CAUSAL INFERENCE
IN STATISTICS

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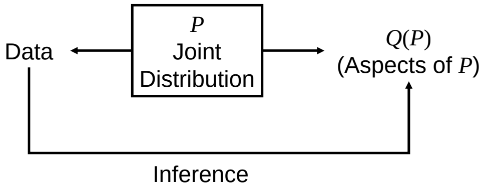
OUTLINE

- Inference: Statistical vs. Causal, distinctions, and mental barriers
- Unified conceptualization of counterfactuals, structural-equations, and graphs
- Inference to three types of claims:
  1. Effect of potential interventions
  2. Attribution (Causes of Effects)
  3. Direct and indirect effects
- Frills

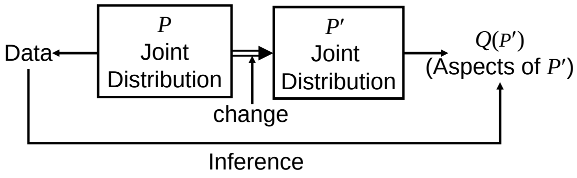
TRADITIONAL STATISTICAL INFERENCE PARADIGM


Data
P
Joint
Distribution
$Q(P)$
(Aspects of P)
Inference

e.g.,
Infer whether customers who bought product A would also buy product B.
$Q = P(B \mid A)$

FROM STATISTICAL TO CAUSAL ANALYSIS:
1. THE DIFFERENCES

Probability and statistics deal with static relations


Data
P
Joint
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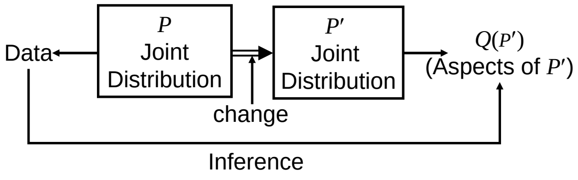
P'
Joint
Distribution
$Q(P')$
(Aspects of P')
change
Inference

What happens when P changes?
e.g.,
Infer whether customers who bought product A would still buy A if we were to double the price.

FROM STATISTICAL TO CAUSAL ANALYSIS:
1. THE DIFFERENCES

What remains invariant when P changes say, to satisfy
$P'(price{=}2){=}1$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Note: $P'(v) \neq P(v \mid price = 2)$

P does not tell us how it ought to change
e.g. Curing symptoms vs. curing diseases
e.g. Analogy: mechanical deformation

FROM STATISTICAL TO CAUSAL ANALYSIS:
1. THE DIFFERENCES (CONT)

1. Causal and statistical concepts do not mix.

CAUSAL	STATISTICAL
Spurious correlation	Regression
Randomization / Intervention	Association / Independence
Confounding / Effect	“Controlling for” / Conditioning
Instrumental variable	Odd and risk ratios
Strong Exogeneity	Collapsibility / Granger causality
Explanatory variables	Propensity score

2.

3.

4.

FROM STATISTICAL TO CAUSAL ANALYSIS: 2. MENTAL BARRIERS

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2. No causes in – no causes out (Cartwright, 1989)

$$\left.\begin{array}{l}\text{statistical assumptions + data}\\ \text{causal assumptions}\end{array}\right\} \Rightarrow \text{causal conclusions}$$

3. Causal assumptions cannot be expressed in the mathematical language of standard statistics.
4.

FROM STATISTICAL TO CAUSAL ANALYSIS: 2. MENTAL BARRIERS

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$$\left.\begin{array}{l}\text{statistical assumptions + data}\\ \text{causal assumptions}\end{array}\right\} \Rightarrow \text{causal conclusions}$$

3. Causal assumptions cannot be expressed in the mathematical language of standard statistics.
4. Non-standard mathematics:
   a) Structural equation models (Wright, 1920; Simon, 1960)
   b) Counterfactuals (Neyman-Rubin (Y_x), Lewis ($x \;\Box\!\!\rightarrow Y$))

WHY CAUSALITY NEEDS SPECIAL MATHEMATICS

Scientific Equations (e.g., Hooke's Law) are non-algebraic

e.g., Length (Y) equals a constant (2) times the weight (X)

Process information	The solution
$Y = 2X$ $X = 1$	$X = 1$ $Y = 2$

WHY CAUSALITY NEEDS SPECIAL MATHEMATICS

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Process information	The solution
$Y = 2X$ $X = 1$	$X = 1$ $Y = 2$

Had X been 3, Y would be 6.
If we raise X to 3, Y would be 6.
Must "wipe out" $X = 1$.

