thead>
<tr>
<th>Aₙ</th>
<th>T</th>
<th>φ₂(Aₙ,T)</th>
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<tbody>
<tr>
<td>true</td>
<td>true</td>
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<td>false</td>
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</tr>
</tbody>
</table>
Series

$\text{Attends}(p_1)$ $\phi_1$ $\phi_1$ $\phi_1$ $\phi_1$

$\text{Attends}(p_2)$ $\phi_2$ $\phi_2$ $\phi_2$ $\phi_2$

$\ldots$

$\text{Attends}(p_N)$ $\phi_1$ $\phi_1$ $\phi_1$ $\phi_1$

Topic
$P(S, A_1, ..., A_N, T) = \frac{1}{Z} \prod_{i=1}^{N} \phi_1(A_i, S) \prod_{i=1}^{N} \phi_2(T, A_i)$
will it become a series?
$P(S) = \frac{1}{Z} \sum_{T} \sum_{A_{1}} \cdots \sum_{A_{N}} \prod_{i=1}^{N} \phi_{1}(A_{i}, S) \prod_{i=1}^{N} \phi_{2}(T, A_{i})$

will it become a series?

$2^{(N+1)}$ terms
\[ \sum \sum \ldots \sum \prod_{i=1}^{N} \phi_1(A_i, S) \prod_{i=1}^{N} \phi_2(T, A_i) \]

2\(^{(N+1)}\) terms
\[
\sum_{T} \left( \sum_{A_1} \phi_1(A_1, S) \phi_2(T, A_1) \right) \ldots \left( \sum_{A_N} \phi_1(A_N, S) \phi_2(T, A_N) \right)
\]

1 for every person
Series

\[ \sum_{T} \left( \sum_{A_1} \phi_1(A_1, S) \phi_2(T, A_1) \right) \cdots \left( \sum_{A_N} \phi_1(A_N, S) \phi_2(T, A_N) \right) \]

N times the same product!
\[ \sum \left( \sum_{A_1} \phi_1(A_1, S) \phi_2(T, A_1) \right) \ldots \left( \sum_{A_N} \phi_1(A_N, S) \phi_2(T, A_N) \right) \]

N times the same sum!
lifted: \[ \sum_{T} \left( \sum_{A} \phi_1(A, S) \phi_2(T, A) \right)^N \]

compute only once!
Series

Attends(p₁)  \quad \phi_1

Attends(p₂)  \quad \phi_1

...  \quad \phi_1

Attends(pₙ)  \quad \phi_1

Topic

\sum T \left( \sum_A \phi_1(A, S) \phi_2(T, A) \right)^N

“lifted sum-out”  \quad “lifted multiplication”
Lifted Variable Elimination

[Poole ’03, …]

- Repeatedly apply certain operators on the model
  - Lifted multiplication
  - Lifted sum-out
  - …
- Until the desired result is found
Lifted Knowledge Compilation
[Van den Broeck et al ‘11,…]

- Compile the model into a “lifted” circuit (“FO d-DNNF”)
  - How? Compilation rules
- Inference = traversing the circuit
  - Time = poly(domain size)
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$S$ 

$A(p_1)$  $\ldots$  $A(p_{N/2})$  $A(p_{N/2+1})$  $\ldots$  $A(p_N)$ 

$T$
$A(p_1)$  $\ldots$  $A(p_{N/2})$  $A(p_{N/2}+1)$  $\ldots$  $A(p_N)$
\[ S(p_1) \quad \ldots \quad A(p_{N/2}) \quad A(p_{N/2+1}) \quad \ldots \quad A(p_N) \]
Bigger groups = more lifting!
Bigger groups = more lifting!
The groups are specified by constraints
Importance of constraints
[Taghipour et al, AISTATS'12]

- Exact lifted algorithms use a particular constraint language
  
  \[ \text{group} \to \text{constraint} \to \text{can it be expressed in the language?} \]

- Often leads to unnecessarily small groups
  \[ \to \text{less lifting} \]
Importance of constraints
[Taghipour et al, AISTATS'12]

- Exact lifted algorithms use a particular constraint language
  
  group → constraint → can it be expressed in the language?

