HOMEWORK 5

1. Determine the equivalence classes of \( \equiv_L \) for each of the following languages \( L \):
   
   a. \( L = \{ w : w \text{ begins with a 1 and ends with a 0} \} \)
   
   b. \( L = \{ w : w \text{ contains at least three 1s} \} \)
   
   c. \( L = \{ w : w \text{ contains 0101 as a substring} \} \)
   
   d. \( L = \{ w : w \text{ has length at least 3 and its third symbol is a 0} \} \)
   
   e. \( L = \{ w : w \text{ does not contain 110 as a substring} \} \)
   
   f. \( L = \{ w : w \text{ has length at most 5} \} \)

2. Use the Myhill-Nerode theorem to prove that the following languages are nonregular:
   
   a. \( L = \{ 0^n1^n2^n : n = 0, 1, 2, 3, \ldots \} \)
   
   b. \( L = \{ www : w \in \{0, 1\}^* \} \)
   
   c. \( L = \{ 0^{2n} : n = 0, 1, 2, 3, \ldots \} \)
   
   d. \( L = \{ uvu : u, v \in \{0, 1\}^* \} \)

3. Construct the smallest possible DFA for each of the following languages, using the Myhill-Nerode theorem in each case to prove that your DFA is indeed the smallest possible:
   
   a. \( L = \{ w : w \text{ ends with 00} \} \)
   
   b. \( L = \{ w : w \text{ contains an even number of 0s, or contains exactly two 1s} \} \)
   
   c. \( L = \{ 0 \} \)
   
   d. \( L = \{ \epsilon \} \)
   
   e. \( L = 0^*1^*0^+ \)
   
   f. \( L = 1^*(001^+)^* \)
   
   g. \( L_k = \{ w : w \text{ is a palindrome of length } k \} \), where \( k \geq 1 \) is a fixed integer.