WHY CAUSALITY NEEDS SPECIAL MATHEMATICS

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Correct notation:

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$Y := 2X$ $X = 1$	$X = 1$ $Y = 2$

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WHY CAUSALITY NEEDS SPECIAL MATHEMATICS

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e.g., Length (Y) equals a constant (2) times the weight (X)

Correct notation:
(or)

Process information	The solution
$Y \leftarrow 2X$ $X = 1$	$X = 1$ $Y = 2$

Had X been 3, Y would be 6.
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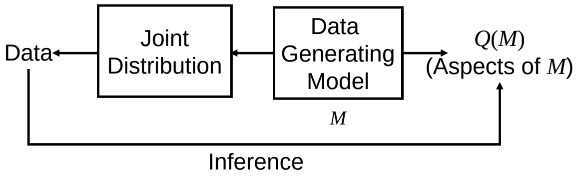
THE STRUCTURAL MODEL PARADIGM



Data

Joint Distribution

Data Generating Model

M

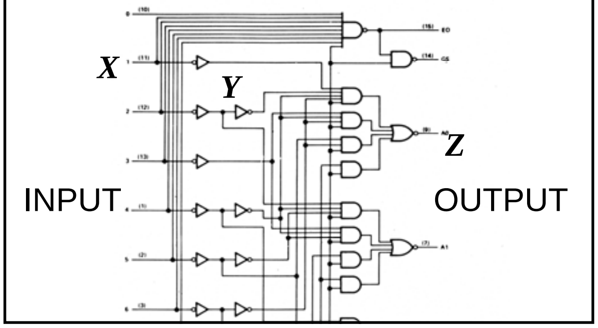
$Q(M)$ (Aspects of M)

Inference

M – Invariant strategy (mechanism, recipe, law, protocol) by which Nature assigns values to variables in the analysis.

"Think Nature, not experiment!"

FAMILIAR CAUSAL MODEL ORACLE FOR MANIPILATION



STRUCTURAL CAUSAL MODELS

Definition: A structural causal model is a 4-tuple $\langle V,U, F, P(u)\rangle$, where

- $V = \{V_1,...,V_n\}$ are endogeneas variables
- $U = \{U_1,...,U_m\}$ are background variables
- $F = \{f_1,..., f_n\}$ are functions determining V,
  $v_i = f_i(v, u)$ e.g., $y = \alpha + \beta x + u_Y$
- $P(u)$ is a distribution over U

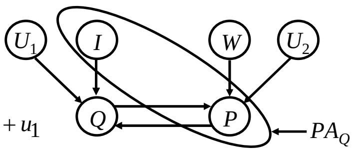
$P(u)$ and F induce a distribution $P(v)$ over observable variables

STRUCTURAL MODELS AND CAUSAL DIAGRAMS

The functions $v_i = f_i(v,u)$ define a graph

$$v_i = f_i(pa_i,u_i) \quad PA_i \subseteq V \setminus V_i \qquad U_i \subseteq U$$

Example: Price – Quantity equations in economics



$$q = b_1 p + d_1 i + u_1$$
$$p = b_2 q + d_2 w + u_2$$

STRUCTURAL MODELS AND INTERVENTION

Let X be a set of variables in V.
The action $do(x)$ sets X to constants x regardless of the factors which previously determined X.
$do(x)$ replaces all functions f_i determining X with the constant functions $X=x$, to create a mutilated model M_x

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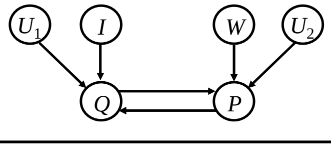

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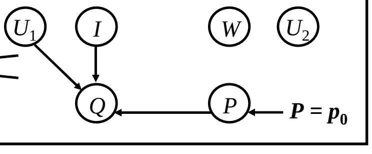
$\boxed{\boldsymbol{M_p}}$

$q = b_1 p + d_1 i + u_1$

~~$p = b_2 q + d_2 w + u_2$~~

$p = p_0$



CAUSAL MODELS AND COUNTERFACTUALS

Definition:
The sentence: “Y would be y (in situation u), had X been x,” denoted $Y_x(u) = y$, means:
The solution for Y in a mutilated model M_x, (i.e., the equations for X replaced by $X = x$) with input $U=u$, is equal to y.

