Robust Power Control Under Location and Channel Uncertainty in Cognitive Radio Networks
Jun Wang, Jianshu Chen, You Lu, Mario Gerla, and Danijela Cabric

Abstract—In this letter, we consider the power control problem in cognitive radio (CR) networks when both primary user (PU) location and wireless channel are unknown. Prior work in power control assumes perfect knowledge of PU and CR locations, which is not practical due to localization errors and node mobility. We assume the distance estimation error in CR–PU links to model location uncertainties and derive the distribution of channel gain with distance-dependent path loss and shadowing. We then proceed to develop an optimization framework for CR power control, which maximizes the CR data rate under PU interference power constraint. Simulation results showing the CR data rate and interference probability to the PUs are presented to demonstrate the superior performance of the proposed algorithm compared with reference schemes.

Index Terms—Power control, cognitive radio networks, location estimation error, channel uncertainty.

I. INTRODUCTION

COGNITIVE radio (CR) is a promising approach to efficiently utilize the scarce RF spectrum resources. In this paradigm, knowledge about spectrum occupancy in time, frequency, and space that is both accurate and timely is crucial in allowing CR networks to opportunistically use the spectrum and avoid interference to a primary user (PU). In particular, information about PU location could enable several key capabilities in CR networks including improved spatio-temporal sensing, intelligent location-aware power control and routing, as well as aiding spectrum policy enforcement.

In this letter, we focus on CR power control algorithms with PU-location awareness. Prior work normally assumes perfect knowledge of both CR and PU locations, and designs algorithms that maximize the achieved CR performance under certain constraints for PU protection (e.g., interference power limit). In [1], power control algorithms that maximize the CR data rate or power efficiency were proposed considering path-loss only channel model. Power control solutions with more practical wireless channel models also exist. For example authors of [2] considered shadowing and small-scale fading, while algorithms that are robust to distribution uncertainties of the signal-to-interference-and-noise ratio (SINR) were developed in [3]. Relay selection and power allocation were jointly studied with known channel conditions in [4].

In a practical CR network, the assumption of perfect location information is not appropriate. In IEEE 802.22 standard which allows CR networks to operate in the TV white space, the CRs obtain PU location information from a central database [5]. However, the database is useful only to obtain location information of the TV base stations which have almost no mobility. For TV receivers which are smaller and possibly mobile devices, it is difficult and costly to keep the database up-to-date. Furthermore, wireless microphones are also the licensed PUs in the TV spectrum. They are more difficult to capture in the database due to more dynamic traffic behavior. As a result, the PU location is usually not perfectly known and needs to be estimated using cooperative localization with CR sensors. The localization process uses different types of measurements from the PUs, such as received-signal-strength (RSS) or direction-of-arrival (DoA). The estimated PU location is subject to errors due to various reasons, such as limited observation time, number of sensors and their coverage, channel variations, node mobility [6]. Therefore, assuming perfect location information makes the previous algorithms less practical.

We propose CR power control algorithm that takes both wireless channel and location estimation uncertainties into consideration, which, to the best of our knowledge, has not been studied in the literature. We start by incorporating the localization error in the distance estimation variance for each communication/interference link. This model also includes uncertainties in the CR self-localization process, and makes the proposed algorithm more comprehensive. We then proceed to model the combined channel variable of path-loss and shadowing as a log-normal variable. The power control algorithm that maximizes the CR data rate subject to CR power limits and PU interference constraints is derived afterwards. Simulation results show that the proposed algorithm obtains comparable CR data rates to the reference schemes, and reduces the interference probability to the PUs by 40% under conditions with medium or large location uncertainty.

