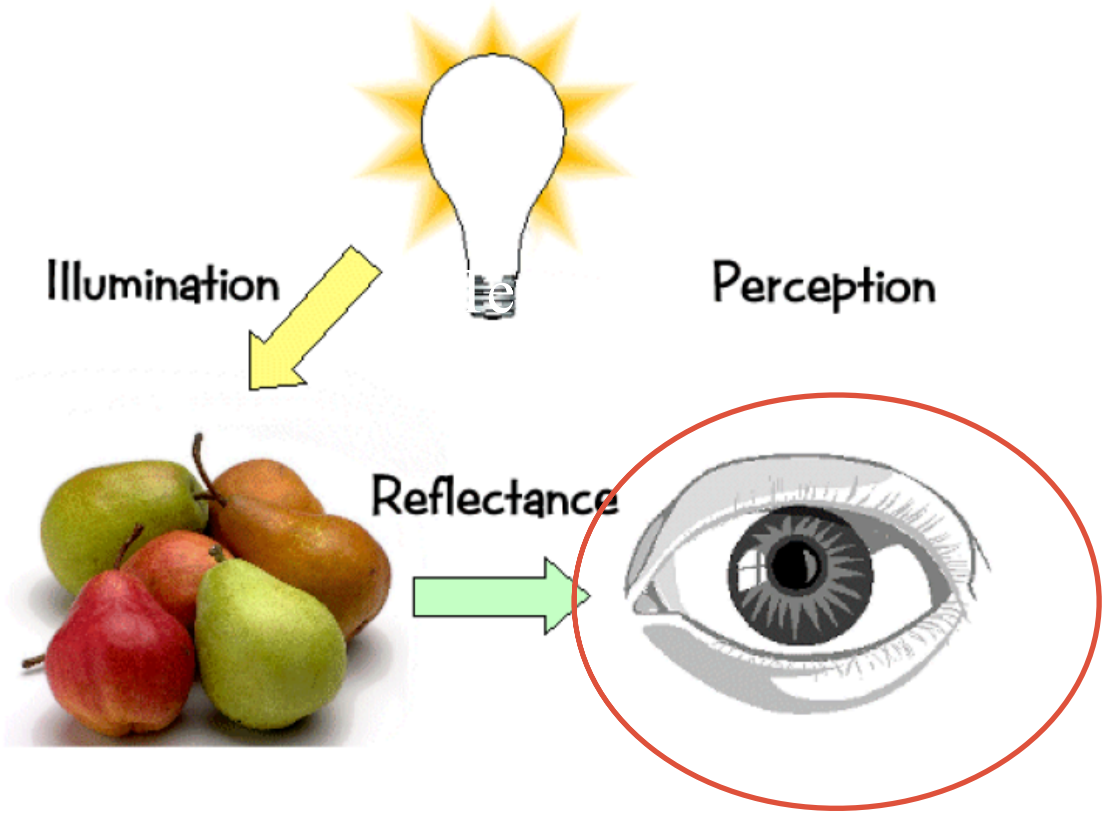
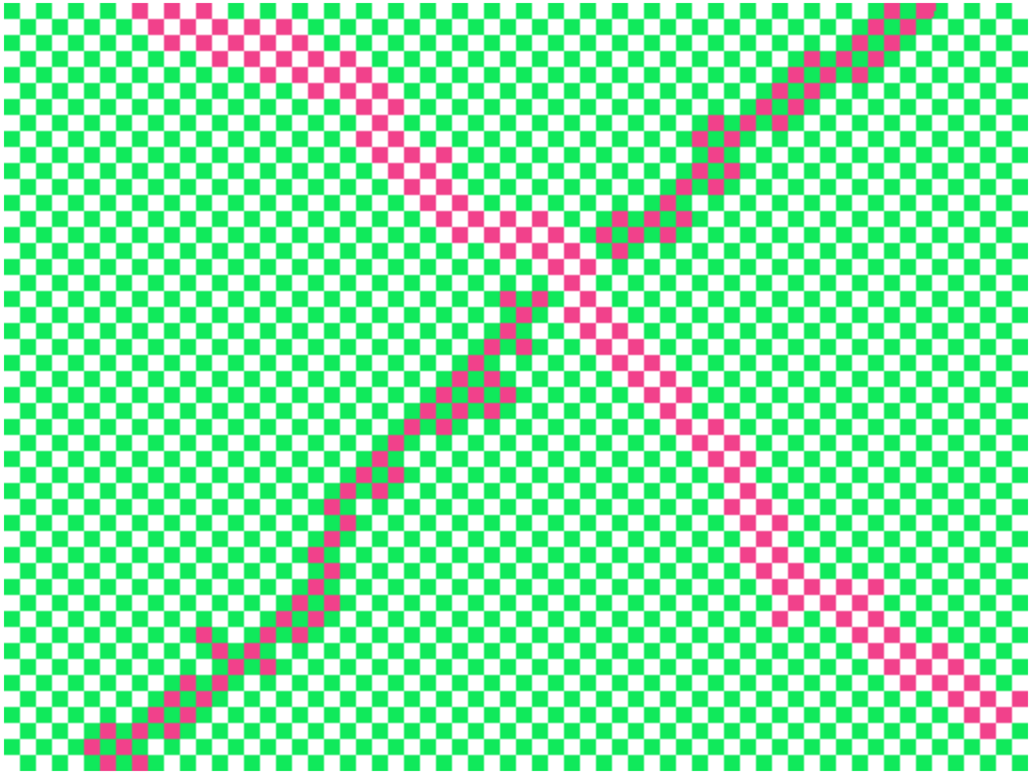
Elements of Color