The Fundamental Equation of Counterfactuals:

$$\boxed{Y_x(u) = Y_{M_x}(u)}$$

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- Joint probabilities of counterfactuals:

$$P(Y_x = y, Z_w = z) = \sum_{u: Y_x(u)=y, Z_w(u)=z} P(u)$$

In particular:

$$P(y \mid do(x)\ ) \overset{\Delta}{=} P(Y_x = y) = \sum_{u: Y_x(u)=y} P(u)$$

$$PN(Y_{x'} = y' \mid x, y) = \sum_{u: Y_{x'}(u)=y'} P(u \mid x, y)$$

REGRESSION VS. STRUCTURAL EQUATIONS (THE CONFUSION OF THE CENTURY)

Regression (claimless, nonfalsifiable):

$$Y = ax + \varepsilon_Y$$

Structural (empirical, falsifiable):

$$Y = bx + u_Y$$

Claim: (regardless of distributions):

$$E(Y \mid do(x)) = E(Y \mid do(x), do(z)) = bx$$

The mothers of all questions:
Q. When would b equal a?
A. When all back-door paths are blocked, $(u_Y \perp\!\!\!\perp X)$

Q. When is b estimable by regression methods?
A. Graphical criteria available

THE FOUR NECESSARY STEPS OF CAUSAL ANALYSIS

Define:	Express the target quantity Q as a function $Q(M)$ that can be computed from any model M.
Assume:	Formulate causal assumptions using ordinary scientific language and represent their structural part in graphical form.
Identify:	Determine if Q is identifiable.
Estimate:	Estimate Q if it is identifiable; approximate it, if it is not.

THE FOUR NECESSARY STEPS FOR EFFECT ESTIMATION

Define:	Express the target quantity Q as a function $Q(M)$ that can be computed from any model M. $\boxed{P(Y_x = y) \quad \text{or} \quad P(y \mid do(x))}$
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THE FOUR NECESSARY STEPS FOR EFFECT ESTIMATION

Define:	Express the target quantity Q as a function $Q(M)$ that can be computed from any model M. $\text{ATE} \overset{\Delta}{=} E(Y \mid do(x_1)) - E(Y \mid do(x_0))$
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THE FOUR NECESSARY STEPS FOR POLICY ANALYSIS

Define: Express the target quantity Q as a function $Q(M)$ that can be computed from any model M.

$$P(Y_{X=g(z)} = y) \text{ or } P(y \mid do(x = g(z))$$

Assume: Formulate causal assumptions using ordinary scientific language and represent their structural part in graphical form.

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$$P(Y_{x,z,u} = y) \text{ or } P(y \mid do(X = x, Z = z, W = w))$$

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INFERRING THE EFFECT OF INTERVENTIONS

The problem:
To predict the impact of a proposed intervention using data obtained prior to the intervention.

The solution (conditional):
Causal Assumptions + Data → Policy Claims

1. Mathematical tools for communicating causal assumptions formally and transparently.
2. Deciding (mathematically) whether the assumptions communicated are sufficient for obtaining consistent estimates of the prediction required.
3. Deriving (if (2) is affirmative) a closed-form expression for the predicted impact

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2. Deciding (mathematically) whether the assumptions communicated are sufficient for obtaining consistent estimates of the prediction required.
4. Suggesting (if (2) is negative) a set of measurements and experiments that, if performed, would render a consistent estimate feasible.

THE FOUR NECESSARY STEPS FROM DEFINITION TO ASSUMPTIONS

Define: Express the target quantity Q as a function $Q(M)$ that can be computed from any model M.

$$P(y \mid do(x))$$

Assume: Formulate causal assumptions using ordinary scientific language and represent their structural part in graphical form.

Identify: Determine if Q is identifiable.

Estimate: Estimate Q if it is identifiable; approximate it, if it is not.

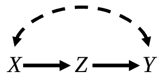
FORMULATING ASSUMPTIONS THREE LANGUAGES

1. English: Smoking (X), Cancer (Y), Tar (Z), Genotypes (U)

2. Counterfactuals:
$$Z_x(u) = Z_{yx}(u),$$
$$X_y(u) = X_{zy}(u) = X_z(u) = X(u),$$
$$Y_z(u) = Y_{zx}(u),$$
$$Z_x \perp\!\!\!\perp \{Y_z, X\}$$

Not too friendly:
Consistent?, complete?, redundant?, arguable?

4. Structural:



IDENTIFIABILITY

Definition:
Let $Q(M)$ be any quantity defined on a causal model M, and let A be a set of assumption.

Q is identifiable relative to A iff

$$P(M_1) = P(M_2) \;\; \Rightarrow \;\; Q(M_1) = Q(M_2)$$

for all M_1, M_2, that satisfy A.

