Determine the equivalence classes of $\equiv_L$ for each of the following languages $L$:

a. $L = \{ w : w \text{ begins with a 1 and ends with a 0} \}$
b. $L = \{ w : w \text{ contains at least three 1s} \}$
c. $L = \{ w : w \text{ contains 0101 as a substring} \}$
d. $L = \{ w : w \text{ has length at least 3 and its third symbol is a 0} \}$
e. $L = \{ w : w \text{ does not contain 110 as a substring} \}$
f. $L = \{ w : w \text{ has length at most 5} \}$

Use the Myhill-Nerode theorem to prove that the following languages are nonregular:

a. $L = \{ 0^n1^n2^n : n = 0, 1, 2, 3, \ldots \}$
b. $L = \{ wvw : w \in \{0, 1\}^* \}$
c. $L = \{ 0^{2^n} : n = 0, 1, 2, 3, \ldots \}$
d. $L = \{ uvu : u, v \in \{0, 1\}^+ \}$

Construct the smallest possible DFA for each of the following languages, using the Myhill-Nerode theorem in each case to prove that your DFA is indeed the smallest possible:

a. $L = \{ w : w \text{ ends with 00} \}$
b. $L = \{ w : w \text{ contains an even number of 0s, or contains exactly two 1s} \}$
c. $L = \{ \epsilon \}$
d. $L = \{ 0^*1^*0^+ \}$
e. $L = 1^*(001^*)^*$
f. $L_k = \{ w : w \text{ is a palindrome of length } k \}$, where $k \geq 1$ is a fixed integer.

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g. $L_k = \{ w : w \text{ is a palindrome of length } k \}$, where $k \geq 1$ is a fixed integer.

Solve Problem 1.63(a) from the textbook.