Scalable Inference and Learning for High-Level Probabilistic Models

Guy Van den Broeck
KU Leuven
Outline

• Motivation
  – Why high-level representations?
  – Why high-level reasoning?

• Intuition: Inference rules

• Liftability theory: Strengths and limitations

• Lifting in practice
  – Approximate symmetries
  – Lifted learning
Outline

• Motivation
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• Liftability theory: Strengths and limitations
• Lifting in practice
  – Approximate symmetries
  – Lifted learning
Graphical Model Learning

Medical Records

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Graphical Model Learning

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Bayesian Network

![Bayesian Network Diagram]

- Asthma
- Smokes
- Cough
Graphical Model Learning

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Bayesian Network

Big data

Medical Records

Bayesian Network

Asthma

Smokes

Cough
# Graphical Model Learning

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## Bayesian Network

- **Asthma**
- **Smokes**
- **Cough**
Graphical Model Learning

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| Frank | 1 | ? | ? |

Frank 1 0.3 0.2

Bayesian Network

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Graphical Model Learning

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Bayesian Network

Asthma → Smokes

Cough

Friends

Brothers
Graphical Model Learning

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### Bayesian Network

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**Friends**

- Frank

**Brothers**

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Graphical Model Learning

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Bayesian Network

Asthma
Smokes
Cough

Rows are independent during learning and inference!
Statistical Relational Representations

Augment graphical model with relations between entities (rows).

Intuition

Markov Logic

+ Friends have similar smoking habits
+ Asthma can be hereditary
Statistical Relational Representations

Augment graphical model with relations between entities (rows).

**Intuition**

- Friends have similar smoking habits
- Asthma can be hereditary

**Markov Logic**

- 2.1  Asthma $\Rightarrow$ Cough
- 3.5  Smokes $\Rightarrow$ Cough
Statistical Relational Representations

Augment graphical model with relations between entities (rows).

**Intuition**

- Asthma
- Smokes
- Cough

+ Friends have similar smoking habits
+ Asthma can be hereditary

**Markov Logic**

2.1 Asthma(x) ⇒ Cough(x)
3.5 Smokes(x) ⇒ Cough(x)

Logical variables refer to entities
Statistical Relational Representations

Augment graphical model with relations between entities (rows).

**Intuition**

- Friends have similar smoking habits
- Asthma can be hereditary

**Markov Logic**

1.9 \( \text{Smokes}(x) \land \text{Friends}(x,y) \Rightarrow \text{Smokes}(y) \)

1.5 \( \text{Asthma}(x) \land \text{Family}(x,y) \Rightarrow \text{Asthma}(y) \)

2.1 \( \text{Asthma}(x) \Rightarrow \text{Cough}(x) \)

3.5 \( \text{Smokes}(x) \Rightarrow \text{Cough}(x) \)
Equivalent Graphical Model

- Statistical relational model (e.g., MLN)
  
  \[
  \text{1.9 } \text{Smokes}(x) \land \text{Friends}(x,y) \Rightarrow \text{Smokes}(y)
  \]

- Ground atom/tuple = random variable in \{true,false\}
  e.g., Smokes(Alice), Friends(Alice,Bob), etc.

- Ground formula = factor in propositional factor graph
Research Overview

Generality

Knowledge Representation

Graphical Models
Bayesian Networks
Research Overview

Knowledge Representation

Generality

Bayesian Networks
Graphical Models
Statistical Relational Models
Probabilistic Databases

- Tuple-independent probabilistic databases

<table>
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<tr>
<td>Brando</td>
<td>0.9</td>
</tr>
<tr>
<td>Cruise</td>
<td>0.8</td>
</tr>
<tr>
<td>Coppola</td>
<td>0.1</td>
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- Query: SQL or First-order logic

```
SELECT Actor.name
FROM Actor, WorkedFor
WHERE Actor.name = WorkedFor.actor
```

- Learned from the web, large text corpora, ontologies, etc., using statistical machine learning.

```
Q(x) = \exists y \text{ Actor}(x) \land \text{WorkedFor}(x, y)
```
Google Knowledge Graph
Google Knowledge Graph

> 570 million entities
> 18 billion tuples
Research Overview

- Probabilistic Programming
- Statistical Relational Models
- Probabilistic Databases
- Graphical Models
- Bayesian Networks

Generality

Knowledge Representation
Probabilistic Programming

• Programming language + random variables
• Reason about distribution over executions
  
  As going from hardware circuits to programming languages

• **ProbLog**: Probabilistic logic programming/datalog

• Example: Gene/protein interaction networks

  Edges (interactions) have probability

  “Does there exist a path connecting two proteins?”

  \[
  \text{path}(X, Y) :- \text{edge}(X, Y).
  \]

  \[
  \text{path}(X, Y) :- \text{edge}(X, Z), \text{path}(Z, Y).
  \]

  Cannot be expressed in first-order logic

  Need a full-fledged programming language!
Research Overview

- Probabilistic Programming
- Statistical Relational Models
- Probabilistic Databases
- Graphical Models
- Bayesian Networks
Research Overview

- Probabilistic Programming
- Statistical Relational Models
- Probabilistic Databases
- Graphical Models
- Bayesian Networks
- Graphical Model Inference
- Program Sampling
- Lifted Inference
Research Overview

- Knowledge
- Reasoning
- Machine Learning

- Probabilistic Programming
- Statistical Relational Models
- Graphical Models
- Probabilistic Databases
- Bayesian Networks
- Program Sampling
- Graphical Model Inference
- Lifted Inference
- Lifted Learning
- Program Induction
- Statistical Relational Learning
- Graphical Model Learning

Generality
Outline

• Motivation
  – Why high-level representations?
  – Why high-level reasoning?