II. SYSTEM MODEL

We assume a single CR pair (i.e. a transmitter and a receiver) operating in a total number of \(N\) distinct frequency bands, where the bandwidth of the \(n\)th band is \(w_n\). The assumption of a single CR pair is used in [1], [4], which is useful to obtain an upper-bound on the achievable CR data rate without interference from within the CR network. The two-dimensional location of the CR transmitter and receiver are denoted as \(t_C = [x_C, y_C]_T\) and \(r_C = [x_C, y_C]_T\), respectively. The CR transmit power in the \(n\)th band is denoted as \(\phi_n\), which is our optimization variable. There are \(M_n\) detected PU pairs in the \(n\)th band, and for the \(m\)th PU pair in the \(n\)th band we denote its transmit power, transmitter and receiver location as \(\varphi_{m,n}, t_m,n = [x_{m,n}, y_{m,n}]_T\) and \(r_m,n = [x_{m,n}, y_{m,n}]_T\), respectively. We
consider three types of links: link from the CR transmitter to the CR receiver, from the PU transmitters to the CR receiver, and from the CR transmitter to the PU receivers. For a given transmitter location \( t \), receiver location \( r \) and transmit power \( P_t \), the receive power is modeled as \( P_r = P_t d^{-\alpha}_t r^{-10^{-\gamma} t / 10} \), where \( d_t r = (x_t - x_r)^2 + (y_t - y_r)^2 \) is the distance of the link, \( \gamma \) is the path-loss exponent, and \( 10^{-\gamma} t / 10 \) is the log-normal shadowing with \( s_l r \sim N(0, \sigma_r^2) \). We assume same path-loss exponent and shadowing variance for all links. The CR network obtains imperfect knowledge of PU locations and thus distances. We model the distance estimates \( d_t r \) as a Gaussian random variable given as \( d_t r \sim N(d_t r, \sigma_r^2) \). The value of \( \sigma_d \) is related to the localization algorithm and channel condition. For unbiased location estimators, \( \sigma_d \) can be set to the root mean square error of the corresponding Cramer–Rao Bound [6]. We assume the same \( \sigma_r^2 \) for all links, however the proposed algorithm can be easily extended to more general cases. We assume the CR network knows \( d_t r \), its variance \( \sigma_d^2 \), and the shadowing variance \( \sigma_r^2 \).

### III. PROPOSED ALGORITHM

The proposed power control algorithm maximizes the total achievable CR data rate summed over all frequency bands, subject to CR transmit power constraints and PU interference power constraints. The algorithm can be generally written as

\[
\max \sum_{n=1}^{N} c_n \quad \text{s.t.} \sum_{n=1}^{N} \phi_n \leq \phi^\text{max}, \phi_n \geq 0, \forall n
\]

(1)

where \( \phi^\text{max} \) is the maximum CR transmit power, \( \psi_{m,n} = \phi_0 d_c^\text{rc}_m r^{-10^{-\gamma} t / 10} \), \( \psi_{m,n} \) and \( \phi_0 \) denote received interference power, maximum tolerable interference power and target (maximum) interference probability at the receiver of the \( m \)th PU pair of the \( n \)th band, respectively, and \( c_n \) is the data rate in the \( n \)th band given as

\[
c_n = w_n \log_2 \left( 1 + \frac{\phi_n \Gamma_c r_c}{\sum_{m=1}^{M} \psi_{m,n} \Gamma_{m,n,r,c} + N_0 w_n} \right)
\]

(3)

where \( \Gamma_c r_c = d_c^\gamma r^{-10^{-\gamma} t / 10} \) and \( \Gamma_{m,n,r,c} = d_{m,n}^\gamma r^{-10^{-\gamma} t / 10} \) are the corresponding channel gains, and \( N_0 \) the noise power spectrum density (PSD). Note that we maximize the total CR rate in (1), similar to [1], [2], rather than attempting to achieve a certain data rate target (or equivalently SINR target) [3], [7]. Due to the dynamics and opportunistic nature of CR network (e.g. random CR and PU placement, unknown PU traffic), it is difficult to find a fixed target CR rate for power control. The power control algorithm may need to be executed several times before finding the feasible target rate, which results in system delay. Therefore maximizing the achievable data rate is a better choice since it provides a best-effort solution to CR network, which is adaptive to the network dynamics. Probabilistic interference power control similar to (2) was also used in [2], [3], however as will be elaborated in later part of this section, we consider uncertainty in distance estimation which is not included in prior work. Given perfect distance and channel knowledge, an ideal power control algorithm (denoted as Ide) will have the rate and interference power constraint given as

\[
c_n, \text{Ide} = w_n \log_2 \left( 1 + \frac{\phi_n d_c^\gamma r^{-10^{-\gamma} t / 10}}{\sum_{m=1}^{M_n} \psi_{m,n} d_{m,n}^\gamma r^{-10^{-\gamma} t / 10} + N_0 w_n} \right)
\]

(4)

where \( \phi_t \) is the instantaneous shadowing variable of a given link. The complete formulation of the ideal algorithm is given by replacing \( c_n \) by \( c_n, \text{Ide} \) in (1), and replacing (2) by (4). However, the ideal algorithm is not implementable since in practice we do not know the true distance \( d_t r \) and the instantaneous shadowing variable \( \phi_t \). Therefore, a major challenge for power control algorithm design and formulation with both channel and distance uncertainties is to obtain the statistical model of the combined channel variable with both path loss and shadowing effect. We propose the following technique to solve this problem.