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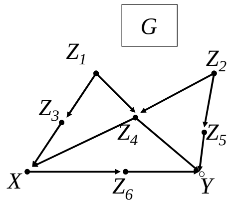
for all M_1, M_2, that satisfy A.

In other words, Q can be determined uniquely from the probability distribution $P(v)$ of the endogenous variables, V, and assumptions A.

A is displayed in graph G.

THE PROBLEM OF CONFOUNDING

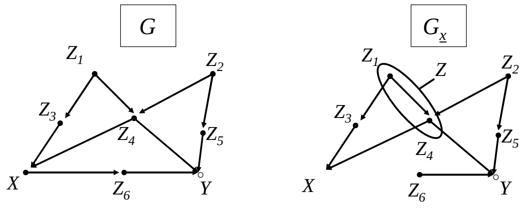
Find the effect of X on Y, $P(y|do(x))$, given the causal assumptions shown in G, where $Z_1,..., Z_k$ are auxiliary variables.



Can $P(y|do(x))$ be estimated if only a subset, Z, can be measured?

ELIMINATING CONFOUNDING BIAS THE BACK-DOOR CRITERION

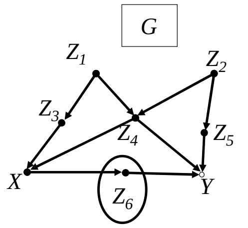
$P(y \mid do(x))$ is estimable if there is a set Z of variables such that Z d-separates X from Y in $G_{\underline{x}}$.



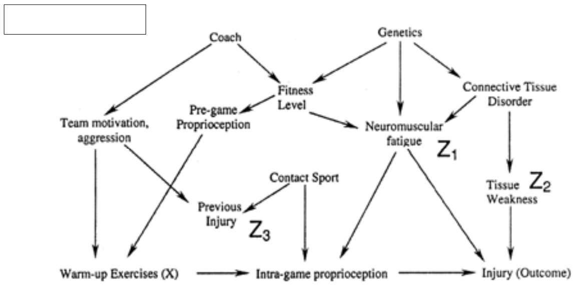
Moreover, $P(y \mid do(x)) = \sum_z P(y \mid x,z)\, P(z)$
("adjusting" for Z)

EFFECT OF INTERVENTION BEYOND ADJUSTMENT

Theorem (Tian-Pearl 2002)
We can identify $P(y|do(x))$ if there is no child Z of X connected to X by a confounding path.



EFFECT OF WARM-UP ON INJURY (After Shrier & Platt, 2008)



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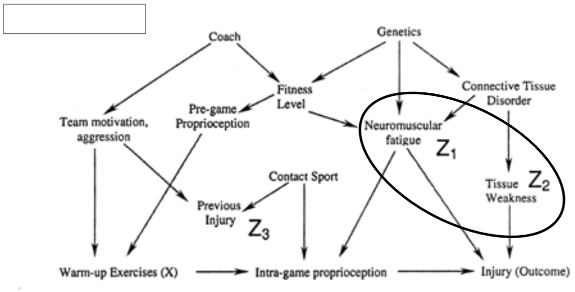

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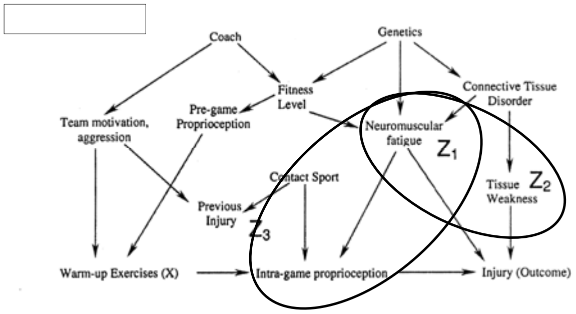

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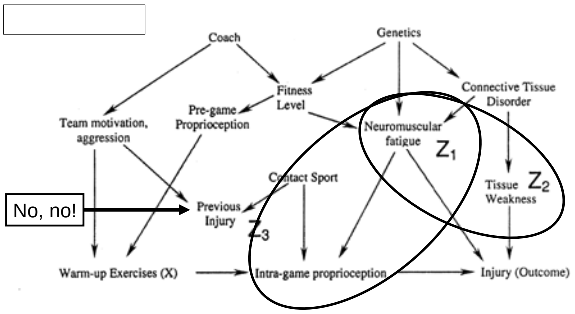


EFFECT OF INTERVENTION COMPLETE IDENTIFICATION

- Complete calculus for reducing $P(y|do(x), z)$ to expressions void of *do*-operators.
- Complete graphical criterion for identifying causal effects (Shpitser and Pearl, 2006).
- Complete graphical criterion for empirical testability of counterfactuals (Shpitser and Pearl, 2007).