- Often leads to unnecessarily small groups → less lifting

- We avoid using a particular constraint language
  Instead: arbitrary constraints + relational algebra
pairwise constraints (C-FOVE)

arbitrary constraints

more evidence
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What is Lifted Inference?

- Propositional inference is intractable
  Solution: lifted inference
    “Exploit symmetries”
    “Reason at first-order level”
    “Reason about groups of objects as a whole”
    “Avoid repeated computations”
    “Mimic resolution in theorem proving”

- There is a common understanding but no formal definition of lifted inference!
What is Lifted Inference?

- What is commonly understood as exact lifted inference?

**Definition: Domain-Lifted Inference**
Complexity of computing $P(q|e)$ in model $m$ is polynomial time in the domain sizes of the logical variables in $q,e,m$
What is Lifted Inference?

- What is commonly understood as exact lifted inference?

**Definition: Domain-Lifted Inference**
Complexity of computing \( P(q|e) \) in model \( m \) is polynomial time in the domain sizes of the logical variables in \( q,e,m \)

- Possibly exponential in the size of \( q,e,m \)
  
  # predicates, # parfactors, # atoms, 
  # arguments, # formulas, # constants in model

[Van den Broeck NIPS11]
What is Lifted Inference?

- Motivation: Large domains lead to intractable propositional inference.
- A formal framework for lifted inference
  - Definition + complexity considerations
  - ~ PAC-learnability (Valiant)
- Other notions, e.g., for approximate inference.
Completeness

- A procedure that is domain-lifted for all models in a class $M$ is called complete for $M$

  All models in $M$ are “liftable”

- There was no completeness result for existing algorithms

  If you give me a model, I cannot say if grounding will be needed, until I run the inference algorithm itself.

[Van den Broeck NIPS11]
Completeness Result

Probabilistic inference in models with
- universal quantifiers $\forall$ and
- 2 logical variables per formula
is domain-liftable.

- A non-trivial class of models
- First completeness results in exact lifted inference
  - Lifted knowledge compilation procedure
  - Lifted variable elimination procedure

[Van den Broeck NIPS11], [Taghipour et al.]
Completeness Game

No domain-lifted inference procedure exists

FOL ∀,∃,= [Jaeger 99]

... [Jaeger 12]

FOL ∀,=, 2 variables [Van den Broeck 11]

Complete domain-lifted inference procedure
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  • Approximate inference
Conditioning

- Task: Probability of query $q$ given evidence $e$: $P(q|e)$
  
  Domain-lifted inference is exponential in the size of $e$.
- Can we compute conditional probabilities efficiently?
  
  Depends on the arity of literals conditioned on:

<table>
<thead>
<tr>
<th>Literal Arity</th>
<th>Complexity of Conditioning</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Polynomial</td>
</tr>
<tr>
<td>1</td>
<td>Polynomial if supported by compilation</td>
</tr>
<tr>
<td>$\geq 2$</td>
<td>$#P$-hard</td>
</tr>
</tbody>
</table>

- Positive and negative result for lifted inference

[Van den Broeck, Davis AAAI12]
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Lifted RCR

• **Practical** usefulness of lifted inference shown for **approximate** inference with lifted BP

• Lifted Relax, Compensate and Recover
  1. Clone all atoms in a model
  2. Relax equivalences between clones
  3. Compensate for removed equivalences
  4. Recover equivalences until model too complex

• Exact lifted inference black box in (3)

[Van den Broeck, Choi, Darwiche]
Lifted RCR

Special case: Lifted BP
Tractable

Exact lifted inference
Intractable

Approximation Error
% KLD of IBP

% Recovered Equivalences

[Van den Broeck, Choi, Darwiche]
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Posters!