The combined channel variable is defined in a general form as

\[
\Gamma_{t,r} = d_t r^{-10^{-\gamma} t / 10}
\]

(5)

for a Gaussian variable \( d_t r \sim N(d_t r, \sigma_d^2) \) with unknown mean \( \bar{d}_t r \) and only one observation \( d_t r \), the maximum likelihood estimate of \( d_t r \) is simply \( d_t r \). Therefore we can reasonably approximate the distribution of \( d_t r \) as \( N(d_t r, \sigma_d^2) \) to obtain the statistical distribution of \( \Gamma_{t,r} \), we first approximate \( d_t r \) as a log-normal variable with the following results discovered in [8]. A Gaussian variable with mean and variance \( \{\mu, \sigma_k^2\} = \{M, 2M\} \) can be well-approximated as a log-normal variable with log-domain mean and variance \( \{\mu_k, \sigma_k^2\} = \{\log(M) - \sigma_k^2 / 2, \log(1 + 2/M)\} \), for sufficiently large \( M \) (e.g. \( M \geq 37 \)). By defining \( M = 2d_t r^2 / \sigma_d^2 \), we can verify that

\[
M d_t r, d_t r \sim N(M, 2M)
\]

and the log-normal approximation can be applied. After several basic manipulations, we obtain the distribution of the channel variable \( \Gamma_{t,r} \) as

\[
\text{ln}\Gamma_{t,r} \sim LN(\mu_k, \sigma_k^2)
\]

(6)

where the mean and variance of \( \Gamma_{t,r} \) in linear-domain is given as

\[
\tilde{\Gamma}_{t,r} = E[\Gamma_{t,r}] = e^{\mu_k + \sigma_k^2 / 2}
\]

(7)

Now we consider the interference constraint (2) by redefining the interference power as \( \psi_{m,n} = \phi_0 \Gamma_{t,r} r^{-10^{-\gamma} t / 10} \) where \( \Gamma_{t,r} = d_t r^{-10^{-\gamma} t / 10} \) is the corresponding channel variable with \( d_t r \sim N(d_t r, \sigma_d^2) \) and \( s_l r \sim N(0, \sigma_r^2) \). We approximate \( \Gamma_{t,r} \) as \( \Gamma_{t,r} \sim LN(\mu_k, \sigma_k^2) \) with the mean and variance obtained from (5) and (6). Then (2) is extended to

\[
\text{Pr} \left[ \log(\psi_{m,n}) \geq \log(\psi_{m,n}^\text{max}) \right] \leq \alpha_{m,n}
\]

(8)

where \( Q(x) \) is the Gaussian Q-function. Similarly for the data rate \( c_n \), the channel variables in (3) can also be approximated as

\[
\text{Pr} \left[ \log(\psi_{m,n}) \geq \log(\psi_{m,n}^\text{max}) \right] \leq \alpha_{m,n}
\]

(9)

To obtain the averaged rate, we first apply first-order Taylor
series expansion to $c_n$ defined in (3) around the channel variables' linear-domain mean value $\bar{\Gamma}_{t_c, nc}$ and $\bar{\Gamma}_{u, nc}$, and then take expectation of the obtained Taylor series. The resulting average rate is the data rate formulation of the proposed algorithm (denoted as $\text{Pro}$), given as

$$c_{n, \text{Pro}} = w_n \log_2 \left( 1 + \frac{\phi_n \bar{\Gamma}_{t_c, nc}}{\sum_{m=1}^{M_n} \frac{1}{\bar{\psi}_{m,n} \bar{\Gamma}_{t_c, nc}} + N_0 w_n} \right). \quad (8)$$

We skip detailed derivations of (8) due to space limitations. The proposed power control algorithm is summarized as

$$\max_{\{\phi_n\}_{n=1}^{N}} \left\{ \sum_{n=1}^{N} \beta_n c_{n, \text{Pro}} \right\} \quad \text{s.t.} \quad \sum_{n=1}^{N} \phi_n \leq \phi_{n, \text{max}}, \quad \phi_n \geq 0, \quad \forall n$$

$$\phi_n \leq \min_m \left\{ \psi_{m,n}^{-1} \left( e^{-\frac{1}{\bar{\psi}_{m,n} \bar{\Gamma}_{t_c, nc}}} \right) \right\}, \quad \forall n \quad (9)$$

where $\beta_n = \prod_{m=1}^{M_n} (1 - \sigma_{m,n})$ accounts for the CR rate reduction when transmitting in parallel with the PUs. The formulation (9) is a maximization of a concave function with linear constraints, which can be solved by standard convex optimization computer softwares. The proposed algorithm does not require frequent estimation of instantaneous channel gains; it only needs statistical information of location estimates and shadowing (i.e. $\sigma_d$ and $\sigma_s$) that can be obtained offline, and provides long-term optimal power allocation.