COUNTERFACTUALS AT WORK ETT – EFFECT OF TREATMENT ON THE TREATED

1. Regret:
   I took a pill to fall asleep.
   Perhaps I should not have?

2. Program evaluation:
   What would terminating a program do to those enrolled?

$$P(Y_x = y \mid x')$$

THE FOUR NECESSARY STEPS EFFECT OF TREATMENT ON THE TREATED

Define: Express the target quantity Q as a function $Q(M)$ that can be computed from any model M.

$$\text{ETT} \overset{\Delta}{=} P(Y_x = y \mid X = x')$$

Assume: Formulate causal assumptions using ordinary scientific language and represent their structural part in graphical form.

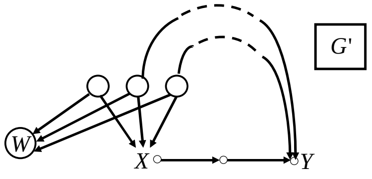
Identify: Determine if Q is identifiable.

Estimate: Estimate Q if it is identifiable; approximate it, if it is not.

ETT - IDENTIFICATION

Theorem (Shpitser-Pearl, 2009)

ETT is identifiable in G iff $P(y \mid do(x),w)$ is identifiable in G'

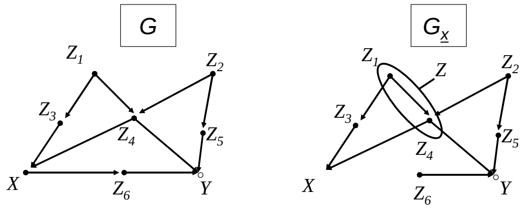


Moreover, ETT $= P(Y_x = y \mid x') = P(y \mid do(x), w) \,\big|_{G'} \big|_{w=x'}$

Complete graphical criterion

ETT - THE BACK-DOOR CRITERION

$P(Y_x = y \mid x')$ is identifiable in G if there is a set Z of variables such that Z d-separates X from Y *in* $G_{\underline{x}}$.



Moreover, ETT $= \sum_z P(y \mid x, z)\, P(z \mid x')$

"Standardized morbidity"

FROM IDENTIFICATION TO ESTIMATION

Define:	Express the target quantity Q as a function $Q(M)$ that can be computed from any model M. $Q = P(y \mid do(x))$
Assume:	Formulate causal assumptions using ordinary scientific language and represent their structural part in graphical form.
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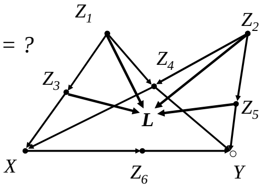
PROPENSITY SCORE ESTIMATOR
(Rosenbaum & Rubin, 1983)

$P(y \mid do(x)) = ?$



$$L(z_1, z_2, z_3, z_4, z_5) \overset{\Delta}{=} P(X = 1 \mid z_1, z_2, z_3, z_4, z_5)$$

Theorem: $\sum_z P(y \mid z, x) P(z) = \sum_l P(y \mid L = l, x) P(L = l)$

Adjustment for L replaces Adjustment for Z

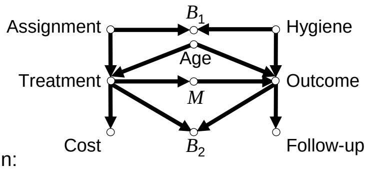
WHAT PROPENSITY SCORE (PS) PRACTITIONERS NEED TO KNOW

$$L(z) = P(X = 1 \mid Z = z)$$

$$\sum_z P(y \mid z, x) P(z) = \sum_l P(y \mid l, x) P(l)$$

1. The assymptotic bias of PS is EQUAL to that of ordinary adjustment (for same Z).
2. Including an additional covariate in the analysis CAN SPOIL the bias-reduction potential of others.
3. Choosing sufficient set for PS, requires knowledge about the model.

WHICH COVARIATES MAY / SHOULD BE ADJUSTED FOR?



Question:

Which of these eight covariates may be included in the propensity score function (for matching) and which should be excluded.

Answer:

Must include:	Age
Must exclude:	B_1, M, B_2, Follow-up, Assignment without Age
May include:	Cost, Hygiene, {Assignment + Age}, {Hygiene + Age + B_1} , more . . .