• Intuition: Inference rules

• Liftability theory: Strengths and limitations

• Lifting in practice
  – Approximate symmetries
  – Lifted learning
A Simple Reasoning Problem

• 52 playing cards
• Let us ask some simple questions

[Van den Broeck; AAAI-KRR’15]
A Simple Reasoning Problem

Probability that Card1 is Q?
A Simple Reasoning Problem

Probability that Card1 is Q? 1/13

[Van den Broeck; AAAI-KRR’15]
A Simple Reasoning Problem

? ?? ?? ?? ???

Probability that Card1 is Hearts?

[Van den Broeck; AAAI-KRR’15]
A Simple Reasoning Problem

Probability that Card1 is Hearts? 1/4
A Simple Reasoning Problem

Probability that Card1 is Hearts given that Card1 is red?

[Van den Broeck; AAAI-KRR’15]
A Simple Reasoning Problem

Probability that Card1 is Hearts given that Card1 is red? 1/2

[Van den Broeck; AAAI-KRR’15]
A Simple Reasoning Problem

Probability that Card52 is Spades given that Card1 is QH?

[Van den Broeck; AAAI-KRR’15]
A Simple Reasoning Problem

Probability that Card52 is Spades given that Card1 is QH? 13/51

[Van den Broeck; AAAI-KRR’15]
Automated Reasoning

Let us automate this:

1. Probabilistic graphical model (e.g., factor graph)

2. Probabilistic inference algorithm (e.g., variable elimination or junction tree)
Classical Reasoning

- Tree
  - Higher treewidth
  - Fewer conditional independencies
  - Slower inference

- Sparse Graph

- Dense Graph
Is There Conditional Independence?

\[ P(\text{Card52} \mid \text{Card1}) \overset{?}{=} P(\text{Card52} \mid \text{Card1}, \text{Card2}) \]
Is There Conditional Independence?

\[ P(\text{Card52} \mid \text{Card1}) \overset{?}{=} P(\text{Card52} \mid \text{Card1, Card2}) \]

\[ ? \overset{?}{=} ? \]
Is There Conditional Independence?

\[
P(\text{Card52} \mid \text{Card1}) \overset{?}{=} P(\text{Card52} \mid \text{Card1, Card2})
\]

13/51 \overset{?}{=} ?
Is There Conditional Independence?

\[
P(\text{Card52} \mid \text{Card1}) \overset{?}{=} P(\text{Card52} \mid \text{Card1}, \text{Card2})
\]

\[
13/51 \overset{?}{=}
\]
Is There Conditional Independence?

\[ P(\text{Card}_{52} \mid \text{Card}_{1}) \not\equiv P(\text{Card}_{52} \mid \text{Card}_{1}, \text{Card}_{2}) \]

\[ \frac{13}{51} \neq \frac{12}{50} \]
Is There Conditional Independence?

\[ \frac{13}{51} \neq \frac{12}{50} \]

\[ P(\text{Card52} \mid \text{Card1}) \neq P(\text{Card52} \mid \text{Card1, Card2}) \]

\[ 13/51 \neq 12/50 \]
Is There Conditional Independence?

\[ P(\text{Card52} \mid \text{Card1}) \neq P(\text{Card52} \mid \text{Card1}, \text{Card2}) \]

\[ 13/51 \neq 12/50 \]

\[ P(\text{Card52} \mid \text{Card1}, \text{Card2}) \neq P(\text{Card52} \mid \text{Card1}, \text{Card2}, \text{Card3}) \]
Is There Conditional Independence?

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P(\text{Card52} \mid \text{Card1}) \neq P(\text{Card52} \mid \text{Card1}, \text{Card2})
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\frac{13}{51} \neq \frac{12}{50}
\]

\[
P(\text{Card52} \mid \text{Card1}, \text{Card2}) \neq P(\text{Card52} \mid \text{Card1}, \text{Card2}, \text{Card3})
\]

\[
\frac{12}{50} \neq \frac{12}{49}
\]
Automated Reasoning

Let us automate this:

1. Probabilistic graphical model (e.g., factor graph) is fully connected!

2. Probabilistic inference algorithm (e.g., variable elimination or junction tree) builds a table with $52^{52}$ rows

[Van den Broeck; AAAI-KRR’15]
What's Going On Here?

Probability that Card52 is Spades given that Card1 is QH?

[Van den Broeck; AAAI-KRR’15]
What's Going On Here?

Probability that Card52 is Spades given that Card1 is QH? 13/51

[Van den Broeck; AAAI-KRR’15]
What's Going On Here?

Probability that Card52 is Spades given that Card2 is QH?
What's Going On Here?

Probability that Card52 is Spades given that Card2 is QH?

13/51

[Van den Broeck; AAAI-KRR’15]
What's Going On Here?

Probability that Card52 is Spades given that Card3 is QH?
What's Going On Here?

Probability that Card52 is Spades given that Card3 is QH?

13/51

[Van den Broeck; AAAI-KRR’15]
Tractable Probabilistic Inference

Which property makes inference tractable?
Traditional belief: Independence
What's going on here?

[Niepert, Van den Broeck; AAAI’14], [Van den Broeck; AAAI-KRR’15]
Tractable Probabilistic Inference

Which property makes inference tractable?

Traditional belief: Independence

What's going on here?