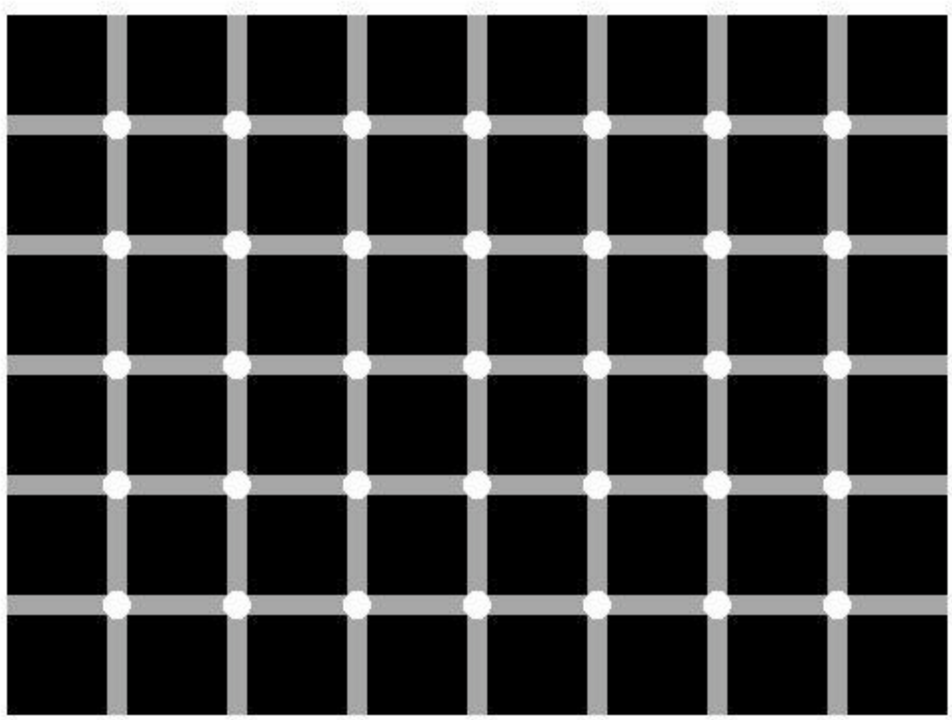
Chapter 11



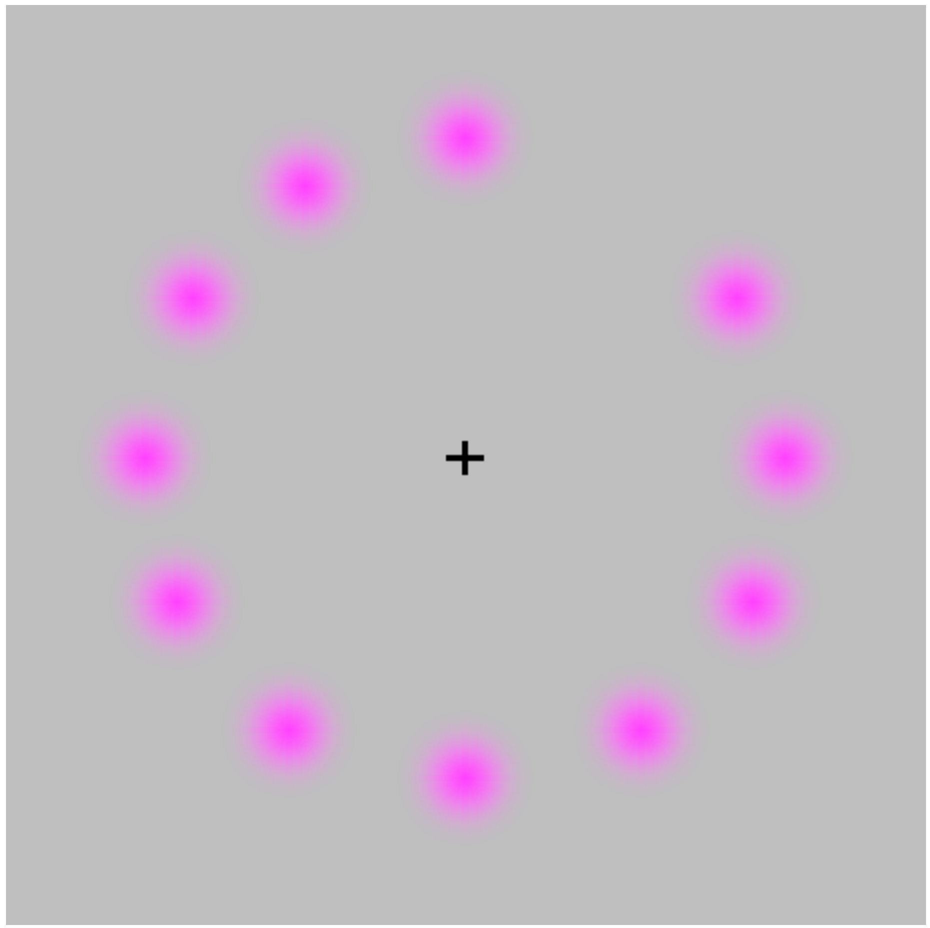
Illusion 1



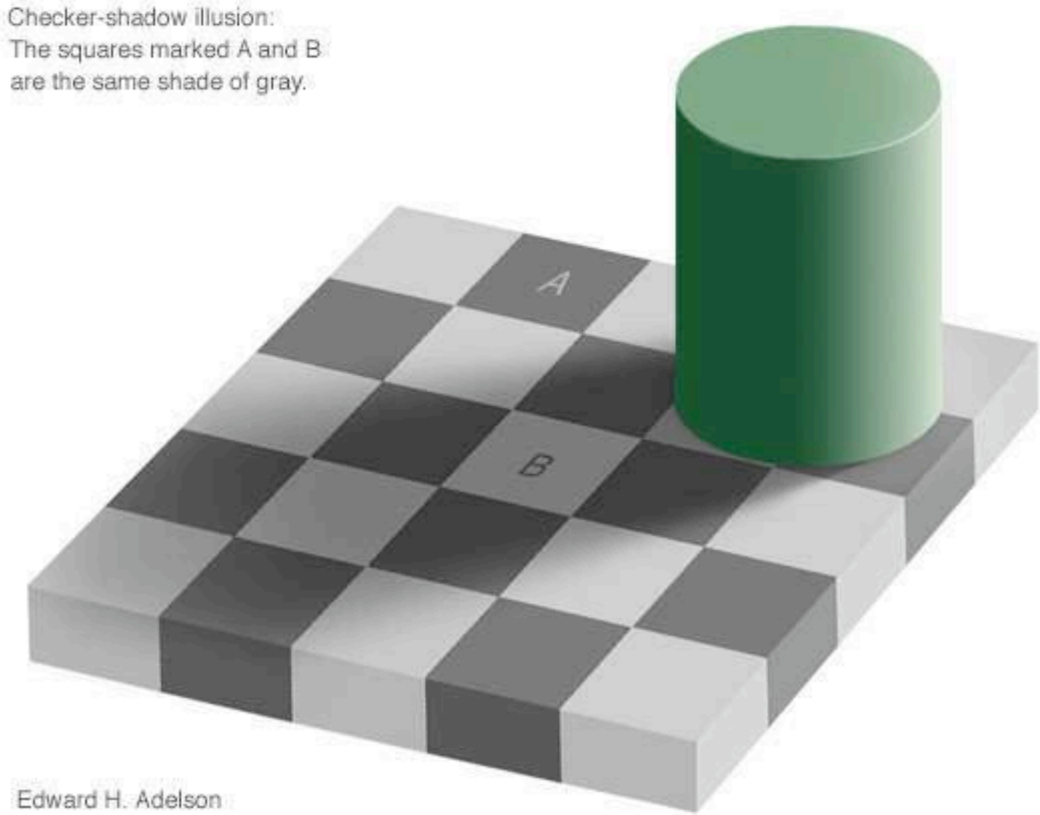
Illusion 2



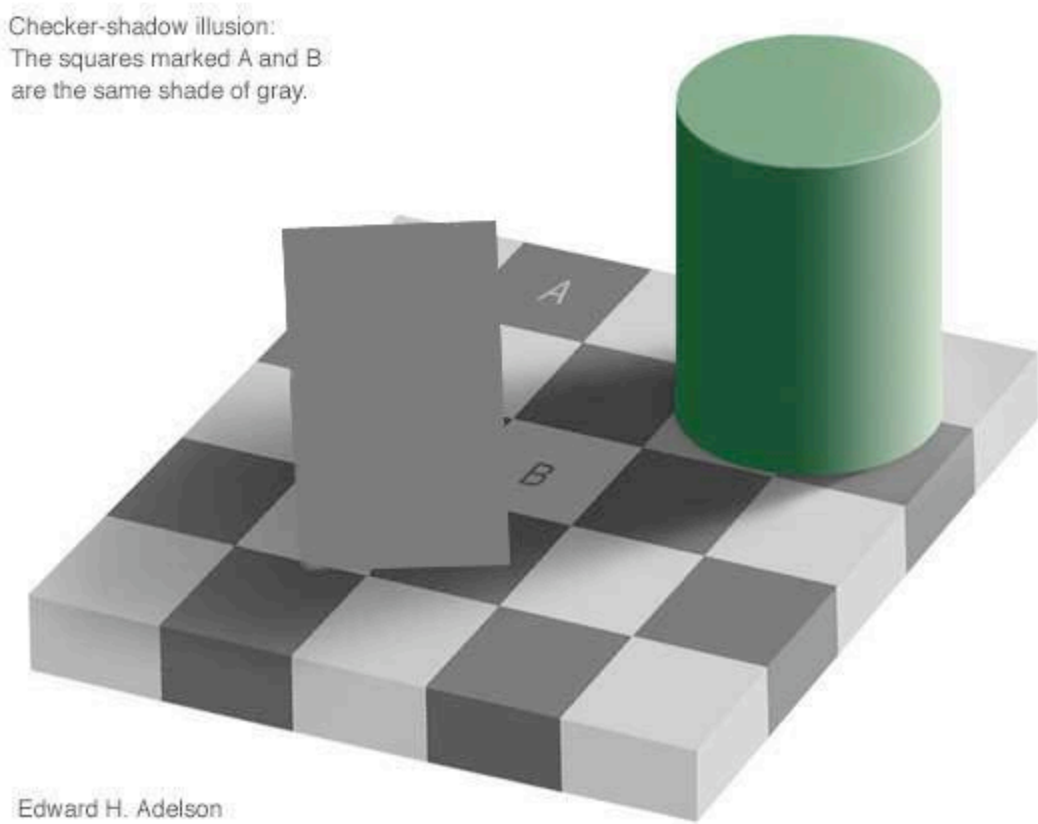
Illusion 3



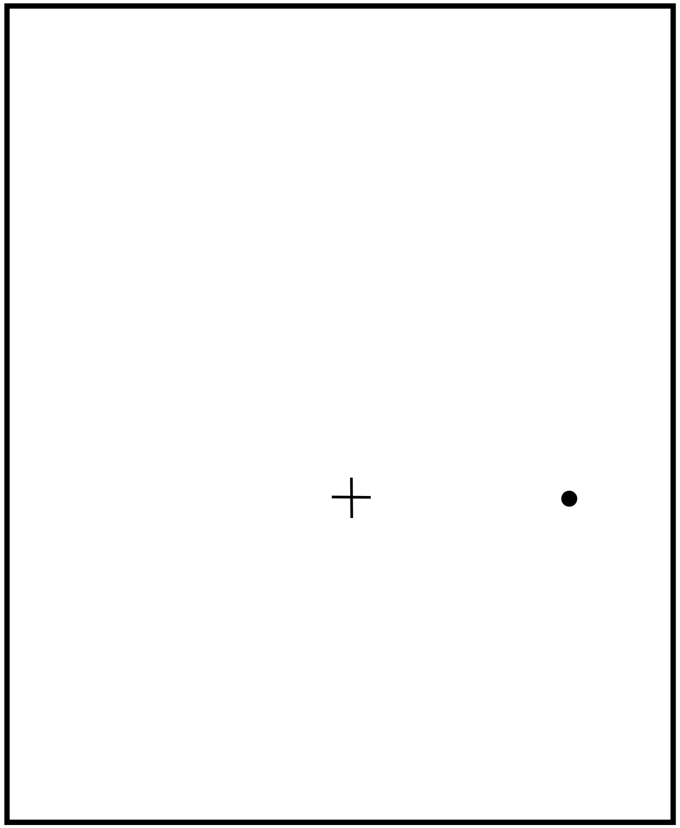
Illusion 4


Checker-shadow illusion:
The squares marked A and B
are the same shade of gray.
A
B
Edward H. Adelson

Illusion 4: the proof



Another interesting phenomenon



Fourier Analysis

- A light wave is described by a function $\psi(x,y,z,t)$ that gives the electric or magnetic field as a function of space (x,y,z) and time t coordinates.

- The function $\psi(x,y,z,t)$ can be characterized in terms of its power spectrum $\varphi(f)$.

- The function $\varphi(f)$ indicates how much of the energy of $\psi(x,y,z,t)$ is associated with waves of frequency f.

Fourier Series

Any periodic and integrable function f(x) in [-pi,pi] can be approximated with a series

$$f(x) \sim a_0 + \sum_{i=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$$

where

$$\begin{aligned} a_0 &= \frac{1}{2\pi} \int_{\pi}^{\pi} f(x) dx, \\ a_n &= \frac{1}{2\pi} \int_{\pi}^{\pi} f(x) \cos(nx) dx, \quad 1 \leq n, \\ b_n &= \frac{1}{2\pi} \int_{\pi}^{\pi} f(x) \sin(nx) dx, \quad 1 \leq n. \end{aligned}$$

Fourier series

- In some sense the function is analyzed in "function coordinates" that happen to be cosines and sines.

Note:

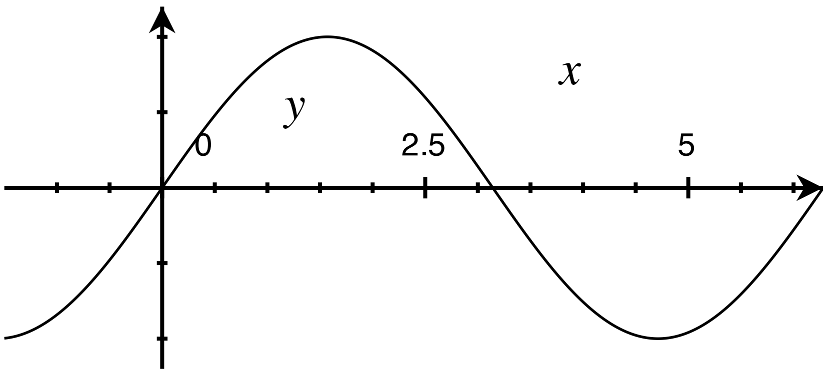
- It is an orthogonal basis

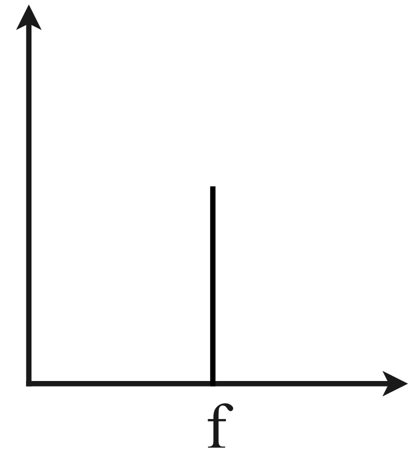
$$\int_{-\pi}^{\pi} \cos(nx)\sin(mx)dx = 0, \quad for \quad m \neq n.$$