WHAT PROPENSITY SCORE (PS) PRACTITIONERS NEED TO KNOW

$$L(z) = P(X = 1 \mid Z = z)$$

$$\sum_z P(y \mid z, x) P(z) = \sum_1 P(y \mid l, x) P(l)$$

1. The assymptotic bias of PS is EQUAL to that of ordinary adjustment (for same Z).
2. Including an additional covariate in the analysis CAN SPOIL the bias-reduction potential of others.
3. Choosing sufficient set for PS, requires knowlege about the model.
4. That any empirical test of the bias-reduction potential of PS, can only be generalized to cases where the causal relationships among covariates, observed and unobserved is the same.

TWO PARADIGMS FOR CAUSAL INFERENCE

Observed: $P(X, Y, Z, ...)$
Conclusions needed: $P(Y_x=y)$, $P(X_y=x \mid Z=z)$...

How do we connect observables, $X,Y,Z,\ldots$
to counterfactuals Y_x, X_z, $Z_y,\ldots$?

<u>*N-R* model</u>	<u>Structural model</u>
Counterfactuals are primitives, new variables	Counterfactuals are derived quantities
Super-distribution $P^*(X, Y, ..., Y_x, X_z, ...)$	Subscripts modify the model and distribution
X, Y, Z constrain $Y_x, Z_y, ...$	$P(Y_x = y) = P_{M_x}(Y = y)$

"SUPER" DISTRIBUTION IN *N-R* MODEL

X	Y	Z	$Y_{x=0}$	$Y_{x=1}$	$X_{z=0}$	$X_{z=1}$	$X_{y=0}\cdots$	U
0	0	0	0	1	0	0	0···	u_1
0	1	1	1	0	1	0	1···	u_2
$\textcircled{0}$	$\textcircled{0}$	0	$\textcircled{1}$	0	0	1	1···	u_3
1	0	0	1	0	0	1	0···	u_4

inconsistency: $x = 0 \Rightarrow Y_{x=0} = Y$ $\quad Y = xY_1 + (1\text{-}x)\, Y_0$

Defines: $P^*(X, Y, Z, ... Y_x, Z_y ... Y_{xz}, Z_{xy}, ... \ ...)$

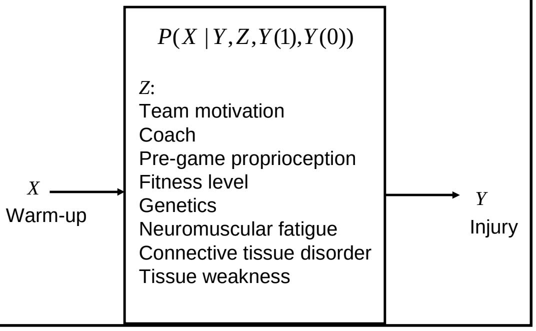
$P^*(Y_x = y \mid Z, X_z)$

$Y_x \perp\!\!\!\perp X \mid Z_y$

THE FOUR NECESSARY STEPS IN POTENTIAL-OUTCOME FRAMEWORK

Define:	Express the target quantity Q as a counterfactual formula
Assume:	Formulate causal assumptions using the distribution: $P(X \mid Y, Z, Y(1), Y(0))$
Identify:	Determine if Q is identifiable.
Estimate:	Estimate Q if it is identifiable; approximate it, if it is not.

EFFECT OF WARM-UP ON INJURY IN POTENTIAL-OUTCOME FRAMEWORK



TYPICAL INFERENCE IN *N-R* MODEL

Find $P^*(Y_x=y)$ given covariate Z,

$$P^*(Y_x = y) = \sum_z P^*(Y_x = y \mid z) P(z)$$

Assume ignorability: $Y_x \perp\!\!\!\perp X \mid Z$
$$= \sum_z P^*(Y_x = y \mid x, z) P(z)$$

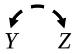
Assume consistency: $X=x \Rightarrow Y_x=Y$
$$= \sum_z P^*(Y = y \mid x, z) P(z)$$
$$= \sum_z P(y \mid x, z) P(z)$$

? Try it: $X \rightarrow Y \rightarrow Z$

Problems:
1) $Y_x \perp\!\!\!\perp X \mid Z$ judgmental & opaque
2) Is consistency the only connection between X, Y and Y_x?

GRAPHICAL – COUNTERFACTUALS SYMBIOSIS

Every causal graph expresses counterfactuals assumptions, e.g., $X \rightarrow Y \rightarrow Z$

1. Missing arrows $Y \leftarrow Z$ $\quad Y_{x,z}(u) = Y_x(u)$
2. Missing arcs  $\quad Y_x \perp\!\!\!\perp Z_y$

consistent, and readable from the graph.