• High-level reasoning
• Symmetry
• Exchangeability

⇒ Lifted Inference

[Niepert, Van den Broeck; AAAI’14], [Van den Broeck; AAAI-KRR’15]
Other Examples of Lifted Inference

- Syllogisms & First-order resolution
- Reasoning about populations

We are investigating a rare disease. The disease is more rare in women, presenting only in one in every two billion women and one in every billion men. Then, assuming there are 3.4 billion men and 3.6 billion women in the world, the probability that more than five people have the disease is

\[
1 - \sum_{n=0}^{5} \sum_{f=0}^{n} \binom{3.6 \times 10^9}{f} \left(1 - 0.5 \times 10^{-9}\right)^{3.6 \times 10^9 - f} \left(0.5 \times 10^{-9}\right)^f \\
\times \binom{3.4 \times 10^9}{n-f} \left(1 - 10^{-9}\right)^{3.4 \times 10^9 - (n-f)} \left(10^{-9}\right)^{n-f}
\]

[Van den Broeck; AAAI-KRR’15], [Van den Broeck; PhD’13]
Equivalent Graphical Model

- Statistical relational model (e.g., MLN)

\[ 3.14 \text{ FacultyPage}(x) \land \text{Linked}(x,y) \Rightarrow \text{CoursePage}(y) \]

- As a probabilistic graphical model:
  - 26 pages; 728 variables; 676 factors
  - 1000 pages; 1,002,000 variables; 1,000,000 factors

- Highly intractable?
  - **Lifted inference** in milliseconds!
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Weighted Model Counting

- Model = solution to a propositional logic formula $\Delta$
- Model counting = $\#SAT$

$\Delta = (\text{Rain} \Rightarrow \text{Cloudy})$

<table>
<thead>
<tr>
<th>Rain</th>
<th>Cloudy</th>
<th>Model?</th>
</tr>
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<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>Yes</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>No</td>
</tr>
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$\#SAT = 3$
Weighted Model Counting

- Model = solution to a propositional logic formula $\Delta$
- Model counting = $\#\text{SAT}$
- Weighted model counting (WMC)
  - Weights for assignments to variables
  - Model weight is product of variable weights $w(.)$

$\Delta = (\text{Rain} \Rightarrow \text{Cloudy})$

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<tr>
<td>T</td>
<td>T</td>
<td>Yes</td>
<td>1 * 3 = 3</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>No</td>
<td>2 * 3 = 0</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>Yes</td>
<td>2 * 3 = 6</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>Yes</td>
<td>2 * 5 = 10</td>
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$\#\text{SAT} = 3$
Weighted Model Counting

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$w(\text{R}) = 1$
$w(\neg\text{R}) = 2$
$w(\text{C}) = 3$
$w(\neg\text{C}) = 5$

$\text{WMC} = 19$
Assembly language for probabilistic reasoning

- Bayesian networks
- Factor graphs
- Probabilistic logic programs
- Markov Logic
- Relational Bayesian networks
- Probabilistic databases
- Weighted Model Counting
Weighted First-Order Model Counting

Model = solution to first-order logic formula $\Delta$

$\Delta = \forall d \ (\text{Rain}(d) \Rightarrow \text{Cloudy}(d))$

Days = \{\text{Monday}\}
Weighted First-Order Model Counting

Model = solution to first-order logic formula $\Delta$

$$\Delta = \forall d \ (\text{Rain}(d) \Rightarrow \text{Cloudy}(d))$$

Days = \{Monday\}

<table>
<thead>
<tr>
<th>Rain(M)</th>
<th>Cloudy(M)</th>
<th>Model?</th>
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<tbody>
<tr>
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<tr>
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$\#\text{SAT} = 3$
**Weighted First-Order Model Counting**

Model = solution to first-order logic formula $\Delta$

$\Delta = \forall d (\text{Rain}(d) \Rightarrow \text{Cloudy}(d))$

Days = \{Monday, Tuesday\}

<table>
<thead>
<tr>
<th>Rain(M)</th>
<th>Cloudy(M)</th>
<th>Rain(T)</th>
<th>Cloudy(T)</th>
<th>Model?</th>
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Weighted First-Order Model Counting

Model = solution to first-order logic formula $\Delta$

$\Delta = \forall d \ (\text{Rain}(d) \Rightarrow \text{Cloudy}(d))$

Days = {Monday, Tuesday}

<table>
<thead>
<tr>
<th>Rain(M)</th>
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</table>

$\#\text{SAT} = 9$
## Weighted First-Order Model Counting

Model = solution to first-order logic formula $\Delta$

### Formula

$$\Delta = \forall d \ (\text{Rain}(d) \Rightarrow \text{Cloudy}(d))$$

### Days

- Days = {Monday, Tuesday}

### Weights

- $w(\text{R}) = 1$
- $w(\neg\text{R}) = 2$
- $w(\text{C}) = 3$
- $w(\neg\text{C}) = 5$

### Model Evaluation

<table>
<thead>
<tr>
<th>Rain(M)</th>
<th>Cloudy(M)</th>
<th>Rain(T)</th>
<th>Cloudy(T)</th>
<th>Model?</th>
<th>Weight</th>
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<tbody>
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<td>T</td>
<td>T</td>
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<td>Yes</td>
<td>1 * 1 * 3 * 3 = 9</td>
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<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>No</td>
<td>2 * 1 * 3 * 3 = 18</td>
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<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>Yes</td>
<td>2 * 1 * 5 * 3 = 30</td>
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<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>Yes</td>
<td>2 * 1 * 5 * 3 = 30</td>
</tr>
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### Weight Calculation

$$\#\text{SAT} = 9$$
Weighted First-Order Model Counting

Model = solution to first-order logic formula $\Delta$

$\Delta = \forall d \ (\text{Rain}(d) \Rightarrow \text{Cloudy}(d))$

<table>
<thead>
<tr>
<th>Days = {Monday Tuesday}</th>
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<tbody>
<tr>
<td>$\text{w}(\text{R})=1$</td>
</tr>
<tr>
<td>$\text{w}(\lnot\text{R})=2$</td>
</tr>
<tr>
<td>$\text{w}(\text{C})=3$</td>
</tr>
<tr>
<td>$\text{w}(\lnot\text{C})=5$</td>
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</table>