Spatial domain

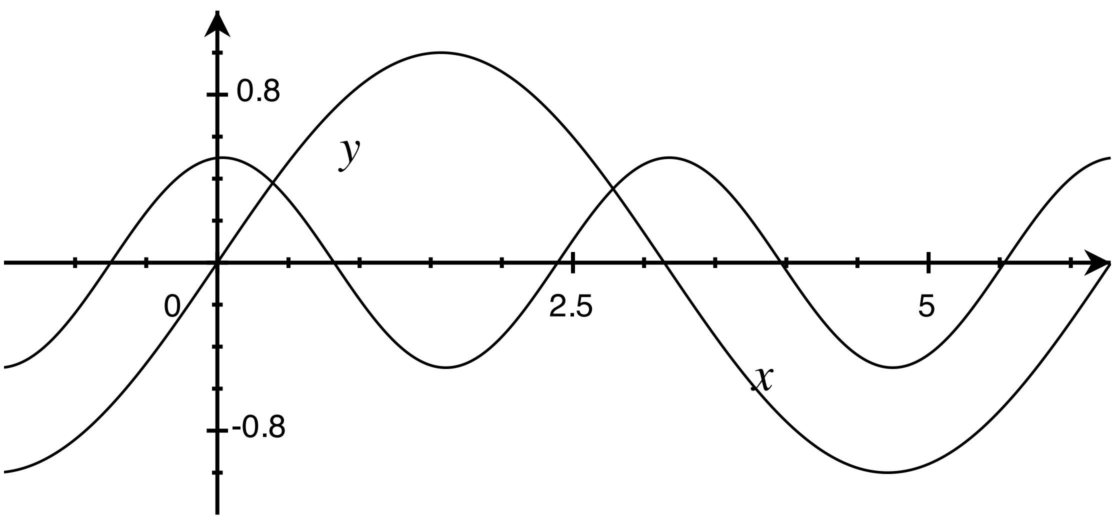
Frequency domain

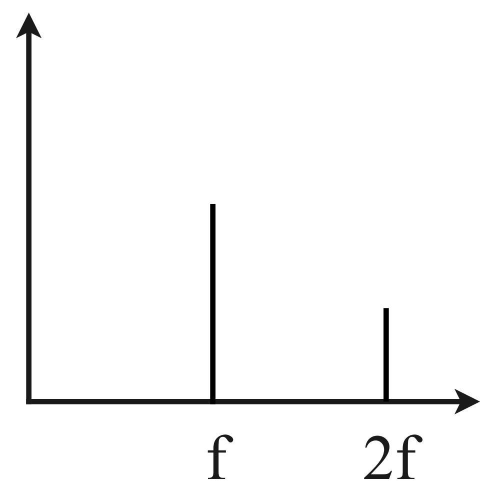
$y = \sin(x)$



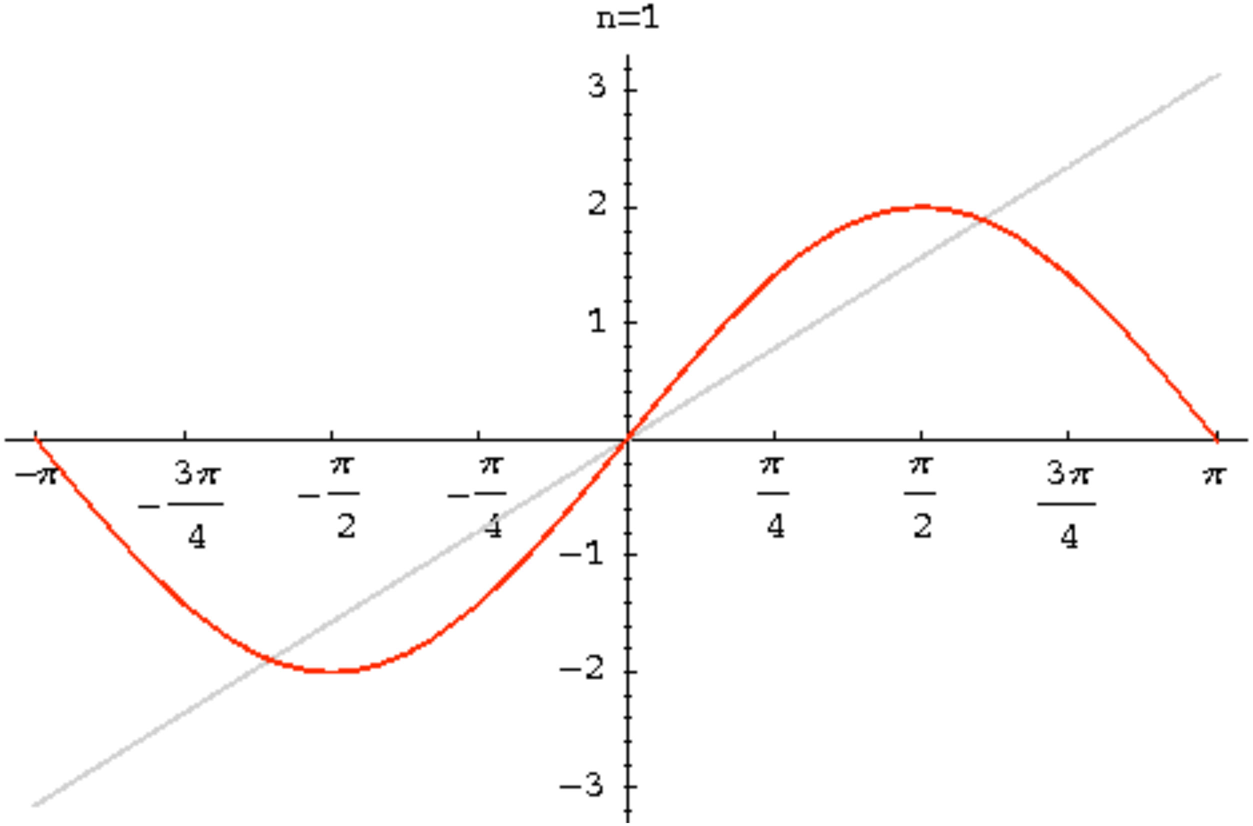


$y = \sin(x) + 0.5\ \sin(2x+1.5)$





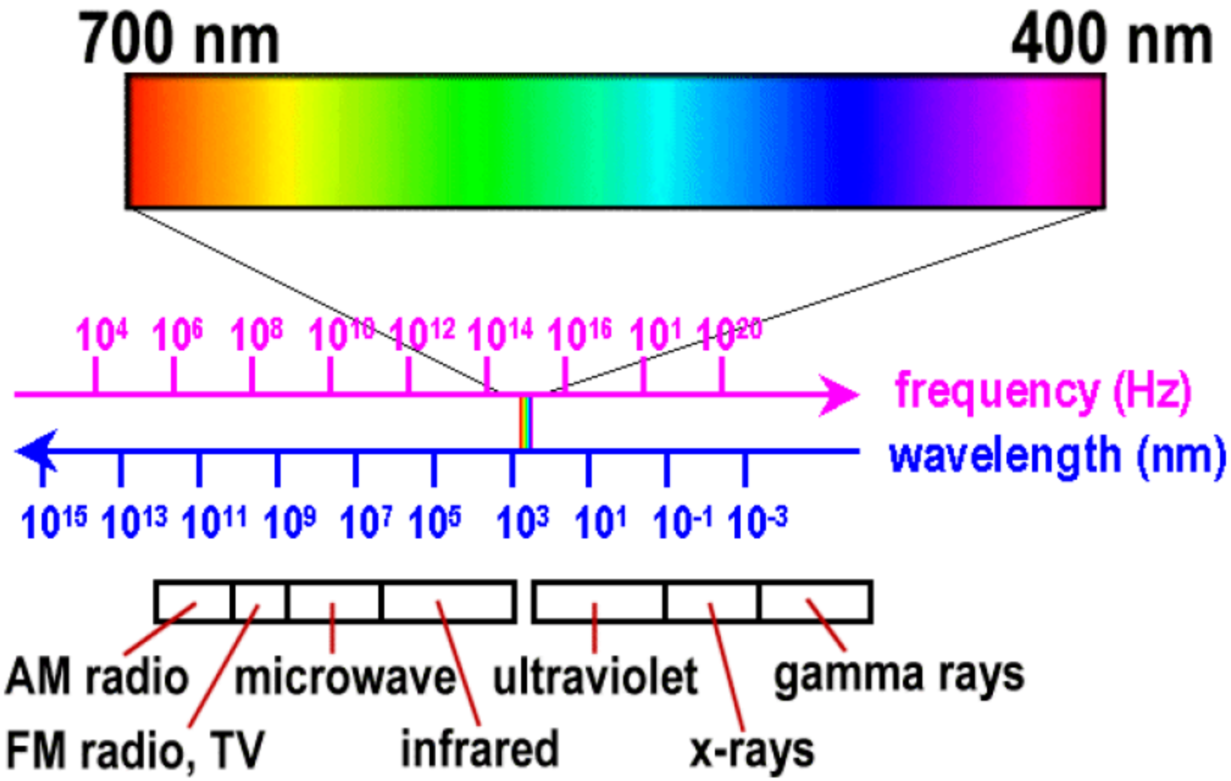
Example



n=1
3
2
1
−1
−2
−3
−π
$-\frac{3\pi}{4}$
$-\frac{\pi}{2}$
$-\frac{\pi}{4}$
$\frac{\pi}{4}$
$\frac{\pi}{2}$
$\frac{3\pi}{4}$
π