IV. REFERENCE ALGORITHMS

In this section we introduce two reference algorithms that will be compared to our work. Reference algorithm 1 (Ref1) only considers channel uncertainties, similar to [2], [3]. The fundamental difference between the proposed algorithm and Ref1 is that Ref1 does not consider location uncertainty and thus treats the distance estimates $d$ as true distances. The data rate and probabilistic interference constraint are given as

$$c_{n, \text{Ref1}} = w_n \log_2 \left( 1 + \frac{\phi_n \bar{\Gamma}_{t_c, nc} \kappa}{\sum_{m=1}^{M_n} \frac{1}{\bar{\psi}_{m,n} \bar{\Gamma}_{t_c, nc}} + N_0 w_n} \right). \quad (10)$$

and $\phi_n \leq \min_m \left\{ \psi_{m,n}^{-1} \left( e^{-\frac{1}{\bar{\psi}_{m,n} \bar{\Gamma}_{t_c, nc}}} \right) \right\}, \quad \forall n$, where $\bar{\psi}_{m,n} = \frac{1}{\bar{\Gamma}_{u, nc}} = \frac{\sigma_d^2}{\bar{\Gamma}_{u, nc}}$ is the mean of the shadowing variable. Reference algorithm 2 (Ref2) ignores uncertainties in both shadowing and distance estimation, therefore it is a path-loss only algorithm similar to [1], [7]. The data rate and interference power constraint are given as

$$c_{n, \text{Ref2}} = w_n \log_2 \left( 1 + \frac{\phi_n \bar{\Gamma}_{t_c, nc} \kappa}{\sum_{m=1}^{M_n} \frac{1}{\bar{\psi}_{m,n} \bar{\Gamma}_{t_c, nc}} + N_0 w_n} \right). \quad (11)$$

and $\phi_n \leq \min_m \left\{ \psi_{m,n}^{-1} \left( e^{-\frac{1}{\bar{\psi}_{m,n} \bar{\Gamma}_{t_c, nc}}} \right) \right\}, \quad \forall n$. The complete formulation of the two reference algorithms are given by replacing $c_n$ in (1) and interference constraint (2) with definitions of each reference algorithm, respectively.

V. NUMERICAL RESULTS

Fig. 1 shows the placement of nodes in our simulation (the coordinates are in meters), where a single CR pair operates in two frequency bands each with bandwidth 2 MHz. The maximum allowable total CR transmit power is 23 dBm. In each frequency band there is one pair of PU. The PU transmit power in the two bands are 20 dBm and 22 dBm, respectively. The distance and power settings are chosen from a typical IEEE 802.11 WLAN system. We use the maximum allowable signal-to-interference-and-noise ratio (SINR) degradation at the PU to characterize interference constraint for the CR. It is straightforward to verify that for a given SINR degradation $e_{\text{max}}^\text{m,n}$, the corresponding interference power constraint is $\psi_{m,n} = (10^{e_{\text{max}}^\text{m,n} / 10} - 1)N_0 w_n$. We assume all PU receivers have a maximum allowable SINR degradation of 0.5 dB. The standard deviation for distance estimation and shadowing are $\sigma_d = 15$ m and $\sigma_s = 2$ dB, respectively. The target maximum interference probability, path-loss exponent and noise PSD are set to $\alpha_{m,n} = 0.2$, $\gamma = 3$ and $N_0 = -100$ dBm, respectively. Each data point in the figures is obtained by averaging 5000 realizations of random variables. In the following subsections, we use these settings unless stated otherwise.

We first compare the proposed algorithm with the ideal algorithm discussed in Section III and the two algorithms discussed in Section IV, for varying distance estimation standard deviation. The algorithms are evaluated in terms of total CR data rate and interference probability to the PU (defined as number of cases that the CR interference power is greater than the maximum allowable value divided by total number of simulations). The results are shown in Fig. 2(a) and (d). We adopt the criteria used in [3] for data rate evaluation, which states that if the CR violates the interference constraint in a certain band, the PU in that band will notify the CR network to stop transmission and the rate of CR in that band is zero. We observe from Fig. 2(a) that the proposed algorithm obtains similar data rate compare to Ref1, and they both outperform Ref2. However, Fig. 2(d) shows that Ref1 obtains the data rate via more aggressive power allocation which increases the interference probability by 40% compared with the proposed algorithm. On the contrary, the proposed algorithm keeps the interference probability strictly under 0.2 for all simulated $\sigma_d$, which agrees with our setting of $\alpha_{m,n} = 0.2$. Note that all the practical algorithms obtain less than half the data rate of the ideal algorithm, due to channel and location uncertainties. With varying shadowing standard deviation $\sigma_s$, the proposed algorithm has similar advantage over Ref1 when $\sigma_s \leq 5$ dB; when $\sigma_s > 5$ dB, shadowing effect becomes dominant and the proposed algorithm obtains similar data rate and interference probability as Ref1. We do not show related results due to space limitation.
Next we study the impact of target maximum interference probability $\alpha_{m,n}$ with results shown in Fig. 2(b) and (e). We observe a tradeoff in setting $\alpha_{m,n}$: if $\alpha_{m,n}$ is small ($\leq 0.1$), the PUs are well-protected but the CR rate is reduced due to power-backup; if $\alpha_{m,n}$ is large ($\geq 0.3$), the CR rate is also reduced due to frequent interference violation caused by aggressive power allocation. In the optimal region of $\alpha_{m,n}$, the proposed algorithm provides similar data rate compared with Ref1, and 3–7% less interference probability. Therefore, for a given system configuration, we can find the optimal setting of $\alpha_{m,n}$ that provides satisfactory CR rate and PU protection.