Every theorem in SCM is a theorem in Potential-Outcome Model, and conversely.

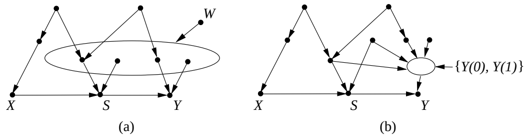
DEMYSTIFYING STRONG IGNORABILITY

$\{Y(0), Y(1)\} \perp\!\!\!\perp X \mid Z$ (Ignorability)

$P(y \mid do(x)) = \sum_z P(y \mid z, x) P(z)$ (Z-admissibility)

$(X \perp\!\!\!\perp Y \mid Z)_{G_{\underline{X}}}$ (Back-door)

Is there a W in G such that $(W \perp\!\!\!\perp X \mid Z)_G \Rightarrow$ Ignorability?



(a) (b)

DETERMINING THE CAUSES OF EFFECTS (The Attribution Problem)

- Your Honor! My client (Mr. A) died BECAUSE he used that drug.

-

DETERMINING THE CAUSES OF EFFECTS (The Attribution Problem)

- Your Honor! My client (Mr. A) died BECAUSE he used that drug.

- Court to decide if it is MORE PROBABLE THAN NOT that A would be alive BUT FOR the drug!

$PN = P(? \mid A \text{ is dead, took the drug}) \geq 0.50$

THE ATTRIBUTION PROBLEM

Definition:

1. What is the meaning of $PN(x,y)$:
   "Probability that event y would not have occurred if it were not for event x, given that x and y did in fact occur."

Answer:

$$PN(x, y) = P(Y_{x'} = y' \mid x, y)$$

Computable from M

THE ATTRIBUTION PROBLEM

Definition:

1. What is the meaning of $PN(x,y)$:
   "Probability that event y would not have occurred if it were not for event x, given that x and y did in fact occur."

Identification:

2. Under what condition can $PN(x,y)$ be learned from statistical data, i.e., observational, experimental and combined.

TYPICAL THEOREMS

(Tian and Pearl, 2000)

- Bounds given combined nonexperimental and experimental data

$$\max\left\{\begin{matrix} 0 \\ \dfrac{P(y) - P(y_{x'})}{P(x,y)} \\ \end{matrix}\right\} \le PN \le \min\left\{\begin{matrix} 1 \\ \dfrac{P(y'_{x'})}{P(x,y)} \\ \end{matrix}\right\}$$

- Identifiability under monotonicity (Combined data)

$$PN = \frac{P(y|x) - P(y|x')}{P(y|x)} + \frac{P(y|x') - P(y_{x'})}{P(x,y)}$$

corrected Excess-Risk-Ratio

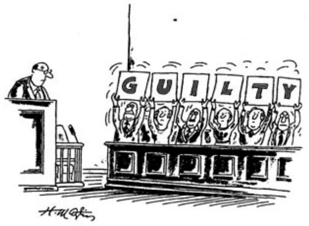
CAN FREQUENCY DATA DECIDE LEGAL RESPONSIBILITY?

	Experimental		Nonexperimental	
	$do(x)$	$do(x')$	x	x'
Deaths (y)	16	14	2	28
Survivals (y')	984	986	998	972
	1,000	1,000	1,000	1,000

- Nonexperimental data: drug usage predicts longer life
- Experimental data: drug has negligible effect on survival
- Plaintiff: Mr. A is special.
  1. He actually died
  2. He used the drug by choice
- Court to decide (given both data):
  Is it more probable than not that A would be alive
  but for the drug?

$$PN \triangleq P(Y_{x'} = y' | x, y) > 0.50$$

SOLUTION TO THE ATTRIBUTION PROBLEM



- WITH PROBABILITY ONE $1 \le P(y'_{x'} | x,y) \le 1$
- Combined data tell more that each study alone

EFFECT DECOMPOSITION (direct vs. indirect effects)

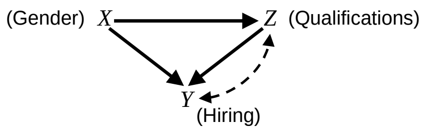
1. Why decompose effects?
2. What is the definition of direct and indirect effects?
3. What are the policy implications of direct and indirect effects?
4. When can direct and indirect effect be estimated consistently from experimental and nonexperimental data?

WHY DECOMPOSE EFFECTS?

1. To understand how Nature works
2. To comply with legal requirements
3. To predict the effects of new type of interventions:
   Signal routing, rather than variable fixing

LEGAL IMPLICATIONS OF DIRECT EFFECT

Can data prove an employer guilty of hiring discrimination?