<table>
<thead>
<tr>
<th>Rain(M)</th>
<th>Cloudy(M)</th>
<th>Rain(T)</th>
<th>Cloudy(T)</th>
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<tr>
<td>T</td>
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<td>$1 \times 1 \times 3 \times 3 = 9$</td>
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<td>T</td>
<td>Yes</td>
<td>$2 \times 1 \times 3 \times 3 = 18$</td>
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<tr>
<td>F</td>
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<td>T</td>
<td>Yes</td>
<td>$2 \times 1 \times 5 \times 3 = 30$</td>
</tr>
<tr>
<td>T</td>
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<td>$1 \times 2 \times 3 \times 3 = 18$</td>
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<td>$2 \times 2 \times 3 \times 5 = 60$</td>
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<td>$2 \times 2 \times 5 \times 5 = 100$</td>
</tr>
</tbody>
</table>

$\text{WFOMC} = 361$
Assembly language for high-level probabilistic reasoning

Parfactor graphs

Probabilistic logic programs

Markov Logic

Relational Bayesian networks

Probabilistic databases

Weighted First-Order Model Counting

[VdB et al.; IJCAI’11, PhD’13, KR’14, UAI’14]
WFOMC Inference: Example

- FO-Model Counting: $w(R) = w(\neg R) = 1$
- Apply inference rules backwards (step 4-3-2-1)
WFOMC Inference: Example

- FO-Model Counting: $w(R) = w(\neg R) = 1$
- Apply inference rules backwards (step 4-3-2-1)

\[ \Delta = (\text{Stress}(\text{Alice}) \Rightarrow \text{Smokes}(\text{Alice})) \]

Domain = \{Alice\}
WFOMC Inference: Example

- FO-Model Counting: $w(R) = w(\neg R) = 1$
- Apply inference rules backwards (step 4-3-2-1)

4. $\Delta = (\text{Stress}(\text{Alice}) \Rightarrow \text{Smokes}(\text{Alice}))$

$\rightarrow$ 3 models

Domain = \{Alice\}
WFOMC Inference: Example

- FO-Model Counting: \( w(R) = w(-R) = 1 \)
- Apply inference rules backwards (step 4-3-2-1)

4. \( \Delta = (\text{Stress(Alice)} \Rightarrow \text{Smokes(Alice)}) \)
   
   \( \rightarrow 3 \text{ models} \)

3. \( \Delta = \forall x, (\text{Stress}(x) \Rightarrow \text{Smokes}(x)) \)
   
   Domain = \{n people\}
WFOMC Inference: Example

- FO-Model Counting: $w(R) = w(\neg R) = 1$
- Apply inference rules backwards (step 4-3-2-1)

4. $\Delta = (\text{Stress(Alice)} \Rightarrow \text{Smokes(Alice)})$
   \hspace{2cm} Domain = \{\text{Alice}\}
   \rightarrow 3$\text{models}$

3. $\Delta = \forall x, (\text{Stress}(x) \Rightarrow \text{Smokes}(x))$
   \hspace{2cm} Domain = \{\text{n people}\}
   \rightarrow 3^n$\text{models}$
WFOMC Inference: Example

3. \( \Delta = \forall x, (\text{Stress}(x) \Rightarrow \text{Smokes}(x)) \)

\( \rightarrow 3^n \) models

Domain = \{n people\}
WFOMC Inference: Example

3. \( \Delta = \forall x, (\text{Stress}(x) \Rightarrow \text{Smokes}(x)) \)  
   \( \rightarrow 3^n \text{ models} \)  
   \( \text{Domain} = \{n \text{ people}\} \)

2. \( \Delta = \forall y, (\text{ParentOf}(y) \land \text{Female} \Rightarrow \text{MotherOf}(y)) \)  
   \( \text{D} = \{n \text{ people}\} \)
WFOMC Inference: Example

3. \( \Delta = \forall x, (\text{Stress}(x) \Rightarrow \text{Smokes}(x)) \)
   \( \text{Domain} = \{n \text{ people}\} \)
   \( \rightarrow 3^n \text{ models} \)

2. \( \Delta = \forall y, (\text{ParentOf}(y) \land \text{Female} \Rightarrow \text{MotherOf}(y)) \)
   \( \text{D} = \{n \text{ people}\} \)
   If Female = true?
   \( \Delta = \forall y, (\text{ParentOf}(y) \Rightarrow \text{MotherOf}(y)) \)
   \( \rightarrow 3^n \text{ models} \)
WFOMC Inference: Example

3. \( \Delta = \forall x, (\text{Stress}(x) \Rightarrow \text{Smokes}(x)) \)

\[ \Rightarrow 3^n \text{ models} \]

Domain = \{n people\}

2. \( \Delta = \forall y, (\text{ParentOf}(y) \land \text{Female} \Rightarrow \text{MotherOf}(y)) \)

If Female = true? \( \Delta = \forall y, (\text{ParentOf}(y) \Rightarrow \text{MotherOf}(y)) \) \( \Rightarrow 3^n \text{ models} \)

If Female = false? \( \Delta = \text{true} \) \( \Rightarrow 4^n \text{ models} \)
WFOMC Inference: Example

3. \[ \Delta = \forall x, (\text{Stress}(x) \Rightarrow \text{Smokes}(x)) \]
   \[ \text{Domain} = \{\text{n people}\} \]
   \[ \rightarrow 3^n \text{ models} \]

