Question #1

Regular Languages

Write regular expressions for each of the following:

a) String over the alphabet \{a,b,c\} with an odd number of a’s

Solution: \((b|c)*.a.(b|c)* . ( (b|c)*.a.(b|c)*.a.(b|c)* )^*\)

b) Binary numbers multiple of 2 and representing a decimal number greater than or equal to 8

Solution: We assume that there might be a leading zero

\(0^*.1 .(0|1)^*.0(1).(0|1).0\)

c) Binary numbers greater than 110011

Solution: We assume that there might be a leading zero.

\(0^*.1.1.0.1.(0|1).0(1)\)
\(0^*.1.1.1.(0|1).0(1).0(1)\)
\(0^*.1.(0|1)^*.0(1).0(1).0(1).0(1).0(1)\)

d) Strings of the kind EPX where E is an integer number, P is a lowercase letter from the alphabet and X is an integer greater than 3 and less than 13. Examples: 143a6, 555b12, etc.

Solution: \(0[(\- ?).[1-9].[0-9]*],[a-z].((1.[0-2])|[3-9])\)

Question #2.

Convert the following regular expressions to nondeterministic finite automata.

a) \(a^* (b|c)^* c\)

Solution:
b) \(((b|a)^*|(c|a)^*cb)^*\)

Solution:

\[
\begin{array}{c}
\text{Start} \\
0 \\
\text{\vspace{0.5cm}} \\
1 \\
\text{\vspace{0.5cm}} \\
2 \\
\text{\vspace{0.5cm}} \\
3 \\
\text{\vspace{0.5cm}} \\
4 \\
\text{\vspace{0.5cm}} \\
5 \\
\text{\vspace{0.5cm}} \\
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7 \\
\text{\vspace{0.5cm}} \\
8 \\
\text{\vspace{0.5cm}} \\
9 \\
\text{\vspace{0.5cm}} \\
10 \\
\text{\vspace{0.5cm}} \\
11 \\
\text{\vspace{0.5cm}} \\
12 \\
\text{\vspace{0.5cm}} \\
13 \\
\text{\vspace{0.5cm}} \\
14 \\
\text{\vspace{0.5cm}} \\
15 \\
\text{\vspace{0.5cm}} \\
16 \\
\text{\vspace{0.5cm}} \\
17 \\
\text{\vspace{0.5cm}} \\
18 \\
\text{\vspace{0.5cm}} \\
19 \\
\text{\vspace{0.5cm}} \\
20 \\
\text{\vspace{0.5cm}} \\
21 \\
\end{array}
\]

c) \(((baa^*)^*bc^*d^*)^*\)

Solution:

\[
\begin{array}{c}
\text{Start} \\
0 \\
\text{\vspace{0.5cm}} \\
1 \\
\text{\vspace{0.5cm}} \\
2 \\
\text{\vspace{0.5cm}} \\
3 \\
\text{\vspace{0.5cm}} \\
4 \\
\text{\vspace{0.5cm}} \\
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\text{\vspace{0.5cm}} \\
6 \\
\text{\vspace{0.5cm}} \\
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9 \\
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10 \\
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11 \\
\text{\vspace{0.5cm}} \\
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\text{\vspace{0.5cm}} \\
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\text{\vspace{0.5cm}} \\
19 \\
\text{\vspace{0.5cm}} \\
20 \\
\text{\vspace{0.5cm}} \\
21 \\
\end{array}
\]
Question #3

Convert the following NFA to DFA. Show each closure and edge in the process. For a) and b) show the state transition table.

a)

Solution:

Closure (0)={0}=A
DFAedge(A,a)=closure(edge(A,a))=closure({1})={1,8}=B-accepting
DFAedge(A,b)=closure(edge(A,b))=closure({2})={2,3,4}=C
DFAedge(B,a)=closure(edge(B,a))=closure({})={}
DFAedge(B,b)=closure(edge(B,b))=closure({})={}
DFAedge(C,a)=closure(edge(C,a))=closure({6})={6,7,8}=D-accepting
DFAedge(C,b)=closure(edge(C,b))=closure({5})={5,7,8}=E-accepting
DFAedge(D,a)=closure(edge(D,a))=closure({})={}
DFAedge(D,b)=closure(edge(D,b))=closure({})={}
DFAedge(E,a)=closure(edge(E,a))=closure({})={}
DFAedge(E,b)=closure(edge(E,b))=closure({})={}