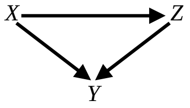
What is the direct effect of X on Y ?

$$E(Y \mid do(x_1), do(z)) - E(Y \mid do(x_0), do(z))$$

(averaged over z) Adjust for Z? No! No!

NATURAL INTRPRETATION OF AVERAGE DIRECT EFFECTS

Robins and Greenland (1992) – “Pure”



$z = f(x, u)$
$y = g(x, z, u)$

Natural Direct Effect of *X* on *Y*: $DE(x_0, x_1; Y)$
The expected change in *Y*, when we change *X* from x_0 to x_1 and, for each *u*, we keep *Z* constant at whatever value it attained before the change.

$$E[Y_{x_1 Z_{x_0}} - Y_{x_0}]$$

In linear models, *DE* = Controlled Direct Effect $= \beta(x_1 - x_0)$

DEFINITION AND IDENTIFICATION OF NESTED COUNTERFACTUALS

Consider the quantity $Q \overset{\Delta}{=} E_u[Y_{xZ_{x^*}(u)}(u)]$

Given $\langle M, P(u)\rangle$, Q is well defined

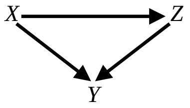
Given u, $Z_{x^*}(u)$ is the solution for Z in M_{x^*}, call it z

$Y_{xZ_{x^*}(u)}(u)$ is the solution for Y in M_{xz}

Can Q be estimated from $\left\{\begin{matrix}\text{experimental}\\ \text{nonexperimental}\end{matrix}\right\}$ data?

Experimental: nest-free expression
Nonexperimental: subscript-free expression

DEFINITION OF INDIRECT EFFECTS



$z = f(x, u)$
$y = g(x, z, u)$

Indirect Effect of *X* on *Y*: $IE(x_0, x_1; Y)$
The expected change in *Y* when we keep *X* constant, say at x_0, and let *Z* change to whatever value it would have attained had *X* changed to x_1.

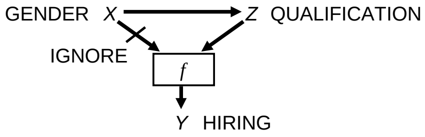
$$E[Y_{x_0 Z_{x_1}} - Y_{x_0}]$$

In linear models, *IE = TE - DE*

POLICY IMPLICATIONS OF INDIRECT EFFECTS

What is the indirect effect of *X* on *Y*?

The effect of Gender on Hiring if sex discrimination is eliminated.


GENDER *X*
Z QUALIFICATION
IGNORE
f
Y HIRING

Blocking a link – a new type of intervention

EXPERIMENTAL IDENTIFICATION OF NATURAL DIRECT EFFECTS

Theorem: If there exists a set *W* such that

$$Y_{xz} \perp\!\!\!\perp Z_{x^*} \mid W \text{ for all } z \text{ and } x$$

Then the average direct effect

$$DE(x, x^*; Y) = E(Y_{x, Z_{x^*}}) - E(Y_{x^*})$$

Is identifiable from experimental data and is given by

$$DE(x, x^*; Y) = \sum_{w,z} [E(Y_{xz} \mid w) - E(Y_{x^*z} \mid w)] P(Z_{x^*} = z \mid w) P(w)$$

GRAPHICAL CONDITION FOR EXPERIMENTAL IDENTIFICATION OF DIRECT EFFECTS

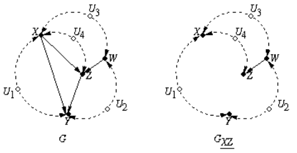
Theorem: If there exists a set *W* such that

$$(Y \perp\!\!\!\perp Z \mid W)_{G_{\underline{XZ}}} \text{ and } W \subseteq ND(X \cup Z)$$

then,

$$DE(x, x^*; Y) = \sum_{w,z} [E(Y_{xz} \mid w) - E(Y_{x^*z} \mid w)] P(Z_{x^*} = z \mid w) P(w)$$

Example:



CONCLUSIONS

I TOLD YOU CAUSALITY IS SIMPLE

CONCLUSIONS

- Formal basis for causal and counterfactual inference (complete)
- Unification of the graphical, potential-outcome and structural equation approaches
- Friendly and formal solutions to century-old problems and confusions.

CONCLUSIONS

He is wise who bases causal inference on an explicit causal structure that is defensible on scientific grounds.

(Aristotle 384-322 B.C.)

From Charlie Poole

QUESTIONS???

They will be answered