Recent advances in lifted inference
@ Leuven

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Outline

- Introduction to lifted inference
- Four contributions
  - Arbitrary constraints
  - Completeness results
  - Conditioning
  - An approximate method
Lifted inference

Exact
  Variable Elimination (2003)
  Knowledge compilation (2011)

Approximate
  Belief propagation (2008)

... and many more!
MLN

1.5 Attends(person) → Series

1.2 Topic → Attends(person)
MLN

1.5 Atends(person) → Series

1.2 Topic → Atends(person)

Series

Attends(p₁)
Attends(p₂)
...
Attends(pₙ)

Topic
MLN

1.5 Attends(person) $\rightarrow$ Series

1.2 Topic $\rightarrow$ Attends(person)

Series

$\phi_1$

$\phi_1$

$\phi_1$

$\phi_1$

Attends($p_1$)

Attends($p_2$)

... 

Attends($p_N$)

$\phi_2$

$\phi_2$

$\phi_2$

$\phi_2$

Topic
MLN

1.5 \text{Attends(person)} \rightarrow \text{Series}

1.2 \text{Topic} \rightarrow \text{Attends(person)}

\begin{align*}
\text{Series} & \rightarrow \phi_1 \\
\text{Attends}(p_1) & \rightarrow \phi_2 \\
\text{Attends}(p_2) & \rightarrow \phi_2 \\
\vdots & \\
\text{Attends}(p_N) & \rightarrow \phi_2
\end{align*}

\begin{tabular}{|c|c|c|}
\hline
A_1 & T & \phi_2(A_1,T) \\
\hline
true & true & 3.3 \\
true & false & 3.3 \\
false & true & 1.0 \\
false & false & 3.3 \\
\hline
\end{tabular}
MLN

1.5 \text{Attends(person)} \rightarrow \text{Series}

1.2 \text{Topic} \rightarrow \text{Attends(person)}

\begin{align*}
\text{Attends}(p_1) & \rightarrow \phi_1 \\
\text{Attends}(p_2) & \rightarrow \phi_1 \\
\vdots & \\
\text{Attends}(p_N) & \rightarrow \phi_1
\end{align*}

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\hline
\end{tabular}

\begin{tabular}{|c|c|c|}
\hline
\text{A}_N & \text{T} & \phi_2(\text{A}_N, \text{T}) \\
\hline
true & true & 3.3 \\
true & false & 3.3 \\
false & true & 1.0 \\
false & false & 3.3 \\
\hline
\end{tabular}
\[ P(S, A_1, ..., A_N, T) = \frac{1}{Z} \prod_{i=1}^{N} \phi_1(A_i, S) \prod_{i=1}^{N} \phi_2(T, A_i) \]
Series

Attends(p₁)

ϕ₁

ϕ₁

ϕ₂

ϕ₁

ϕ₂

ϕ₂

ϕ₂

Topic

\[ P(S) = \frac{1}{Z} \sum_{T} \sum_{A_1} \ldots \sum_{A_N} \prod_{i=1}^{N} \phi_1(A_i, S) \prod_{i=1}^{N} \phi_2(T, A_i) \]

will it become a series?
Series

Attends(p₁)

Attends(p₂)

...

Attends(pₙ)

Topic

\[ P(S) = \frac{1}{Z} \sum_{T} \sum_{A_1} \ldots \sum_{A_N} \prod_{i=1}^{N} \phi_1(A_i, S) \prod_{i=1}^{N} \phi_2(T, A_i) \]

will it become a series?  

2^{(N+1)} terms
\[ \sum \sum \ldots \sum \prod_{i=1}^{N} \phi_1(A_i, S) \prod_{i=1}^{N} \phi_2(T, A_i) \]

2\(^{N+1}\) terms
\[
\sum_{T} \left( \sum_{A_1} \phi_1(A_1, S) \phi_2(T, A_1) \right) \ldots \left( \sum_{A_N} \phi_1(A_N, S) \phi_2(T, A_N) \right)
\]

1 for every person
\[
\sum_{T} \left( \sum_{A_{i}} \phi_{1}(A_{i}, S) \phi_{2}(T, A_{i}) \right) \cdots \left( \sum_{A_{N}} \phi_{1}(A_{N}, S) \phi_{2}(T, A_{N}) \right)
\]

N times the same product!
\[ \sum_{T} \left( \sum_{A_1} \phi_1(A_1, S) \phi_2(T, A_1) \right) \ldots \left( \sum_{A_N} \phi_1(A_N, S) \phi_2(T, A_N) \right) \]

\text{N times the same sum !}
Series

\[ \sum \phi_1(p_1) \phi_2 \sum \phi_1(p_2) \phi_2 \ldots \sum \phi_1(p_N) \phi_2 \]

\text{Topic}

\text{lifted: } \sum_T \left( \sum_A \phi_1(A, S) \phi_2(T, A) \right)^N

\text{compute only once!}
lifted: \[ \sum_T \left( \sum_A \phi_1(A, S) \phi_2(T, A) \right)^N \]

“lifted sum-out”

“lifted multiplication”
Lifted Variable Elimination

[Poole ’03,…]