We finally compare the algorithms with different maximum total CR transmit power $\phi_{\text{max}}$, with results shown in Fig. 2(c) and (f). We can divide $\phi_{\text{max}}$ into two regions according to the behaviors of the algorithms. In the power-limited region ($\leq 16$ dBm), the CR interference is much smaller than the constraint due to limited CR transmit power. In this region all algorithms obtain similar performance. In the interference-limited region ($\geq 18$ dBm), the CR transmitter has the capability to reach the PU interference limit, therefore we need to carefully design power control algorithm. In the interference-limited region, the proposed algorithm has clear advantage over the two reference algorithms, especially in PU protection. Note that beyond 20 dBm, increasing the maximum allowable total CR transmit power is not beneficial since the interference constraint dominates the power allocation. Therefore the performance of all algorithms saturates afterwards.

VI. CONCLUSION

In this letter we presented a robust CR power control algorithm that considers both location and channel uncertainties. We first modeled the combined path-loss with distance estimation error and shadowing as a log-normal variable, which made the power control algorithm design feasible. We then presented the proposed algorithm that maximizes the total CR data rate summed over all frequency bands, subject to the interference power constraints by the PUs. It was shown in our simulations that the proposed algorithm obtains comparable CR data rates with the reference schemes, and provides much lower interference level to the PUs.

REFERENCES

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consider three types of links: link from the CR transmitter to the CR receiver, from the PU transmitters to the CR receiver, and from the CR transmitter to the PU receivers. For a given transmitter location \( t \), receiver location \( r \) and transmit power \( P_t \), the receive power is modeled as \( P_r = P_t d_{t,r}^{-\gamma} 10^{-\alpha_t/10} \), where \( d_{t,r} = \sqrt{(x_t-x_r)^2 + (y_t-y_r)^2} \) is the distance of the link, \( \gamma \) is the path-loss exponent, and \( 10^{-\alpha_t/10} \) is the log-normal shading with \( s_{t,r} \sim N(0, \sigma_s^2). \) We assume same path-loss exponent and shading variance for all links. The CR network obtains imperfect knowledge of PU locations and thus distances. We model the distance estimates \( d_{t,r} \) as a Gaussian random variable given as \( d_{t,r} \sim N(d_{t,r}, \sigma_d^2). \) The value of \( \sigma_d \) is related to the localization algorithm and channel condition. For unbiased location estimators, \( \sigma_d \) can be set to the root mean square error of the corresponding Cramer–Rao Bound [6]. We denote the distance estimate at a specific estimation period as \( d_{t,r} \), which is a realization of the random variable \( d_{t,r} \). Note that we assume the same \( \sigma_d^2 \) for all links, however the proposed algorithm can be easily extended to more general cases. We assume the CR network knows \( d_{t,r} \), its variance \( \sigma_d^2 \), and the shadowing variance \( \sigma_s^2 \).

### III. PROPOSED ALGORITHM

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\]

where \( \phi_{\max} \) is the maximum CR transmit power, \( \psi_{m,n} = \phi_n d_{t,c_{m,n}}^{-\gamma} 10^{-\alpha_{c_{m,n}}/10} \), \( \psi_{m,n} \) and \( \alpha_{m,n} \) denote received interference power, maximum tolerable interference power and target (maximum) interference probability at the receiver of the \( m \)th PU pair of the \( n \)th band, respectively, and \( c_n \) is the data rate in the \( n \)th band given as

\[
c_n = w_n \log \left( 1 + \frac{\phi_n \Gamma_{c_{m,n}} d_{t,c_{m,n}}^{-\gamma} 10^{-\alpha_{c_{m,n}}/10}}{\sum_{m=1}^{N} \psi_{m,n} \Gamma_{c_{m,n}} + N_0 w_n} \right)
\]

