1 Ways to Number

There are ten fingers on two hands. But counting the spaces between the fingers gives a total of eight . When counting on our fingers the tenth starts again. But some in the Amazon basin count on the spaces in between fingers. That would make the eighth place the *start-again* location.

A lot of things aren't grouped *ten-at-a-time. Dozen,* a word like *ten,* groups eggs. *A minute* groups sixty seconds. *A week* groups seven days.

Grouping by a special number (ten, twelve, sixty, seven) defines a *base.* A base is always one more than the largest value shown by a single digit. It is also always the number of single digit numbers, where we always begin with zero for none or nothing. A number n written when the base is b could use digits drawn from $0, 1, ..., b-1$. Suppose the string representing n to base b is called x. That could be written:

$$n_{10} = x_b \tag{1}$$

Computers and other digital systems use a different base than ten. These three are most useful:

Binary or base 2 – numbers written using just $0, 1$
Example:

$$10_2 = 2_{10} \tag{2}$$

Octal or base 8 – numbers written using just
$0, 1, 2, 3, 4, 5, 6, 7$;
Example:

$$543_8 = 320 + 32 + 3 = 355_{10} \tag{3}$$

Hexadecimal or base 16 – numbers written using just
$0, 1, 2, 3, 4, 5, 6, 7, 8, 9, X, L, C, D, E, F$ or $0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F$
Example:

$$27D_{12} = 2 \times 256 + 7 \times 16 + 13 = 637_{10} \tag{4}$$

The next question tests one's understanding of bases.

The square of 24 in base b equals 554 in base b. What is base b? [1]

Sample Solution: The statement is equivalent to this relationship:

$$(24_b)(24_b) = 554_b \tag{5}$$

That implies:

$$(4_b)(4_b) - b = 4_b \tag{6}$$

A reasonable guess is $b = 12$.

Writing each of the three expressions in the first equation in base 10, using $b = 12$ we have:

$$(2 \times 12 + 4)(2 \times 12 + 4) = 5 \times 144 + 5 \times 12 + 4 \tag{7}$$

Multiplying the four terms on the left hand side in base ten yields

$$4 \times 144 + (8 + 8) \times 12 + 16 = 576 + 192 + 16 = 784 \tag{8}$$

Evaluating the right hand side yields $720 + 60 + 4$ for the same value, 784.

References

[1] Don Christiansen. Problem. *IEEE-USA TODAY'S ENGINEER*, page 4, 2006.