Visible Spectrum

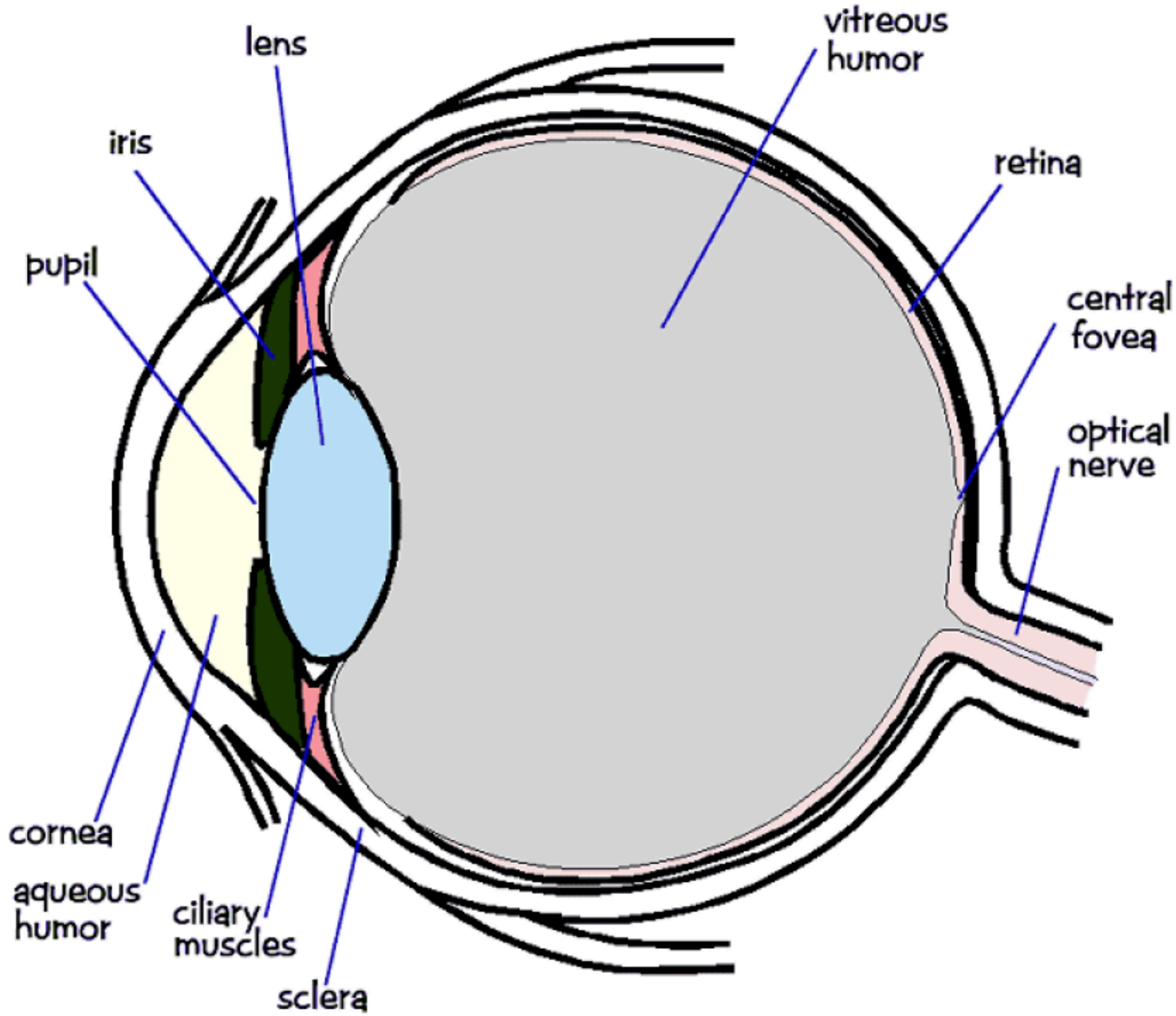
We percieve electromagnetic energy having wavelengths in the range 400-700 nm as *visible light*.



The Eye

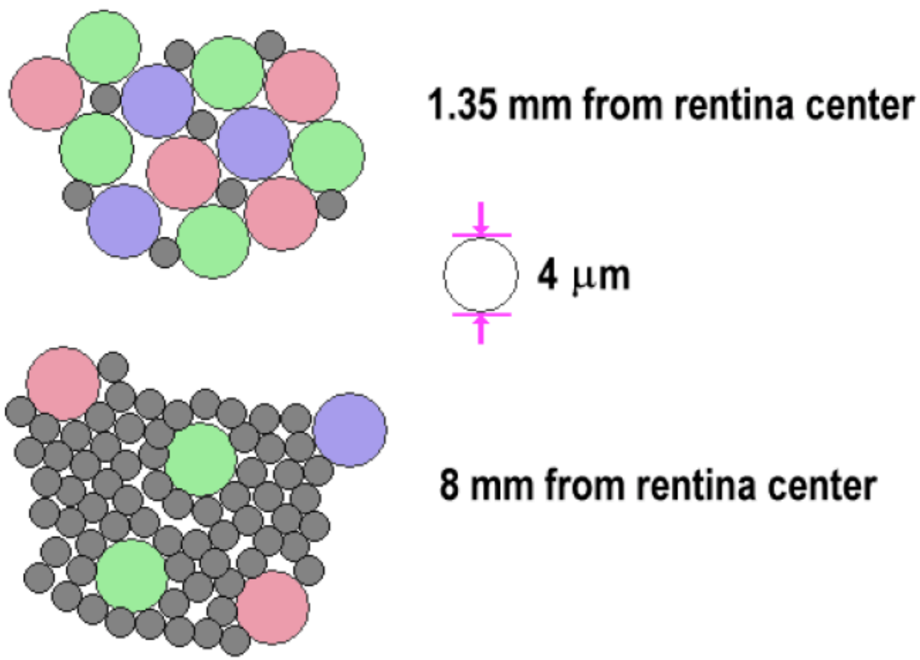
The photosensitive part of the eye is called the *retina*.

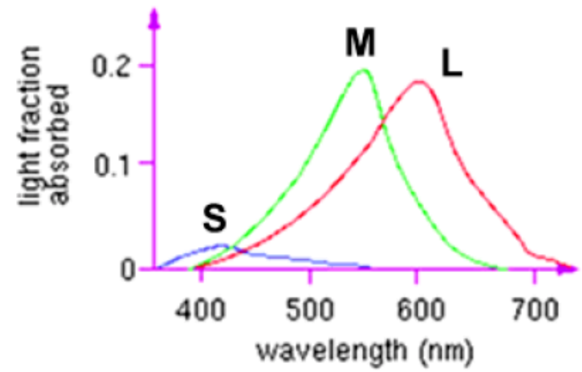
The retina is largely composed of two types of cells, called *rods* and *cones*. Only the cones are responsible for color perception.



The Fovea

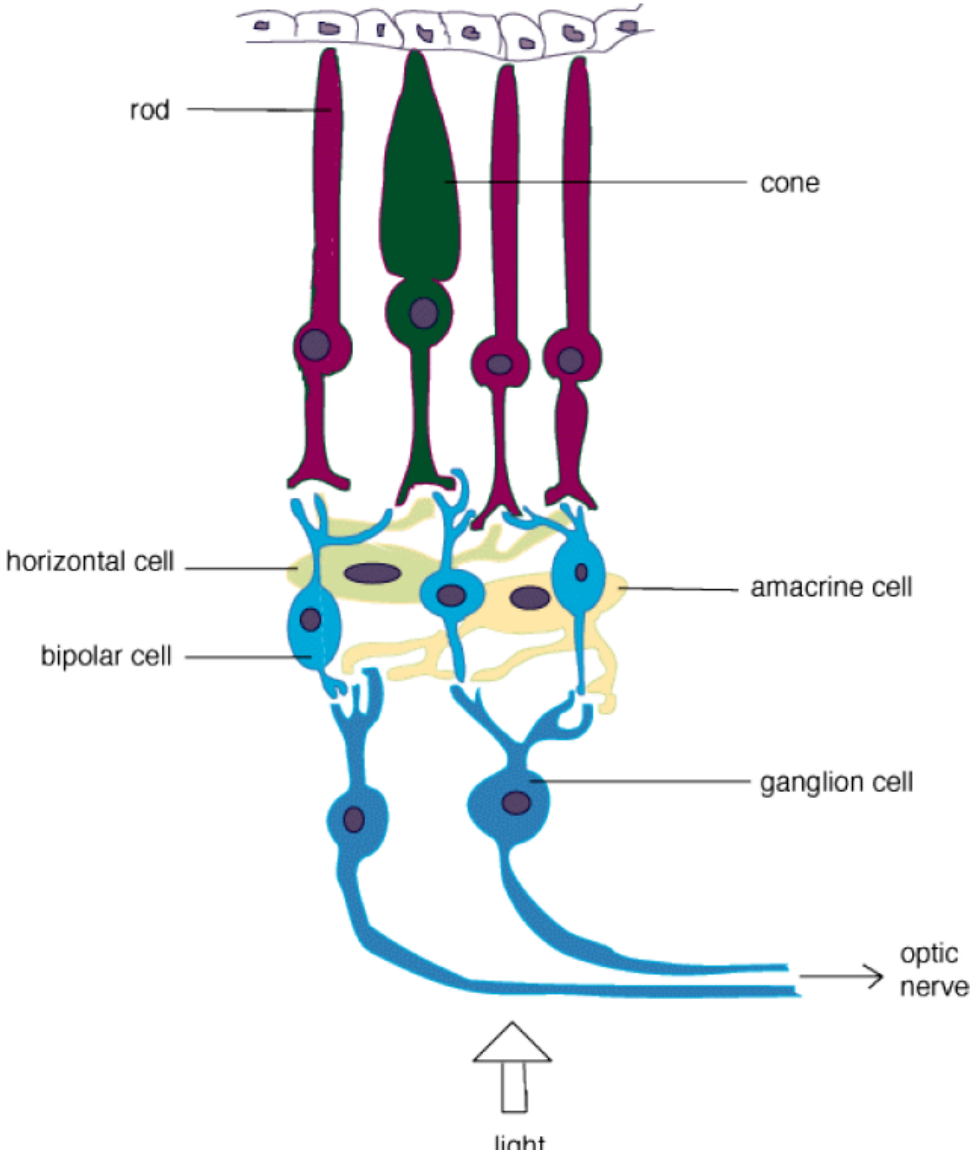
Cones are most densely packed within a region of the eye called the *fovea*.

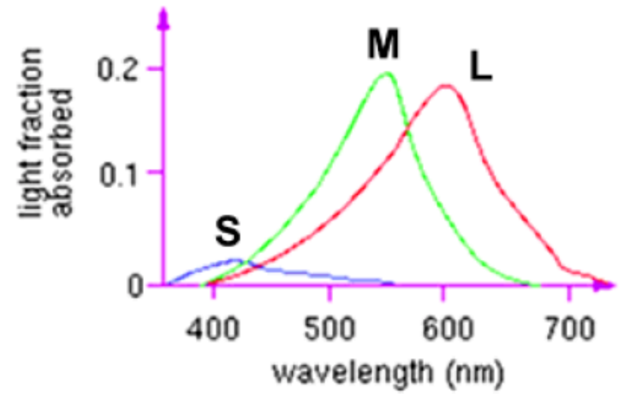




There are three types of cones, referred to as S, M, and L. They are roughly equivalent to blue, green, and red sensors, respectively. Their peak sensitivities are located at approximately 430nm, 560nm, and 610nm for the "average" observer.