where \( \Gamma_{c_{m,n}} = d_{t,c_{m,n}}^{-\gamma} 10^{-\alpha_{c_{m,n}}/10} \) and \( \Gamma_{t,c_{m,n}} = d_{t,c_{m,n}}^{-\gamma} 10^{-\alpha_{c_{m,n}}/10} \) are the corresponding channel gains, and \( N_0 \) is the noise power spectral density (PSD). Note that we maximize the total CR rate in (1), similar to [1], [2], rather than attempting to achieve a certain data rate target (or equivalently SINR target) [3], [7]. Due to the dynamics and opportunistic nature of CR network (e.g. random CR and PU placement, unknown PU traffic), it is difficult to find a fixed target CR rate for power control. The power control algorithm may need to be executed several times before finding the feasible target rate, which results in system delay. Therefore maximizing the achievable data rate is a better choice since it provides a best-effort solution to CR network, which is adaptive to the network dynamics. Probabilistic interference power control similar to (2) was also used in [2], [3], however as will be elaborated in later part of this section, we consider uncertainty in distance estimation which is not included in prior work. Given perfect distance and channel knowledge, an ideal power control algorithm (denoted as Ide) will have the data rate and interference power constraint given as

\[
c_{n,\text{Ide}} = w_n \log \left( 1 + \frac{\phi_n d_{t,c_{m,n}}^{-\gamma} 10^{-\alpha_{c_{m,n}}/10}}{\sum_{m=1}^{N} \psi_{m,n} d_{t,c_{m,n}}^{-\gamma} 10^{-\alpha_{c_{m,n}}/10} + N_0 w_n} \right)
\]

where \( \phi_{\max} \) is the instantaneous shadowing variable of a given link. The complete formulation of the ideal algorithm is given by replacing \( c_n \) by \( c_{n,\text{Ide}} \) in (1), and replacing (2) by (4). However, the ideal algorithm is not implementable since in practice we do not know the true distance \( d_{t,r} \) and the instantaneous shadowing variable \( s_{t,r} \). Therefore, a major challenge for power control algorithm design and formulation with both channel and distance uncertainties is to obtain the statistical model of the combined channel variable with both path loss and shadowing effect. We propose the following technique to solve this problem.

The combined channel variable is defined in a general form as \( \Gamma_{t,r} = d_{t,r}^{-\gamma} 10^{-\alpha_{t,r}/10} \). For a Gaussian variable \( d_{t,r} \sim N(d_{t,r}, \sigma_d^2) \) with unknown mean \( d_{t,r} \) and only one observation \( d_{t,r} \), the maximum likelihood estimate of \( d_{t,r} \) is simply \( d_{t,r} \). Therefore we can reasonably approximate the distribution of \( d_{t,r} \) as \( \sim N(d_{t,r}, \sigma_d^2) \). To obtain the statistical distribution of \( \Gamma_{t,r} \), we first approximate \( d_{t,r} \) as a log-normal variable with the following results discovered in [8]. A Gaussian variable with mean and variance \( \{\mu_N, \sigma_N^2\} = \{M, 2M\} \) can be well-approximated as a log-normal variable with log-domain mean and variance \( \{\mu_c, \sigma_c^2\} = \{\log(M) - \sigma_N^2/2, \log(1+2/M)\} \), for sufficiently large \( M \) (e.g. \( M \geq 37 \)). By defining \( M = 2d_{t,r}^2/\sigma_d^2 \), we can verify that \( M d_{t,r}/\sigma_d^2 \sim N(M, 2M) \) and the log-normal approximation can be applied. After several basic manipulations, we obtain the distribution of the channel variable \( \Gamma_{t,r} \) as \( \Gamma_{t,r} \sim \mathcal{L}N(\mu_c, \sigma_c^2) \), where

\[
\mu_c = \frac{1}{2} \log \left( \frac{d_{t,r}^2 + \sigma_d^2}{d_{t,r}^2} \right),
\]

\[
\sigma_c^2 = \gamma^2 \log \left( \frac{d_{t,r}^2 + \sigma_d^2}{d_{t,r}^2} \right) + \left( \frac{10}{10} \right)^2 \sigma_d^2,
\]

and the mean of \( \Gamma_{t,r} \) in linear-domain is given as \( \bar{\Gamma}_{t,r} = \mathbb{E}[\Gamma_{t,r}] = e^{\mu_c + \sigma_c^2/2} \). Now we consider the interference constraint (2) by redefining the interference power as \( \psi_{m,n} = \phi_n \Gamma_{c_{m,n}} \) where \( \Gamma_{c_{m,n}} = d_{t,c_{m,n}}^{-\gamma} 10^{-\alpha_{c_{m,n}}/10} \) is the corresponding channel variable with \( d_{t,c_{m,n}} \sim N(d_{t,c_{m,n}}, \sigma_d^2) \) and \( s_{c_{m,n}} \sim N(0, \sigma_s^2) \). We approximate \( \Gamma_{c_{m,n}} \) as \( \Gamma_{c_{m,n}} \sim \mathcal{L}N(\mu_{c_{m,n}}, \sigma_{c_{m,n}}^2) \) with the mean and variance obtained from (5) and (6). Then (2) is extended to