2. \[ \Delta = \forall y, (\text{ParentOf}(y) \land \text{Female} \Rightarrow \text{MotherOf}(y)) \]
   \[ \text{D} = \{\text{n people}\} \]
   If Female = true?
   \[ \Delta = \forall y, (\text{ParentOf}(y) \Rightarrow \text{MotherOf}(y)) \]
   \[ \rightarrow 3^n \text{ models} \]
   If Female = false?
   \[ \Delta = \text{true} \]
   \[ \rightarrow 4^n \text{ models} \]

\[ \rightarrow 3^n + 4^n \text{ models} \]
WFOMC Inference: Example

1. \[ \Delta = \forall x, (\text{ParentOf}(x,y) \land \text{Female}(x) \Rightarrow \text{MotherOf}(x,y)) \] \hspace{1cm} D = \{n \text{ people}\}

2. \[ \Delta = \forall y, (\text{ParentOf}(y) \land \text{Female} \Rightarrow \text{MotherOf}(y)) \] \hspace{1cm} D = \{n \text{ people}\}

   If Female = true? \[ \Delta = \forall y, (\text{ParentOf}(y) \Rightarrow \text{MotherOf}(y)) \] \hspace{1cm} \rightarrow 3^n \text{ models}

   If Female = false? \[ \Delta = \text{true} \] \hspace{1cm} \rightarrow 4^n \text{ models}

   \rightarrow 3^n + 4^n \text{ models}

3. \[ \Delta = \forall x, (\text{Stress}(x) \Rightarrow \text{Smokes}(x)) \] \hspace{1cm} \text{Domain} = \{n \text{ people}\}

   \rightarrow 3^n \text{ models}
**WFOMC Inference: Example**

### 1.  
\[ \Delta = \forall x, y, (\text{ParentOf}(x, y) \land \text{Female}(x) \Rightarrow \text{MotherOf}(x, y)) \]  
\[ \text{D} = \{n \text{ people}\} \]  
\[ \Rightarrow (3^n + 4^n)^n \text{ models} \]

### 2.  
\[ \Delta = \forall y, (\text{ParentOf}(y) \land \text{Female} \Rightarrow \text{MotherOf}(y)) \]  
\[ \text{D} = \{n \text{ people}\} \]  

- If Female = true? \[ \Delta = \forall y, (\text{ParentOf}(y) \Rightarrow \text{MotherOf}(y)) \]  
  \[ \Rightarrow 3^n \text{ models} \]

- If Female = false? \[ \Delta = \text{true} \]  
  \[ \Rightarrow 4^n \text{ models} \]

\[ \Rightarrow 3^n + 4^n \text{ models} \]

### 3.  
\[ \Delta = \forall x, (\text{Stress}(x) \Rightarrow \text{Smokes}(x)) \]  
\[ \text{Domain} = \{n \text{ people}\} \]  
\[ \Rightarrow 3^n \text{ models} \]
Atom Counting: Example

\[ \Delta = \forall x, y, (\text{Smokes}(x) \land \text{Friends}(x, y) \Rightarrow \text{Smokes}(y)) \]

Domain = \{n people\}
Atom Counting: Example

\[ \Delta = \forall x, y, (\text{Smokes}(x) \land \text{Friends}(x, y) \Rightarrow \text{Smokes}(y)) \]

Domain = \{n people\}

- If we know precisely who smokes, and there are \( k \) smokers?

**Database:**

<table>
<thead>
<tr>
<th></th>
<th>Smokes</th>
<th>Friends</th>
<th>Smokes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smokes(Alice)</td>
<td>1</td>
<td></td>
<td>k</td>
</tr>
<tr>
<td>Smokes(Bob)</td>
<td>0</td>
<td></td>
<td>k</td>
</tr>
<tr>
<td>Smokes(Charlie)</td>
<td>0</td>
<td></td>
<td>k</td>
</tr>
<tr>
<td>Smokes(Dave)</td>
<td>1</td>
<td></td>
<td>k</td>
</tr>
<tr>
<td>Smokes(Eve)</td>
<td>0</td>
<td></td>
<td>k</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Atom Counting: Example

\[ \Delta = \forall x, y, (\text{Smokes}(x) \land \text{Friends}(x, y) \Rightarrow \text{Smokes}(y)) \]

**Domain** = \{n people\}

- If we know precisely who smokes, and there are \( k \) smokers?

**Database:**
- \( \text{Smokes}(\text{Alice}) = 1 \)
- \( \text{Smokes}(\text{Bob}) = 0 \)
- \( \text{Smokes}(\text{Charlie}) = 0 \)
- \( \text{Smokes}(\text{Dave}) = 1 \)
- \( \text{Smokes}(\text{Eve}) = 0 \)
- ...

```plaintext
\( \text{Smokes} \)  \( \text{Friends} \)  \( \text{Smokes} \)

\( k \)  \( \cdot \)  \( k \)

\( n-k \)  \( \cdot \)  \( n-k \)
```
Atom Counting: Example

\[ \Delta = \forall x, y, (\text{Smokes}(x) \land \text{Friends}(x, y) \Rightarrow \text{Smokes}(y)) \]

Domain = \{n people\}

- If we know precisely who smokes, and there are \( k \) smokers?

**Database:**

- Smokes(Alice) = 1
- Smokes(Bob) = 0
- Smokes(Charlie) = 0
- Smokes(Dave) = 1
- Smokes(Eve) = 0
- ...
Atom Counting: Example

\[ \Delta = \forall x, y, (\text{Smokes}(x) \land \text{Friends}(x, y) \Rightarrow \text{Smokes}(y)) \]

- If we know precisely who smokes, and there are \( k \) smokers?

**Database:**
- Smokes(Alice) = 1
- Smokes(Bob) = 0
- Smokes(Charlie) = 0
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- ...