The Fovea

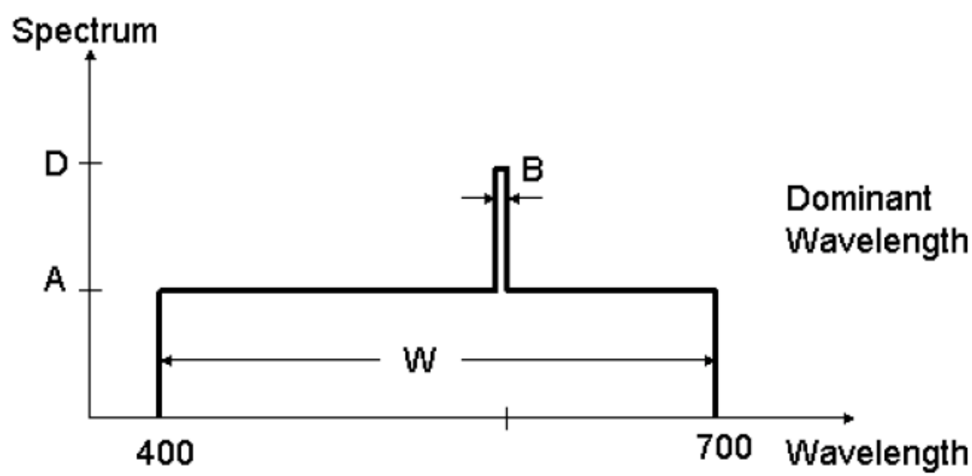




Colorblindness results from a deficiency of one cone type.

Dominant Wavelength

- Location of **dominant wavelength** specifies the **hue** of the color
- The **luminance** is the total power of the light (area under curve) and is related to **brightness**
- The **saturation** (purity) is percentage of luminance in the dominant wavelength

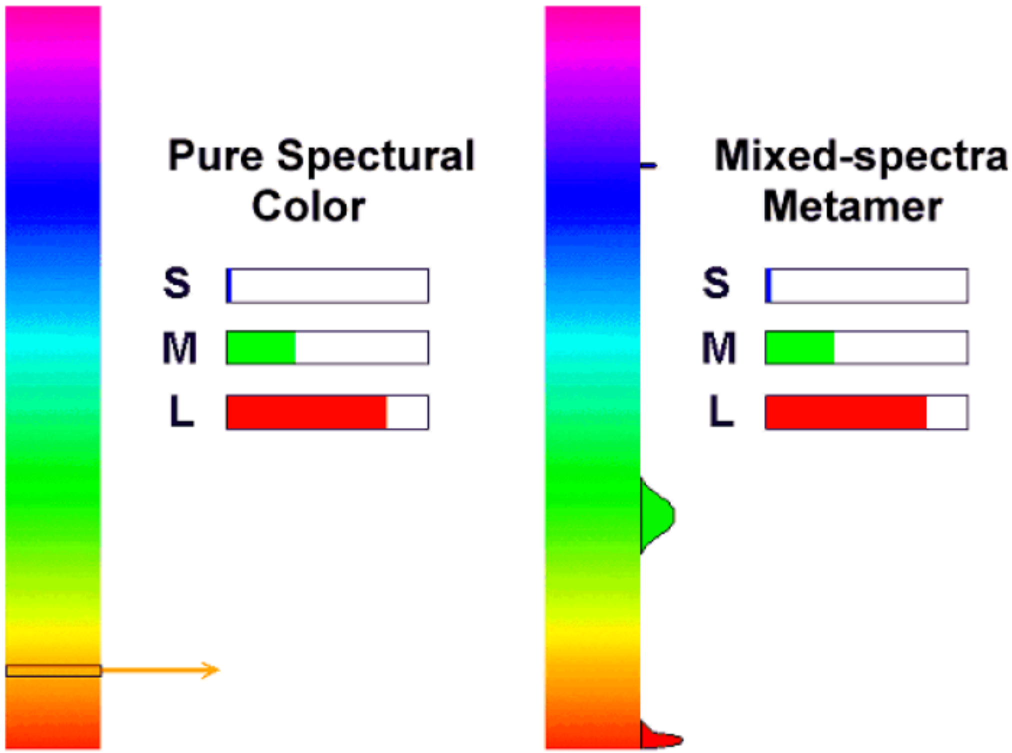


$L = (D - A)B + A\,W$ luminance

$S = (D-A)B / L$ saturation

Color Perception

- Different spectra can result in a perceptually identical sensations called *metamers*
- Color perception results from the simultaneous stimulation of 3 cone types (*trichromat*)
- Our perception of color is also affected by surround effects and adaptation



Color Algebra

- $S = P$, means spectrum S and spectrum P are perceived as the same color
- if $(S = P)$ then $(N + S = N + P)$
- if $(S = P)$ then $aS = aP$, for scalar a
- It is meaningful to write linear combinations of colors $T = aA + bB$
- Color percepion is three-dimensional, any color C can be constructed as the superposition of three primaries:

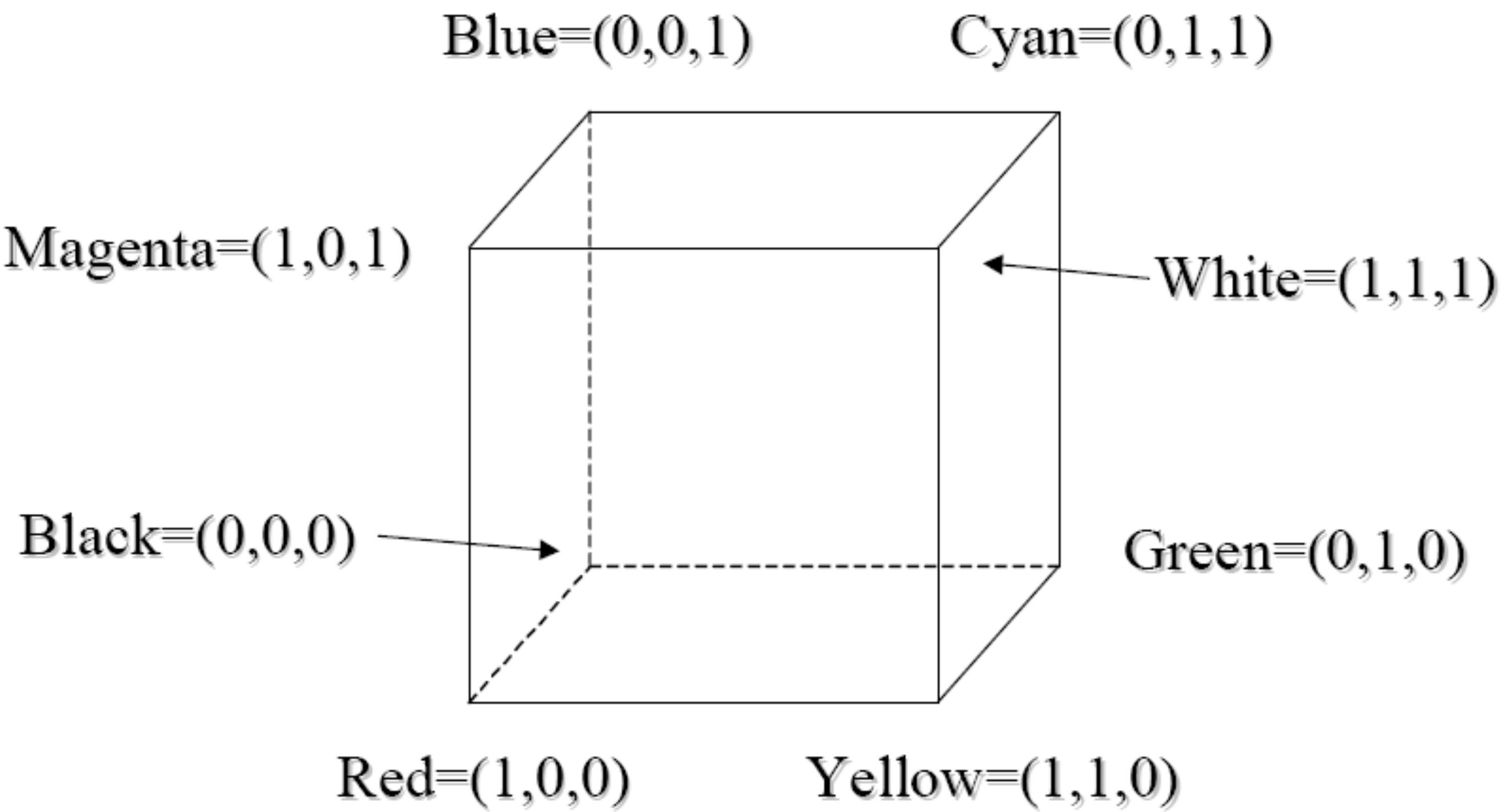
$$C = rR + gG + bB$$

- Focus on "unit brightness" colors, for which $r+g+b=1$, these lie on a plane in 3D color space

The RGB Color Model

- Represent colors as combinations of red, green and blue primaries. “Additive Primaries”.
- Any composite color is defined by three weights: w_r, w_g and w_b.
- Each weight lies in the range [0..1].
- The space of colors form a cube in three dimensions.

The RGB Color Model



Complementary Colors

- Cyan = White - Red.

  $(0,1,1) = (1,1,1) – (1,0,0)$

- Magenta = White - Green.

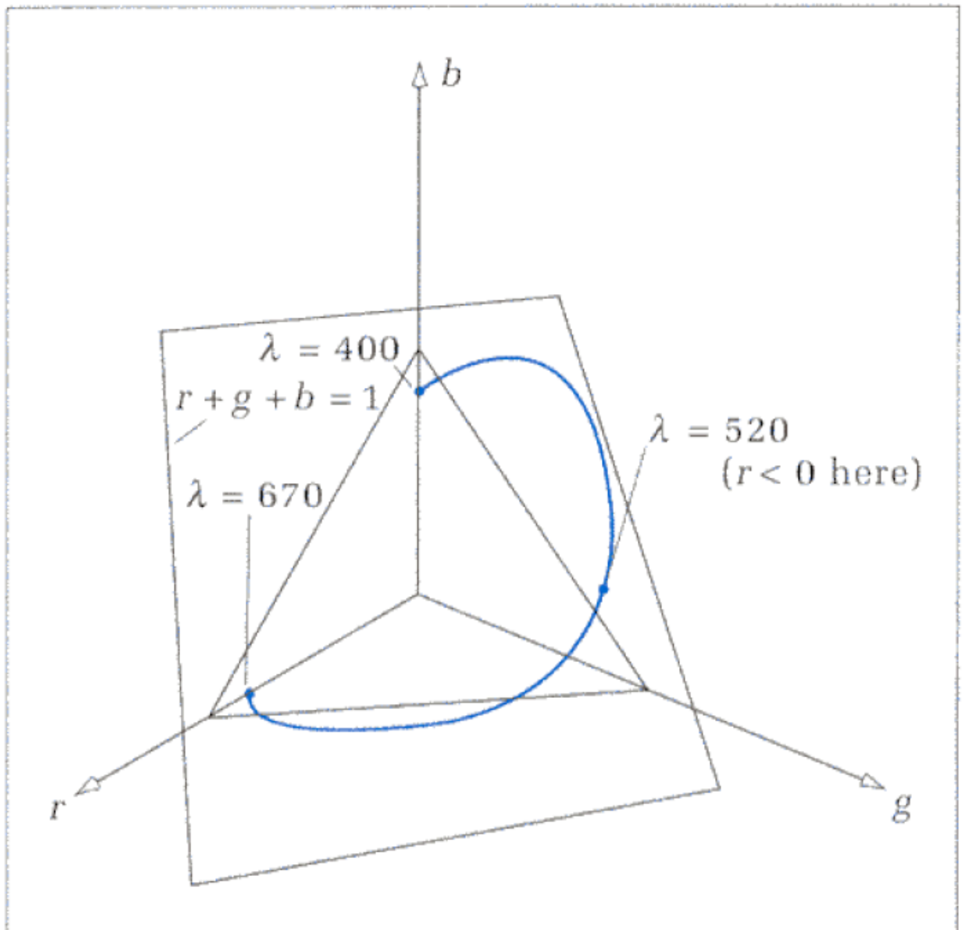
  $(1,0,1) = (1,1,1) – (0,1,0)$

- Yellow = White - Blue.