\[
\Pr \left[ \log \psi_{m,n} \geq \log \psi_{m,n}^{\max} \right] \leq \alpha_{m,n}
\]

\[\Leftrightarrow \phi_n \leq \psi_{m,n}^{\max} e^{-\left[\mu_{c_{m,n}} + \sigma_{c_{m,n}}^2/2\right]} Q^{-1}(\alpha_{m,n}) \],

where \( Q(x) \) is the Gaussian Q-function. Similarly for the data rate \( c_n \), the channel variables in (3) can also be approximated as \( \Gamma_{c_{m,n}} \sim \mathcal{L}N(\mu_{c_{m,n}}, \sigma_{c_{m,n}}^2) \) and \( \Gamma_{t,c_{m,n}} \sim \mathcal{L}N(\mu_{t,c_{m,n}}, \sigma_{t,c_{m,n}}^2) \). To obtain the averaged rate, we first apply first-order Taylor
series expansion to $c_n$ defined in (3) around the channel variables’ linear-domain mean value $\Gamma_{t_c, \tau_c}$ and $\Gamma_{t_{0,n}, \tau_c}$, and then take expectation of the obtained Taylor series. The resulting average rate is the data rate formulation of the proposed algorithm (denoted as Pro), given as

$$c_{n, \text{Pro}} = w_n \log_2 \left( 1 + \frac{\phi_n \Gamma_{t_c, \tau_c}}{\sum_{m=1}^{M_c} \phi_{m,n} \tau_m + N_0 w_n} \right). \tag{8}$$

We skip detailed derivations of (8) due to space limitations. The proposed power control algorithm is summarized as

$$\begin{align*}
\max \{ & \phi_n \}_{n=1}^{N} \beta_n c_{n, \text{Pro}} \\
\text{s.t.} \sum_{n=1}^{N} & \phi_n \leq \phi^\text{max}_n, \phi_n \geq 0, \forall n \\
\phi_n \leq & \min_m \left\{ \psi_{m,n}^{\max,1} e^{-[\mu_t_{c,m,n} + \sigma_t_{c,m,n}]Q^{-1}(\alpha_{m,n})} \right\}, \forall n
\end{align*} \tag{9}$$

where $\beta_n = \prod_{m=1}^{M_c} (1 - \alpha_{m,n})$ accounts for the CR rate reduction when transmitting in parallel with the PUs. The formulation (9) is a maximization of a concave function with linear constraints, which can be solved by standard convex optimization computer softwares. The proposed algorithm does not require frequent estimation of instantaneous channel gains; it only needs statistical information of location estimates and shadowing (i.e. $\sigma_d$ and $\sigma_s$) that can be obtained offline, and provides long-term optimal power allocation.

IV. Reference Algorithms

In this section we introduce two reference algorithms that will be compared to our work. Reference algorithm 1 (Ref1) only considers channel uncertainties, similar to [2], [3]. The fundamental difference between the proposed algorithm and Ref1 is that Ref1 does not consider location uncertainty and thus treats the distance estimates $\hat{d}$ as true distances. The data rate and probabilistic interference constraint are given as

$$c_{n, \text{Ref1}} = w_n \log_2 \left( 1 + \frac{\phi_n \hat{d}_{c,m,n}^{-\gamma} \sum_{m=1}^{M_c} \psi_{m,n} \tau_m + N_0 w_n} {\sum_{m=1}^{M_c} \phi_{m,n} \tau_m} \right), \tag{10}$$

and $\phi_n \leq \min_m \left\{ \psi_{m,n}^{\max,2} e^{-[\mu_t_{c,m,n} + \sigma_t_{c,m,n}]Q^{-1}(\alpha_{m,n})} \right\}, \forall n$, where $\sum_{m=1}^{M_c} \psi_{m,n} \tau_m$ and $\kappa = e^{\log(10)^2} - \sigma_s^2$ is the mean of the shadowing variable. Reference algorithm 2 (Ref2) ignores uncertainties in both shadowing and distance estimation, therefore it is a path-loss only algorithm similar to [1], [7]. The data rate and interference power constraint are given as

$$c_{n, \text{Ref2}} = w_n \log_2 \left( 1 + \frac{\phi_n \hat{d}_{c,m,n}^{-\gamma} \sum_{m=1}^{M_c} \psi_{m,n} \tau_m + N_0 w_n} {\sum_{m=1}^{M_c} \phi_{m,n} \tau_m} \right), \tag{11}$$

and $\phi_n \leq \min_m \left\{ \psi_{m,n}^{\max,2} \hat{d}_{c,m,n}^{-\gamma} \right\}, \forall n$. The complete formulation of the two reference algorithms are given by replacing $c_n$ in (1) and interference constraint (2) with definitions of each reference algorithm, respectively.