Domain = \{n people\}
Atom Counting: Example

\[ \Delta = \forall x, y, \ (\text{Smokes}(x) \land \text{Friends}(x, y) \Rightarrow \text{Smokes}(y)) \]

- If we know precisely who smokes, and there are \( k \) smokers?

**Database:**
- Smokes(Alice) = 1
- Smokes(Bob) = 0
- Smokes(Charlie) = 0
- Smokes(Dave) = 1
- Smokes(Eve) = 0
- ...

**Domain** = \{n people\}
Atom Counting: Example

\[ \Delta = \forall x, y, (\text{Smokes}(x) \land \text{Friends}(x, y) \Rightarrow \text{Smokes}(y)) \]

Domain = \{n people\}

- If we know precisely who smokes, and there are \( k \) smokers?

**Database:**
- Smokes(Alice) = 1
- Smokes(Bob) = 0
- Smokes(Charlie) = 0
- Smokes(Dave) = 1
- Smokes(Eve) = 0
- ...
Atom Counting: Example

$\Delta = \forall x, y, (\text{Smokes}(x) \land \text{Friends}(x, y) \Rightarrow \text{Smokes}(y))$

Domain = \{n people\}

- If we know precisely who smokes, and there are $k$ smokers?

**Database:**
- Smokes(Alice) = 1
- Smokes(Bob) = 0
- Smokes(Charlie) = 0
- Smokes(Dave) = 1
- Smokes(Eve) = 0
- ...

```
        Smokes       Friends       Smokes
        k            k              k
        n-k          n-k
```
Atom Counting: Example

\[ \Delta = \forall x, y, (\text{Smokes}(x) \land \text{Friends}(x, y) \Rightarrow \text{Smokes}(y)) \]

Domain = \{ n \text{ people} \}

- If we know precisely who smokes, and there are \( k \) smokers?

**Database:**
- Smokes(Alice) = 1
- Smokes(Bob) = 0
- Smokes(Charlie) = 0
- Smokes(Dave) = 1
- Smokes(Eve) = 0
- ...

![Diagram showing atom counting with a database and relationships]

\[ \text{Smokes} \quad \text{Friends} \quad \text{Smokes} \]

\[ k \quad k \]

\[ n-k \quad n-k \]
Atom Counting: Example

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Domain = \{n people\}
Atom Counting: Example

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$\Rightarrow 2^{n^2-k(n-k)}$ models

Domain = \{n people\}
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- ...

\[ \Rightarrow 2^{n^2 - k(n-k)} \] models

- If we know that there are \( k \) smokers?
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\[ 2^{n^2 - k(n-k)} \] models

- If we know that there are \( k \) smokers?

\[ \binom{n}{k} 2^{n^2 - k(n-k)} \] models
Atom Counting: Example

\[ \Delta = \forall x, y, (\text{Smokes}(x) \land \text{Friends}(x,y) \implies \text{Smokes}(y)) \]

- If we know precisely who smokes, and there are \( k \) smokers?

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- In total...

Domain = \{n people\}
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If we know that there are \( k \) smokers?

\[\binom{n}{k} 2^{n^2 - k(n-k)} \text{ models}\]

In total...

\[
\sum_{k=0}^{n} \binom{n}{k} 2^{n^2 - k(n-k)} \text{ models}
\]
First-Order Knowledge Compilation

Markov Logic 3.14 \( \text{Smokes}(x) \land \text{Friends}(x,y) \Rightarrow \text{Smokes}(y) \)

[Van den Broeck et al.; IJCAI’11, NIPS’11, PhD’13, KR’14]
First-Order Knowledge Compilation

Markov Logic

3.14 Smokes(x) ∧ Friends(x,y) ⇒ Smokes(y)

Weight Function

- \( w(\text{Smokes}) = 1 \)
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\[ \forall x, y, F(x, y) \leftrightarrow [ \text{Smokes}(x) \land \text{Friends}(x, y) \Rightarrow \text{Smokes}(y) ] \]

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Compile

First-Order d-DNNF Circuit

Weight Function

\[ w(\text{Smokes}) = 1 \]
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Domain

Alice
Bob
Charlie

[Van den Broeck et al.; IJCAI’11, NIPS’11, PhD’13, KR’14]
First-Order Knowledge Compilation

Markov Logic

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First-Order d-DNNF Circuit

Domain

Alice
Bob
Charlie

\[ Z = \text{WFOMC} = 1479.85 \]

[Van den Broeck et al.; IJCAI’11, NIPS’11, PhD’13, KR’14]
Let us automate this:

- **Relational** model

\[
\forall p, \exists c, \text{Card}(p,c) \\
\forall c, \exists p, \text{Card}(p,c) \\
\forall p, \forall c, \forall c', \text{Card}(p,c) \land \text{Card}(p,c') \Rightarrow c = c'
\]

- **Lifted** probabilistic inference algorithm
Playing Cards Revisited

Let us automate this:

\[
\forall p, \exists c, \text{Card}(p,c) \\
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Playing Cards Revisited

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\[ \#\text{SAT} = \sum_{k=0}^{n} \binom{n}{k} \sum_{l=0}^{n} \binom{n}{l} (l + 1)^k (-1)^{2n-k-l} = n! \]

Playing Cards Revisited

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\[\forall p, \exists c, \text{Card}(p,c)\]
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Computed in time polynomial in \(n\)

Outline

• Motivation
  – Why high-level representations?
  – Why high-level reasoning?
• Intuition: Inference rules
• Liftability theory: Strengths and limitations
• Lifting in practice
  – Approximate symmetries
  – Lifted learning
Theory of Inference

Goal: Understand complexity of probabilistic reasoning

• Low-level graph-based concepts (treewidth) ⇒ inadequate to describe high-level reasoning
• Need to develop “liftability theory”
• Deep connections to
  – database theory, finite model theory, 0-1 laws,
  – counting complexity

[Van den Broeck.; NIPS’11], [Van den Broeck, Jaeger.; StarAI’12]
Lifted Inference: Definition

- **Informal** [Poole’03, etc.]
  