  $(1,1,0) = (1,1,1) – (0,0,1)$

Saturated Color Curve in RGB

- Plot the saturated colors (pure spectral colors) in the r+g+b=1 plane



What are the rgb coordinates of the pure spectral colors?

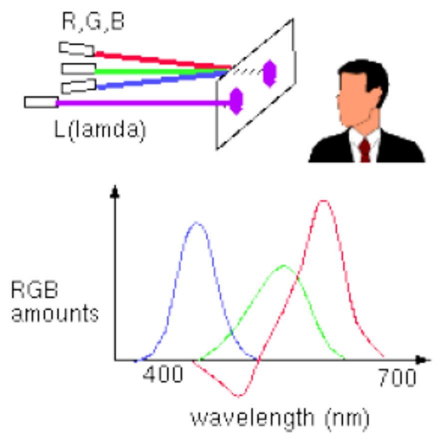
Perception of color is largely a result of a psycho-physical process.

The question can only be answered experimentally.

This was done long time ago through an experiment of "Color Matching".

Color Matching

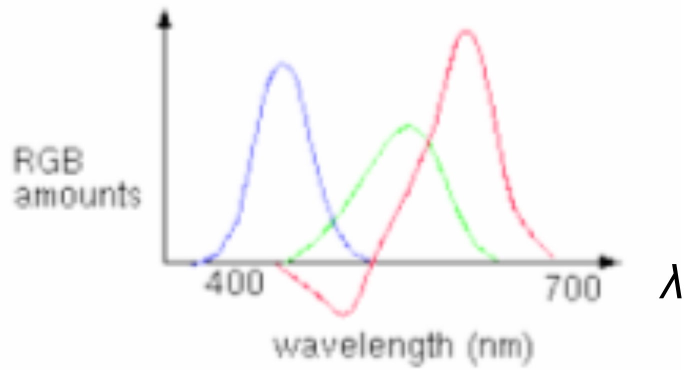
In order to define the perceptual 3D space in a "standard" way, a set of experiments can (and have been) carried by having observers try and match color of a given wavelength, lambda, by mixing three other pure wavelengths, such as R=700nm, G=546nm, and B=436nm in the following example. Note that the phosphours of color TVs and other CRTs do not emit pure red, green, or blue light of a single wavelength, as is the case for this experiment.



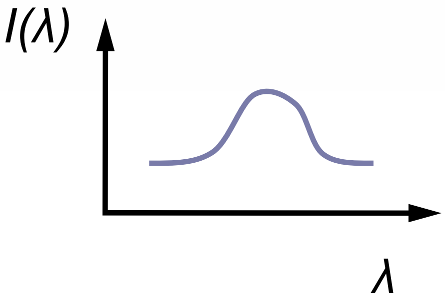
Matching functions

Weight functions, not spectrums.

$f_i : i = r,g,b$



Given a spectrum *I(λ)*, we can use this functions to compute its color:



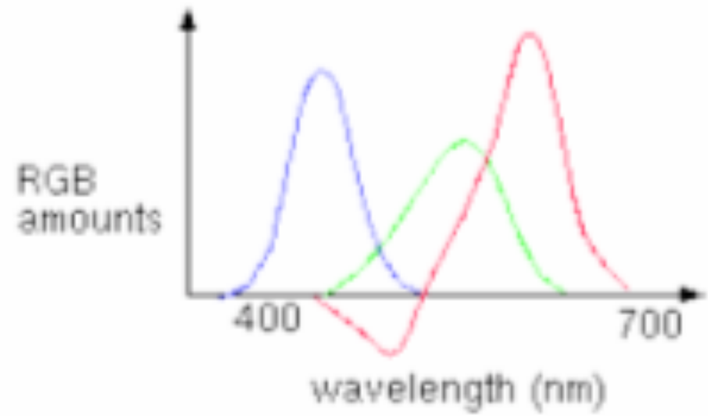
$$R = k\int_{\lambda} f_r(\lambda) I(\lambda) d\lambda$$

$$G = k\int_{\lambda} f_g(\lambda) I(\lambda) d\lambda$$

$$B = k\int_{\lambda} f_b(\lambda) I(\lambda) d\lambda$$

$C = R\mathbf{R} + B\mathbf{B} + G\mathbf{G}$ where $\mathbf{R}, \mathbf{G}, \mathbf{B}$ are the unit vectors.

Problem



Negative coefficients

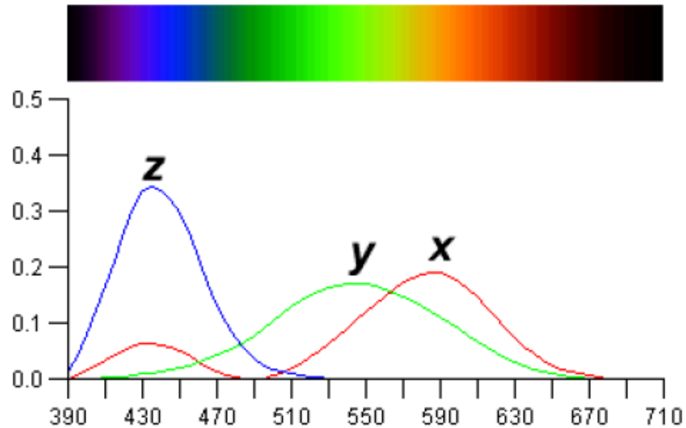
This will happen with any choice of visible primaries.

Adding colors creates a less saturated color.

Solution: affine transformation of (r,g,b)

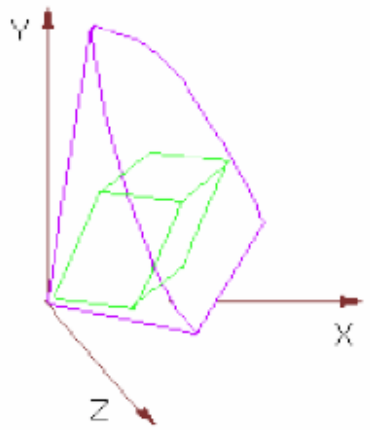
CIE Color Space

In order to achieve a representation which uses only positive mixing coefficients, the CIE ("Commission Internationale d'Eclairage") defined three new hypothetical light sources, x, y, and z, which yield positive matching curves:



If we are given a spectrum and wish to find the corresponding X, Y, and Z quantities, we can do so by integrating the product of the spectral power and each of the three matching curves over all wavelengths. The weights X,Y,Z form the three-dimensional CIE XYZ space, as shown below.

The RGB cube in XYZ



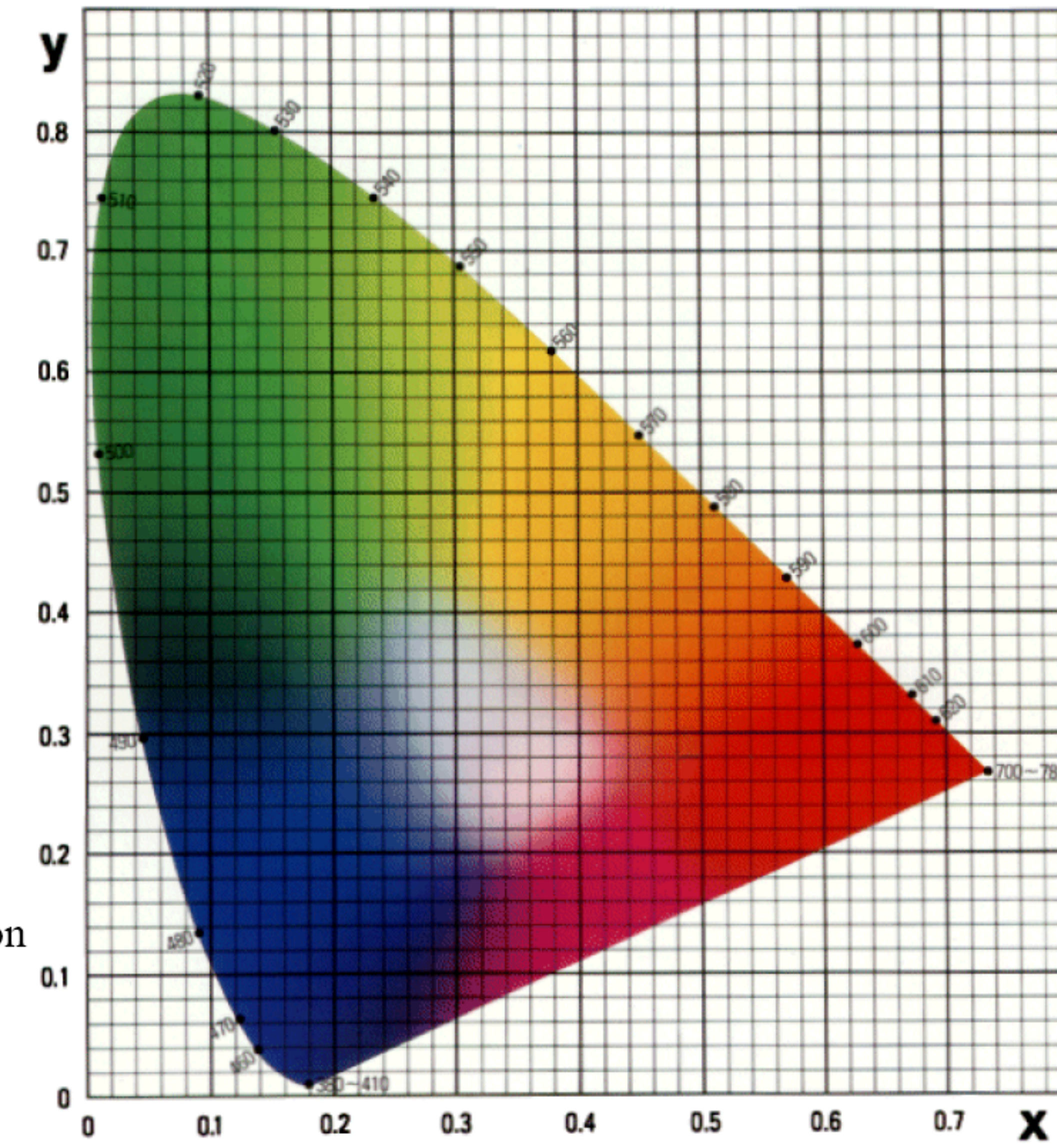
CIE Chromaticity Diagram

Often it is convenient to work in a 2D color space. This is commonly done by projecting the 3D color space onto the plane X+Y+Z=1, yielding a *CIE chromaticity diagram*. The projection is defined as:

$$x = \frac{X}{X+Y+Z} \qquad y = \frac{Y}{X+Y+Z}$$

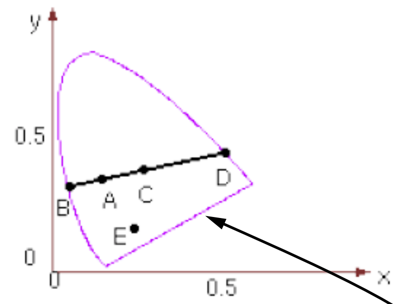
$$z = \frac{Z}{X+Y+Z} = 1 - x - y$$

Giving the chromaticity diagram shown on the right.

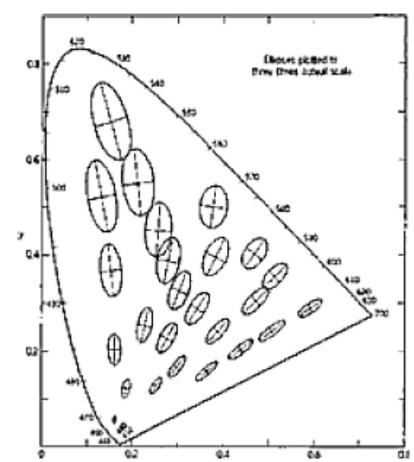


Definitions:

- Spectrophotometer
- Illuminant C
- Complementary colors



- Dominant wavelength
- Non-spectral colors
- Perceptually uniform color space



Working in XYZ

$$mono(\lambda) = (X(\lambda)\mathbf{X}, Y(\lambda)\mathbf{Y}, Z(\lambda)\mathbf{Z}) = (X, Y, Z)$$

From XYZ to (x,y,Y)

$$x = X/(X + Y + Z),$$

$$y = Y/(X + Y + Z),$$

$$z = Z/(X + Y + Z)$$

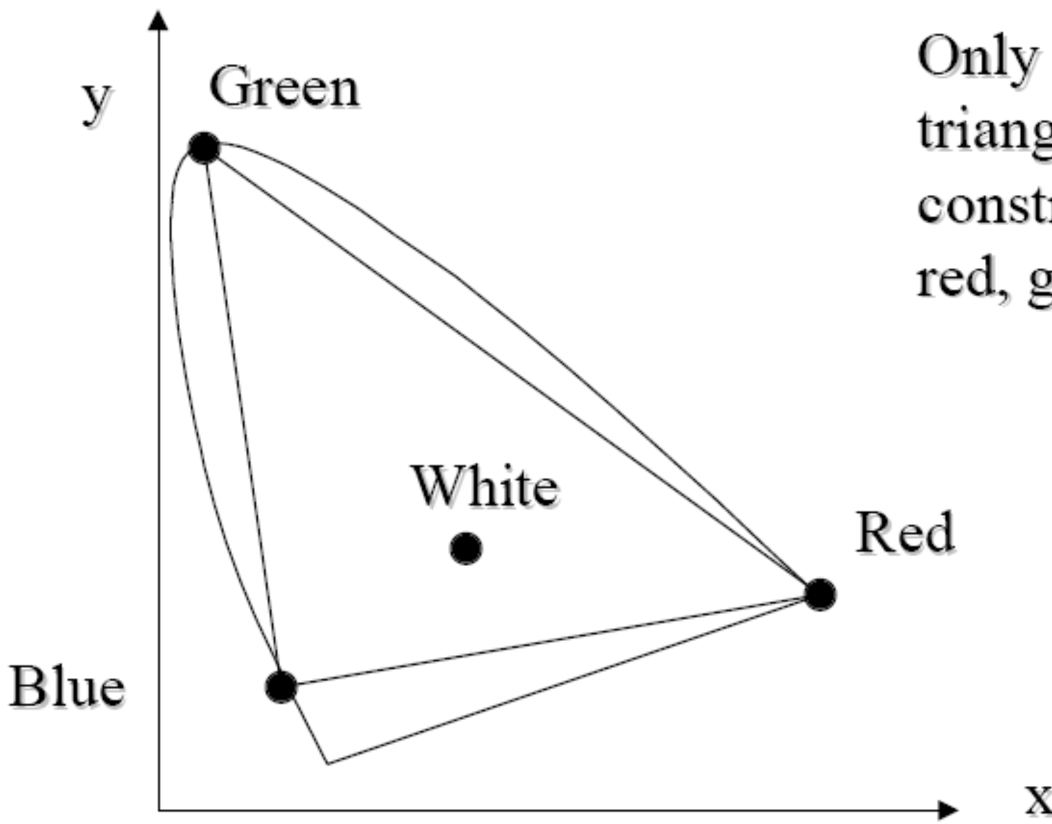
From (x,y,Y) to XYZ

$$X = xY/y,$$

$$Y = Y,$$

$$Z = (1 - x - y)Y/z$$

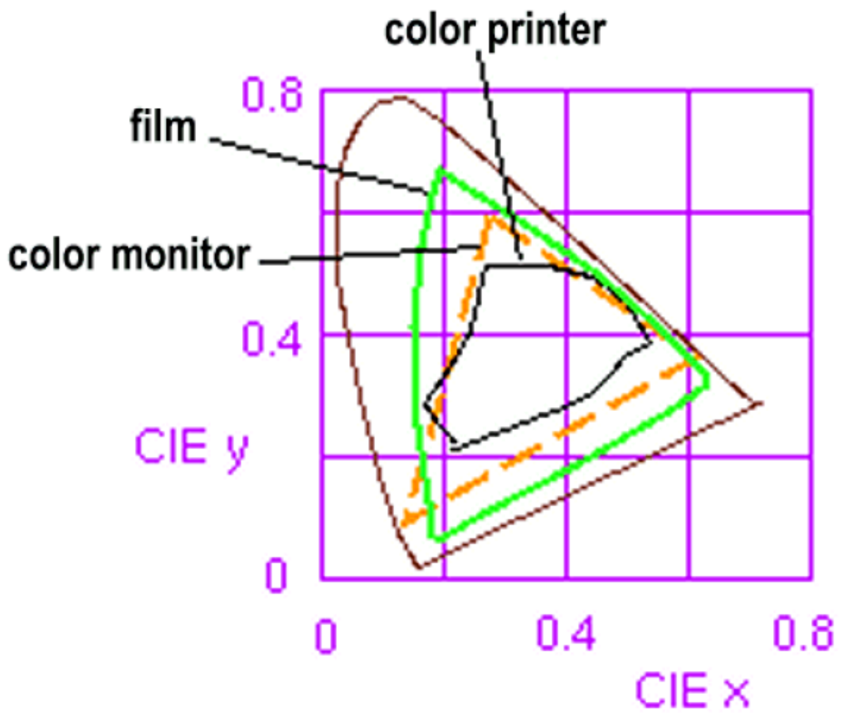
RGB Color Gamut



Only colors within the triangle can be constructed by mixing red, green and blue.

Color Gamuts

The chromaticity diagram can be used to compare the "gamuts" of various possible output devices (i.e., monitors and printers). Note that a color printer cannot reproduce all the colors visible on a color monitor.



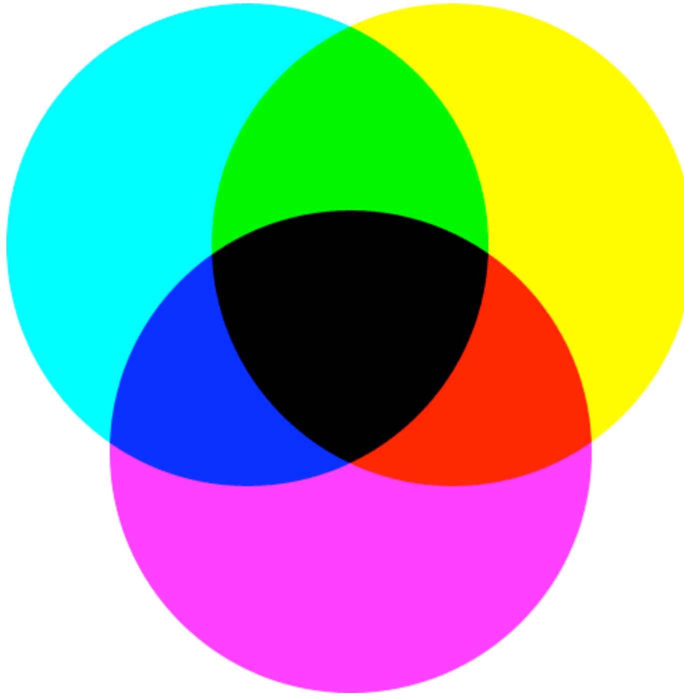
Additive v. Subtractive Color

- Color CRT monitors are best understood using the RGB model.
  - All three RGB electron beams off: Black screen.
  - All three RGB electron beams on: White screen.
- Color printers are best understood using the CMY model.
  - All three CMY pigments absent: White paper.
  - All three CMY pigments present: Black paper.