State transition table

<table>
<thead>
<tr>
<th>State</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>B</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>C</td>
<td>D</td>
<td>E</td>
</tr>
<tr>
<td>D</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>E</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Solution:
Closure (0)={0,1,2,4,7}=A
DFAEdge(A,a)=closure(edge(A,a))=closure({3})={1,2,3,4,6,7}=B
DFAEdge(A,b)=closure(edge(A,b))=closure({5})={1,2,4,5,6,7}=C
DFAEdge(A,c)=closure(edge(A,c))=closure({8})={8,9,12,15}=D
DFAEdge(B,a)=closure(edge(B,a))=closure({3})={1,2,3,4,6,7}=B
DFAEdge(B,b)=closure(edge(B,b))=closure({5})={1,2,4,5,6,7}=C
DFAEdge(B,c)=closure(edge(B,c))=closure({8})={8,9,12,15}=D
DFAEdge(C,a)=closure(edge(C,a))=closure({3})={1,2,3,4,6,7}=B
DFAEdge(C,b)=closure(edge(C,b))=closure({5})={1,2,4,5,6,7}=C
DFAEdge(C,c)=closure(edge(C,c))=closure({8})={8,9,12,15}=D
DFAEdge(D,a)=closure(edge(D,a))=closure({10})={10}=E
DFAEdge(D,b)=closure(edge(D,b))=closure({13})={8,9,12,13,14,15}=F
DFAEdge(D,c)=closure(edge(D,c))=closure({16})={16}=G-accepting
DFAEdge(E,a)=closure(edge(E,a))=closure({})={}
DFAEdge(E,b)=closure(edge(E,b))=closure({11})={8,9,11,12,14,15}=H
DFAEdge(E,c)=closure(edge(E,c))=closure({})={}
DFAEdge(F,a)=closure(edge(F,a))=closure({10})={10}=E
DFAEdge(F,b)=closure(edge(F,b))=closure({13})={8,9,12,13,14,15}=F
DFAEdge(F,c)=closure(edge(F,c))=closure({16})={16}=G-accepting
DFAEdge(G,a)=closure(edge(G,a))=closure({})={}
DFAEdge(G,b)=closure(edge(G,b))=closure({})={}
DFAEdge(G,c)=closure(edge(G,c))=closure({})={}
DFAEdge(H,a)=closure(edge(H,a))=closure({10})={10}=E
DFA_{edge}(H,b) = \text{closure}(\text{edge}(H,b)) = \text{closure}(\{13\}) = \{8,9,12,13,14,15\} = F
DFA_{edge}(H,c) = \text{closure}(\text{edge}(H,c)) = \text{closure}(\{16\}) = \{16\} = \text{G-accepting}

### State transition table

<table>
<thead>
<tr>
<th>State/Input</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>B</td>
<td>B</td>
<td>C</td>
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<tr>
<td>C</td>
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<tr>
<td>D</td>
<td>E</td>
<td>F</td>
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<td>F</td>
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<tr>
<td>G</td>
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<td></td>
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</tr>
<tr>
<td>H</td>
<td>E</td>
<td>F</td>
<td>G</td>
</tr>
</tbody>
</table>

![State transition diagram]
Solution:

Closure(1)={1,6,13}=A
DFAedge(A,c)=closure(edge(A,c))=closure({2,7,17})={2,7,17}=B
DFAedge(B,o)=closure(edge(B,o))=closure({3,8,18})={3,8,18}=C
DFAedge(C,o)=closure(edge(C,o))=closure({4,9})={4,9}=D
DFAedge(C,m)=closure(edge(C,m))=closure({14})={14}=E
DFAedge(C,d)=closure(edge(C,d))=closure({19})={19}=F
DFAedge(D,l)=closure(edge(D,l))=closure({5,10})={5,10}=G-accepting
DFAedge(E,p)=closure(edge(E,p))=closure({15})={15}=H
DFAedge(F,e)=closure(edge(F,e))=closure({20})={20}=I-accepting
DFAedge(G,e)=closure(edge(G,e))=closure({11})={11}=J
DFAedge(H,i)=closure(edge(H,i))=closure({9})={9}=K
DFAedge(H,u)=closure(edge(H,u))=closure({16})={16}=L
DFAedge(J,r)=closure(edge(J,r))=closure({12})={12}=M-accepting
DFAedge(K,l)=closure(edge(K,l))=closure({10})={10}=N
DFAedge(L,t)=closure(edge(L,t))=closure({10})={10}=N
DFAedge(N,e)=closure(edge(N,e))=closure({11})={11}=J