  Exploit symmetries, Reason at first-order level, Reason about groups of objects, Scalable inference, High-level probabilistic reasoning, etc.

- **A formal definition**: Domain-lifted inference

  Inference runs in time **polynomial** in the number of entities in the domain.

[Van den Broeck.; NIPS’11]
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  - Polynomial in #rows, #entities, #people, #webpages, #cards
  - ~ data complexity in databases

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[Van den Broeck.; NIPS’11]
First-Order Knowledge Compilation

Markov Logic

- **FOL Sentence**
  - $\forall x, y, F(x, y) \leftrightarrow [\text{Smokes}(x) \land \text{Friends}(x, y) \Rightarrow \text{Smokes}(y)]$

  Weight Function
  - $w(\text{Smokes}) = 1$
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  Compile?

[Van den Broeck.; NIPS'11]
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Compile?

First-Order d-DNNF Circuit

Domain

- Alice
- Bob
- Charlie

Z = WFOMC = 1479.85

Evaluation in time polynomial in domain size

[Van den Broeck.; NIPS'11]
First-Order Knowledge Compilation

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First-Order \( d \)-DNNF Circuit

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Evaluation in time polynomial in domain size

Domain-lifted!

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[Van den Broeck.; NIPS'11]
What Can Be Lifted?

**Theorem:** WFOMC for $\text{FO}^2$ is liftable

[Van den Broeck.; NIPS’11], [Van den Broeck et al.; KR’14]
What Can Be Lifted?

**Theorem:** WFOMC for FO$^2$ is liftable

**Corollary:** Markov logic with two logical variables per formula is liftable.

[Van den Broeck.; NIPS’11], [Van den Broeck et al.; KR’14]
What Can Be Lifted?

**Theorem:** WFOMC for FO$^2$ is liftable

**Corollary:** Markov logic with two logical variables per formula is liftable.

**Corollary:** Tight probabilistic logic programs with two logical variables are liftable.

...
$\text{FO}^2$ is liftable!
$\mathsf{FO}^2$ is liftable!
“Smokers are more likely to be friends with other smokers.”
“Colleagues of the same age are more likely to be friends.”
“People are either family or friends, but never both.”
“If X is family of Y, then Y is also family of X.”
“If X is a parent of Y, then Y cannot be a parent of X.”
FO² is liftable!

Medical Records

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Statistical Relational Model in FO²

2.1 Asthma(x) ⇒ Cough(x)
3.5 Smokes(x) ⇒ Cough(x)
1.9 Smokes(x) ∧ Friends(x,y) ⇒ Smokes(y)
1.5 Asthma (x) ∧ Family(x,y) ⇒ Asthma (y)

Frank | 1 | 0.2 | 0.6

[Van den Broeck.; NIPS’11], [Van den Broeck et al.; KR’14]
**FO² is liftable!**

### Medical Records

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### Statistical Relational Model in FO²

- 2.1 \( \text{Asthma}(x) \Rightarrow \text{Cough}(x) \)
- 3.5 \( \text{Smokes}(x) \Rightarrow \text{Cough}(x) \)
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- 1.5 \( \text{Asthma}(x) \land \text{Family}(x,y) \Rightarrow \text{Asthma}(y) \)

### Big data

| Frank | 1     | 0.2   | 0.6   |

[Van den Broeck.; NIPS’11], [Van den Broeck et al.; KR’14]
Can Everything Be Lifted?

[Beame, Van den Broeck, Gribkoff, Suciu; PODS’15]
Can Everything Be Lifted?

Theorem: There exists an FO³ sentence Θ₁ for which first-order model counting is #P₁-complete in the domain size.

[Beame, Van den Broeck, Gribkoff, Suciu; PODS’15]
Can Everything Be Lifted?

**Theorem:** There exists an FO³ sentence $\Theta_1$ for which first-order model counting is $\#P_1$-complete in the domain size.

A counting Turing machine is a nondeterministic TM that prints the number of its accepting computations.

The class $\#P_1$ consists of all functions computed by a polynomial-time counting TM with unary input alphabet.

**Proof:** Encode a universal $\#P_1$-TM in FO³

[Beame, Van den Broeck, Gribkoff, Suciu; PODS’15]
Fertile Ground

- Monadic
- Y-acyclic CQs
- Safe monotone CNF
- Safe type-1 CNF

[VD; NIPS’11], [VD et al.; KR’14], [Gribkoff, VDB, Suciu; UAI’15], [Beame, VDB, Gribkoff, Suciu; PODS’15], etc.
Fertile Ground

\[ \Delta = \forall x, y, z, \text{Friends}(x, y) \land \text{Friends}(y, z) \Rightarrow \text{Friends}(x, z) \]

[VdB; NIPS’11], [VdB et al.; KR’14], [Gribkoff, VdB, Suciu; UAI’15], [Beame, VdB, Gribkoff, Suciu; PODS’15], etc.
# Statistical Properties

1. **Independence**

   \[
   P(\text{Alice}) = P(\text{Bob}) \times P(\text{Charlie})
   \]

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2. **Partial Exchangeability**

   \[
   P(\text{Alice}) = P(\text{Bob}) \times P(\text{Charlie})
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3. **Independent and identically distributed (i.i.d.)**

   \[
   = \text{Independence} + \text{Partial Exchangeability}
   \]
Statistical Properties for Tractability

- Tractable classes independent of representation
- Traditionally:
  - Tractable learning from \textit{i.i.d. data}
  - Tractable inference when \textit{cond. independence}
- New understanding:
  - Tractable learning from \textit{exchangeable data}
  - Tractable inference when
    - Conditional independence
    - Conditional exchangeability
    - A combination

[Niepert, Van den Broeck; AAAI’14]
Outline

• Motivation
  – Why high-level representations?
  – Why high-level reasoning?