RGB vs CMY

RGB

CMY



The CMY Color Model

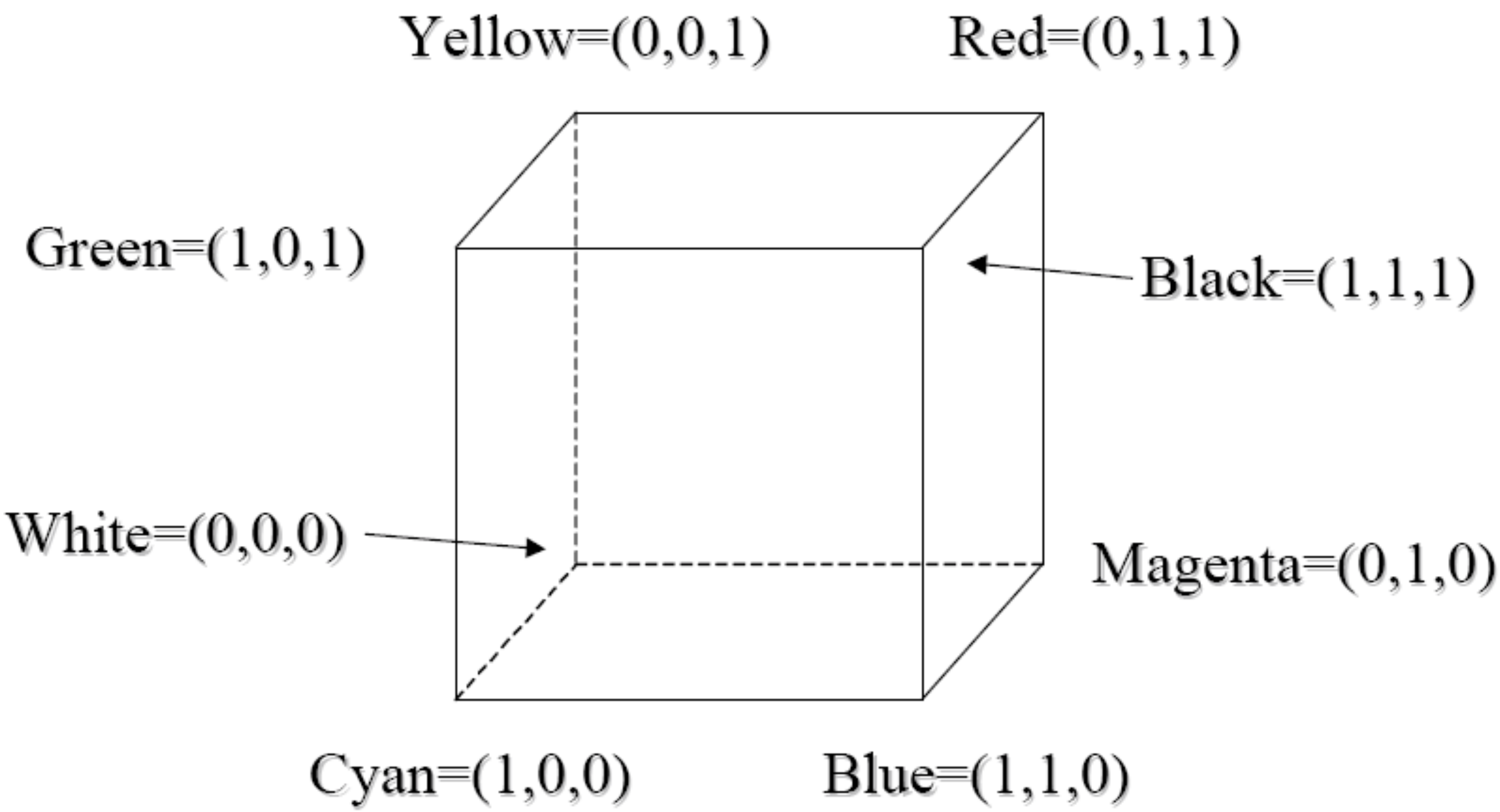
- Represent colors as combinations of cyan, magenta and yellow primaries. “Subtractive Primaries”.
- Any composite color is defined by three weights: w_c, w_m and w_y.
- Each weight lies in the range [0..1].
- The space of colors form a cube in three dimensions.

Complementary Colors

- Red = White - Cyan.
- Green = White - Magenta.
- Blue = White - Yellow.

Affine transformation

The CMY Color Model



Color Printing

Green paper is green because it reflects green and absorbs other wavelengths. The following table summarizes the properties of the four primary types of printing ink.

dye color	*absorbs*	*reflects*
cyan	red	blue and green
magenta	green	blue and red
yellow	blue	red and green
black	all	none

To produce blue, one would mix cyan and magenta inks, as they both reflect blue while each absorbing one of green and red. Unfortunately, inks also interact in non-linear ways. This makes the process of converting a given monitor color to an equivalent printer color a challenging problem.

Black ink is used to ensure that a high quality black can always be printed, and is often referred to as to K. Printers thus use a CMYK color model.

Equivalent colors between monitors (color conversion)

Monitor 1 has phosphors with colors:

$\mathbf{R}_1 = (X^1_r, Y^1_r, Z^1_r)$

$\mathbf{G}_1 = (X^1_g, Y^1_g, Z^1_g)$

$\mathbf{B}_1 = (X^1_b, Y^1_b, Z^1_b)$

Monitor 2 has phosphors with colors :

$\mathbf{R}_2 = (X^2_r, Y^2_r, Z^2_r)$

$\mathbf{G}_2 = (X^2_g, Y^2_g, Z^2_g)$

$\mathbf{B}_2 = (X^2_b, Y^2_b, Z^2_b)$

Given color $C_1 = (R^1_c, G^1_c, B^1_c)$ in monitor 1 what is the equivalent color $C_2 = (R^2_c, G^2_c, B^2_c)$ in monitor 2 ?

Color in monitor one

Given color $C_1 = (R^1_c, G^1_c, B^1_c)$ in monitor one its coordinates $C = (X_c, Y_c, Z_c)$ in XYZ-space are:

$$
\begin{array}{l}
C = R_c^1 \mathbf{R}_1 + G_c^1 \mathbf{G}_1 + B_c^1 \mathbf{B}_1 \rightarrow \\
\left[\begin{array}{c} X_c \\ Y_c \\ Z_c \end{array}\right] = \left[\begin{array}{ccc} X_r^1 & X_g^1 & X_b^1 \\ Y_r^1 & Y_g^1 & Y_b^1 \\ Z_r^1 & Z_g^1 & Z_b^1 \end{array}\right] \left[\begin{array}{c} R_c^1 \\ G_c^1 \\ B_c^1 \end{array}\right] \rightarrow \\
\mathbf{C} = \mathrm{M}_1 \mathbf{C}_1
\end{array}
\qquad (1)
$$

Equivalent color in monitor 2

Similarly for monitor 2:

$C = M_2 C_2$

Putting both together:

$$\begin{cases} C = M_1 C_1 \\ C = M_2 C_2 \end{cases} \rightarrow \begin{cases} C = M_1 C_1 \\ C_2 = M_2^{-1} C \end{cases} \rightarrow$$

$$C_2 = M_2^{-1} M_1 C_1$$

Other Color Systems

Several other color models also exist. Models such as HSV (hue, saturation, value) and HLS (hue, luminosity, saturation) are designed for intuitive understanding. Using these color models, the user of a paint program would quickly be able to select a desired color.

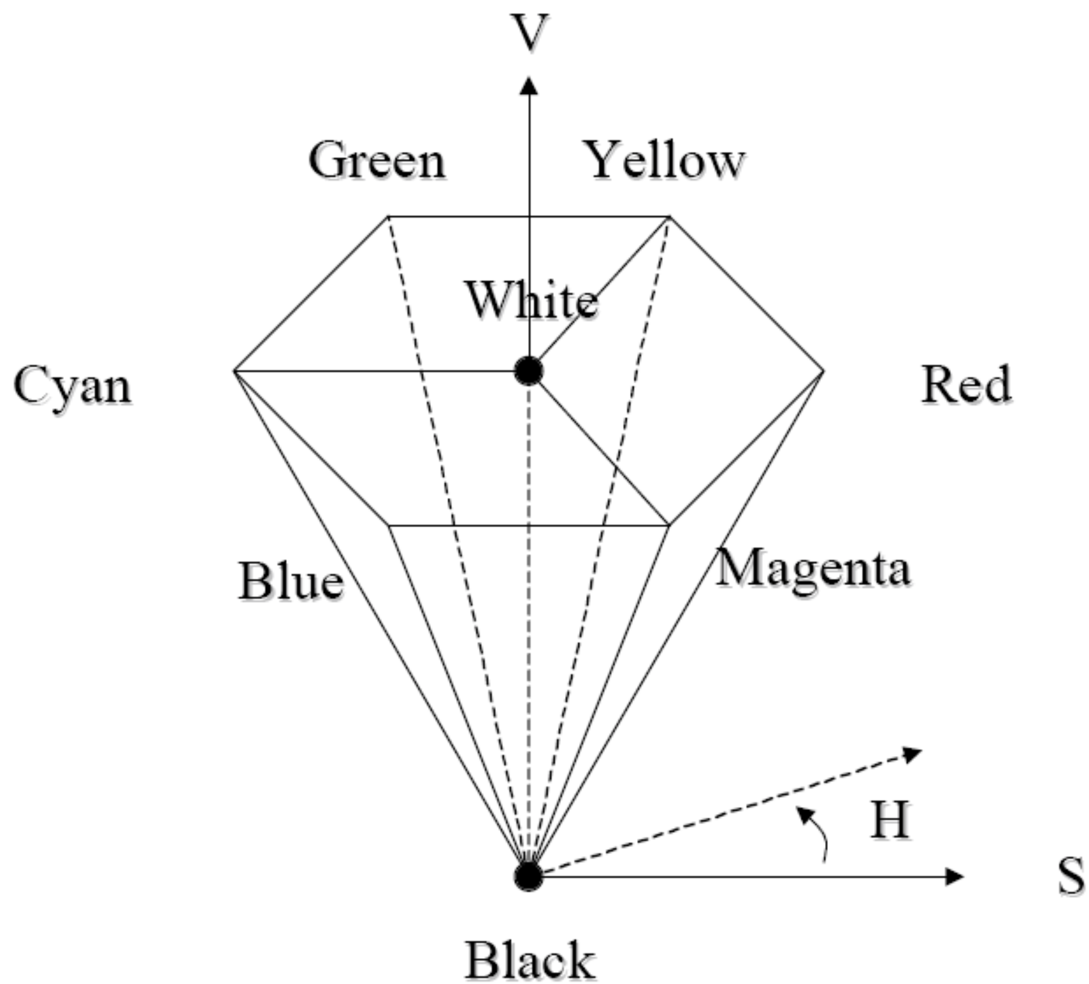
Example: **NTSC YIQ color space**

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.30 & 0.59 & 0.11 \\ 0.60 & -0.28 & -0.32 \\ 0.21 & -0.52 & 0.31 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

The HSV Color Model

- Hue: Our qualitative idea of color.
- Saturation: The amount of white light mixed in with a pure hue.
- Value: The overall lightness or brightness.

The HSV Color Model



Visualization of the HSV