V. Numerical Results

Fig. 1 shows the placement of nodes in our simulation (the coordinates are in meters), where a single CR pair operates in two frequency bands each with bandwidth 2 MHz. The maximum allowable total CR transmit power is 23 dBm. In each frequency band there is one pair of PU. The PU transmit power in the two bands are 20 dBm and 22 dBm, respectively. The distance and power settings are chosen from a typical IEEE 802.11 WLAN system. We use the maximum allowable signal-to-interference-and-noise ratio (SINR) degradation at the PU to characterize interference constraint for the CR. It is straightforward to verify that for a given SINR degradation $\gamma_{\text{max}}$, the corresponding interference power constraint is $\psi_{m,n}^{\max,2} = (10^{\text{dB} / 10} - 1)N_0 w_n$. We assume all PU receivers have a maximum allowable SINR degradation of 0.5 dB. The standard deviation for distance estimation and shadowing are $\sigma_d = 15$ m and $\sigma_s = 2$ dBm, respectively. The target maximum interference probability, path-loss exponent and noise PSD are set to $\alpha_{m,n} = 0.2$, $\gamma = 3$ and $N_0 = -100$ dBm, respectively. Each data point in the figures is obtained by averaging 5000 realizations of random variables. In the following subsections, we use these settings unless stated otherwise.

We first compare the proposed algorithm with the ideal algorithm discussed in Section III and the two algorithms discussed in Section IV, for varying distance estimation standard deviation. The algorithms are evaluated in terms of total CR data rate and interference probability to the PU (defined as number of cases that the CR interference power is greater than the maximum allowable value divided by total number of simulations). The results are shown in Fig. 2(a) and (d). We adopt the criteria used in [3] for data rate evaluation, which states that if the CR violates the interference constraint in a certain band, the PU in that band will notify the CR network to stop transmission and the rate of CR in that band is zero. We observe from Fig. 2(a) that the proposed algorithm obtains similar data rate compare to Ref1, and they both outperform Ref2. However, Fig. 2(d) shows that Ref1 obtains the data rate via more aggressive power allocation which increases the interference probability by 40% compared with the proposed algorithm. On the contrary, the proposed algorithm keeps the interference probability strictly under 0.2 for all simulated $\sigma_d$, which agrees with our setting of $\alpha_{m,n} = 0.2$. Note that all the practical algorithms obtain less than half the data rate of the ideal algorithm, due to channel and location uncertainties. With varying shadowing standard deviation $\sigma_s$, the proposed algorithm has similar advantage over Ref1 when $\sigma_s \leq 5$ dB; when $\sigma_s > 5$ dB, shadowing effect becomes dominant and the proposed algorithm obtains similar data rate and interference probability as Ref1. We do not show related results due to space limitation.
Next we study the impact of target maximum interference probability $\alpha_m$ with results shown in Fig. 2(b) and (e). We observe a tradeoff in setting $\alpha_m$: if $\alpha_m$ is small ($\leq 0.1$), the PUs are well-protected but the CR rate is reduced due to power-backup; if $\alpha_m$ is large ($\geq 0.3$), the CR rate is also reduced due to frequent interference violation caused by aggressive power allocation. In the optimal region of $\alpha_m$, the proposed algorithm provides similar data rate compared with Ref1, and 3–7% less interference probability. Therefore, for a given system configuration, we can find the optimal setting of $\alpha_m$ that provides satisfactory CR rate and PU protection.

We finally compare the algorithms with different maximum total CR transmit power $\phi_{\max}$ with results shown in Fig. 2(c) and (f). We can divide $\phi_{\max}$ into two regions according to the behaviors of the algorithms. In the power-limited region ($\leq 16$ dBm), the CR interference is much smaller than the constraint due to limited CR transmit power. In this region all algorithms obtain similar performance. In the interference-limited region ($\geq 18$ dBm), the CR transmitter has the capability to reach the PU interference limit, therefore we need to carefully design power control algorithm. In the interference-limited region, the proposed algorithm has clear advantage over the two reference algorithms, especially in PU protection. Note that beyond 20 dBm, increasing the maximum allowable total CR transmit power is not beneficial since the interference constraint dominates the power allocation. Therefore the performance of all algorithms saturates afterwards.

VI. CONCLUSION

In this letter we presented a robust CR power control algorithm that considers both location and channel uncertainties. We first modeled the combined path-loss with distance estimation error and shadowing as a log-normal variable, which made the power control algorithm design feasible. We then presented the proposed algorithm that maximizes the total CR data rate summed over all frequency bands, subject to the interference power constraints by the PUs. It was shown in our simulations that the proposed algorithm obtains comparable CR data rates with the reference schemes, and provides much lower interference level to the PUs.

REFERENCES