• Intuition: Inference rules

• Liftability theory: Strengths and limitations

• Lifting in practice
  – Approximate symmetries
  – Lifted learning
Approximate Symmetries

• What if not liftable? Asymmetric graph?
• Exploit approximate symmetries:
  – Exact symmetry $g$: $\Pr(x) = \Pr(x^g)$
    
    E.g. Ising model without external field
  – Approximate symmetry $g$: $\Pr(x) \approx \Pr(x^g)$
    
    E.g. Ising model with external field

[Van den Broeck, Darwiche; NIPS’13], [Van den Broeck, Niepert; AAAI’15]
Example: Statistical Relational Model

• WebKB: Classify pages given links and words
• Very large Markov logic network
  
  1.3  \( \text{Page}(x, \text{Faculty}) \Rightarrow \text{HasWord}(x, \text{Hours}) \)
  
  1.5  \( \text{Page}(x, \text{Faculty}) \land \text{Link}(x, y) \Rightarrow \text{Page}(y, \text{Course}) \)

  and 5000 more ...

• No symmetries with evidence on Link or Word
• Where do approx. symmetries come from?

[Van den Broeck, Darwiche; NIPS’13], [Van den Broeck, Niepert; AAAI’15]
Over-Symmetric Approximations

• OSA makes model more symmetric
• E.g., low-rank Boolean matrix factorization

Link (“aaai.org”, “google.com”)
Link (“google.com”, “aaai.org”)
Link (“google.com”, “gmail.com”)
Link (“ibm.com”, “aaai.org”)

Link (“aaai.org”, “google.com”)
Link (“google.com”, “aaai.org”)
- Link (“google.com”, “gmail.com”)
+ Link (“aaai.org”, “ibm.com”)
Link (“ibm.com”, “aaai.org”)

googel.com and ibm.com become symmetric!

[Van den Broeck, Darwiche; NIPS’13]
Experiments: WebKB

[Van den Broeck, Niepert; AAAI’15]
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Lifted Weight Learning

• **Given:** A set of first-order logic formulas

\[ w \text{ FacultyPage}(x) \land \text{Linked}(x,y) \Rightarrow \text{CoursePage}(y) \]

A set of training databases

• **Learn:** The associated maximum-likelihood weights

\[
\frac{\partial}{\partial w_j} \log \Pr_w(db) = n_j(db) - \mathbb{E}_w[n_j]
\]

Count in databases
Efficient

Expected counts
Requires inference

\[
\mathbb{E}_w[n_F] = \Pr(F\theta_1) + \cdots + \Pr(F\theta_m)
\]

• **Idea:** Lift the computation of \( \mathbb{E}_w[n_j] \)

[Van den Broeck et al.; StarAI’13]
Learning Time

\[ w \text{ Smokes}(x) \land \text{Friends}(x,y) \Rightarrow \text{Smokes}(y) \]

Big data

Learns a model over 900,030,000 random variables

[Van den Broeck et al.; StarAI’13]
Learning Time

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Learns a model over 900,030,000 random variables

[Van den Broeck et al.; StarAI’13]
Lifted Structure Learning

- **Given:** A set of training **databases**
- **Learn:** A set of first-order logic **formulas**
  The associated maximum likelihood **weights**
- **Idea:** Learn liftable models (regularize with symmetry)

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[VHaaren, Van den Broeck, et al.;’15]
Outline

• Motivation
  – Why high-level representations?
  – Why high-level reasoning?
• Intuition: Inference rules
• Liftability theory: Strengths and limitations
• Lifting in practice
  – Lifted learning
  – Approximate symmetries
Conclusions

- A radically new reasoning paradigm
- Lifted inference is frontier and integration of AI, KR, ML, DBs, theory, etc.
- We need
  - relational databases and logic
  - probabilistic models and statistical learning
  - algorithms that scale
- Many theoretical open problems
- It works in practice
Long-Term Outlook

Probabilistic inference and learning exploit

~ 1988: conditional independence

~ 2000: contextual independence (local structure)
Long-Term Outlook

Probabilistic inference and learning exploit

~ 1988: conditional independence

~ 2000: contextual independence (local structure)

~ 201?: symmetry & exchangeability
# Collaborators

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<thead>
<tr>
<th>KU Leuven</th>
<th>UCLA</th>
<th>Indiana Univ.</th>
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<td>Luc De Raedt</td>
<td>Adnan Darwiche</td>
<td>Sriraam Natarajan</td>
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<td>Wannes Meert</td>
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| Af. Cort.        |                |                        |
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| Univ. Washington |                |                        |
|-----------------|----------------|                        |
| Mathias Niepert |                |                        |
| Dan Suciu       |                |                        |
| Eric Gribkoff   |                |                        |
| Paul Beame      |                |                        |

| Trento Univ.    |                |                        |
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Prototype Implementation

.MLN  WMC  FOPI  d-DNNF  FastM  Dimacs
Read  exec  WFOMC  Compile  Learn weights

Pr(Friends(Guy, Warnes)) = 0.70
Pr(Friends(Jesse, Guy)) = 0.75
...

2.3 Smokes(x) ∧ Friends(x,y) → Smokes(y)

http://dtai.cs.kuleuven.be/wfomc
Thanks