- Repeatedly apply certain operators on the model
  - Lifted multiplication
  - Lifted sum-out
  - ...
- Until the desired result is found
Lifted Knowledge Compilation
[Van den Broeck et al ‘11,…]

- Compile the model into a “lifted” circuit (“FO d-DNNF”)
  - How? Compilation rules
- Inference = traversing the circuit
  - Time = \( poly(\text{domain size}) \)
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$A(p_1)$, $\ldots$, $A(p_{N/2})$, $A(p_{N/2}+1)$, $\ldots$, $A(p_N)$
Bigger groups = more lifting!
Bigger groups = more lifting!
The groups are specified by constraints.
Importance of constraints
[Taghipour et al, AISTATS'12]

- Exact lifted algorithms use a particular constraint language

  group $\rightarrow$ constraint $\rightarrow$ can it be expressed in the language?

- Often leads to unnecessarily small groups $\rightarrow$ less lifting
Importance of constraints
[Taghipour et al, AISTATS'12]

- Exact lifted algorithms use a particular constraint language

  group → constraint → can it be expressed in the language?

- Often leads to unnecessarily small groups
  → less lifting

- We avoid using a particular constraint language
  Instead: arbitrary constraints
    + relational algebra
pairwise constraints (C-FOVE)

runtime (log)

more evidence

arbitrary constraints
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What is Lifted Inference?

- Propositional inference is intractable
  
  Solution: lifted inference
  
  “Exploit symmetries”
  “Reason at first-order level”
  “Reason about groups of objects as a whole”
  “Avoid repeated computations”
  “Mimic resolution in theorem proving”

- There is a common understanding but no formal definition of lifted inference!
What is Lifted Inference?

• What is commonly understood as exact lifted inference?

Definition: Domain-Lifted Inference
Complexity of computing $P(q|e)$ in model $m$ is polynomial time in the domain sizes of the logical variables in $q,e,m$

1.5  Attends(person) → Series
1.2  Topic → Attends(person)

[Van den Broeck NIPS11]
What is Lifted Inference?

- What is commonly understood as exact lifted inference?

   **Definition: Domain-Lifted Inference**
   Complexity of computing $P(q|e)$ in model $m$ is polynomial time in the domain sizes of the logical variables in $q, e, m$

- Possibly exponential in the size of $q, e, m$
   
   # predicates, # parfactors, # atoms, # arguments, # formulas, # constants in model

[Van den Broeck NIPS11]
What is Lifted Inference?

- Motivation: Large domains lead to intractable propositional inference.
- A formal framework for lifted inference
  - Definition + complexity considerations
  - \(\sim\) PAC-learnability (Valiant)
- Other notions, e.g., for approximate inference.
Completeness

• A procedure that is domain-lifted for all models in a class $M$ is called complete for $M$

  All models in $M$ are “liftable”

• There was no completeness result for existing algorithms

  If you give me a model,
  I cannot say if grounding will be needed,
  until I run the inference algorithm itself.

[Van den Broeck NIPS11]
Completeness Result

Probabilistic inference in models with

- universal quantifiers $\forall$ and
- 2 logical variables per formula

is domain-liftable.

- A non-trivial class of models
- First completeness results in exact lifted inference
  - Lifted knowledge compilation procedure
  - Lifted variable elimination procedure

[Van den Broeck NIPS11], [Taghipour et al.]
Completeness Game

No domain-lifted inference procedure exists

FOL $\forall, \exists, =$ [Jaeger 99]

... [Jaeger 12]

[FOL $\forall, =$, 2 variables] [Van den Broeck 11]

Complete domain-lifted inference procedure
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• Introduction to lifted inference
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  • Completeness result
  • **Conditioning**
  • Approximate inference
Conditioning

- Task: Probability of query $q$ given evidence $e$: $P(q|e)$
  Domain-lifted inference is exponential in the size of $e$.
- Can we compute conditional probabilities efficiently?
  Depends on the arity of literals conditioned on:

<table>
<thead>
<tr>
<th>Literal Arity</th>
<th>Complexity of Conditioning</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Polynomial</td>
</tr>
<tr>
<td>1</td>
<td>Polynomial if supported by compilation #P-hard</td>
</tr>
<tr>
<td>$\geq 2$</td>
<td></td>
</tr>
</tbody>
</table>

- Positive and negative result for lifted inference

[Van den Broeck, Davis AAAI12]
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Lifted RCR

- **Practical** usefulness of lifted inference shown for **approximate** inference with lifted BP
- Lifted Relax, Compensate and Recover
  1. Clone all atoms in a model
  2. Relax equivalences between clones
  3. Compensate for removed equivalences
  4. Recover equivalences until model too complex
- **Exact** lifted inference black box in (3)

[Van den Broeck, Choi, Darwiche]
Lifted RCR

Special case: Lifted BP
Tractable

Exact lifted inference
Intractable

Approximation Error
% KLD of IBP

% Recovered Equivalences

[Van den Broeck, Choi, Darwiche]
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Posters!

Website & Implementation: http://dtai.cs.kuleuven.be/ml/systems/wfomc