# CS6220: DATA MINING TECHNIQUES

Matrix Data: Clustering: Part 1

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October 19, 2015

#### **Announcements**

- Homework 1 grades out
- Re-grading policy:
  - If you have doubts in your grading, please submit a regrading form (via emails to both TAs and CC to the Instructor) indicating clearly the reason why you think it should be regraded
  - The deadline of the regrading form should be submitted within one week after you receive your score
  - We will regrade the whole homework/exam
- Homework 3 out tomorrow

### **Methods to Learn**

IVICTIONS to Ecuiti							
	Matrix Data	Text Data	Set Data	Sequence Data	Time Series	Graph & Network	Images
Classification	Decision Tree; Naïve Bayes; Logistic Regression SVM; kNN			HMM		Label Propagation*	Neural Network
Clustering	K-means; hierarchical clustering; DBSCAN; Mixture Models; kernel k- means*	PLSA				SCAN*; Spectral Clustering*	
Frequent Pattern Mining			Apriori; FP-growth	GSP; PrefixSpan			
Prediction	Linear Regression				Autoregression		
Similarity Search					DTW	P-PageRank	

Ranking

PageRank

# **Matrix Data: Clustering: Part 1**

- Cluster Analysis: Basic Concepts
- Partitioning Methods
- Hierarchical Methods
- Density-Based Methods
- Evaluation of Clustering
- Summary

# What is Cluster Analysis?

- Cluster: A collection of data objects
  - similar (or related) to one another within the same group
  - dissimilar (or unrelated) to the objects in other groups
- Cluster analysis (or clustering, data segmentation, ...)
  - Finding similarities between data according to the characteristics found in the data and grouping similar data objects into clusters
- Unsupervised learning: no predefined classes (i.e., learning by observations vs. learning by examples: supervised)
- Typical applications
  - As a stand-alone tool to get insight into data distribution
  - As a preprocessing step for other algorithms

# **Applications of Cluster Analysis**

- Data reduction
  - Summarization: Preprocessing for regression, PCA, classification, and association analysis
  - Compression: Image processing: vector quantization
- Prediction based on groups
  - Cluster & find characteristics/patterns for each group
- Finding K-nearest Neighbors
  - Localizing search to one or a small number of clusters
- Outlier detection: Outliers are often viewed as those "far away" from any cluster

## **Clustering: Application Examples**

- Biology: taxonomy of living things: kingdom, phylum, class, order, family, genus and species
- Information retrieval: document clustering
- Land use: Identification of areas of similar land use in an earth observation database
- Marketing: Help marketers discover distinct groups in their customer bases, and then use this knowledge to develop targeted marketing programs
- City-planning: Identifying groups of houses according to their house type, value, and geographical location
- Earth-quake studies: Observed earth quake epicenters should be clustered along continent faults
- Climate: understanding earth climate, find patterns of atmospheric and ocean

#### **Basic Steps to Develop a Clustering Task**

- Feature selection
  - Select info concerning the task of interest
  - Minimal information redundancy
- Proximity measure
  - Similarity of two feature vectors
- Clustering criterion
  - Expressed via a cost function or some rules
- Clustering algorithms
  - Choice of algorithms
- Validation of the results
  - Validation test (also, clustering tendency test)
- Interpretation of the results
  - Integration with applications

# **Requirements and Challenges**

- Scalability
  - Clustering all the data instead of only on samples
- Ability to deal with different types of attributes
  - Numerical, binary, categorical, ordinal, linked, and mixture of these
- Constraint-based clustering
  - User may give inputs on constraints
  - Use domain knowledge to determine input parameters
- Interpretability and usability
- Others
  - Discovery of clusters with arbitrary shape
  - Ability to deal with noisy data
  - Incremental clustering and insensitivity to input order
  - High dimensionality

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## **Partitioning Algorithms: Basic Concept**

Partitioning method: Partitioning a dataset **D** of **n** objects into a set of **k** clusters, such that the sum of squared distances is minimized (where c<sub>i</sub> is the centroid or medoid of cluster C<sub>i</sub>)

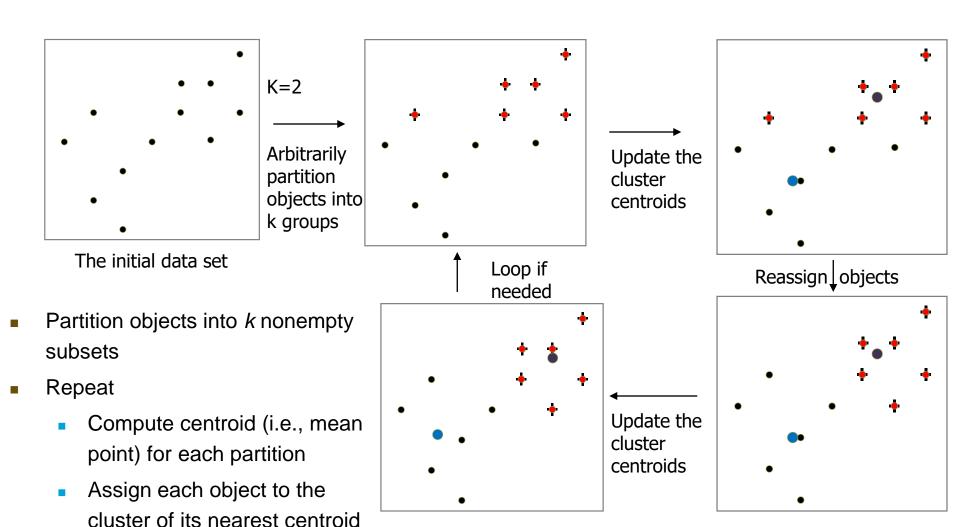
$$E = \sum_{i=1}^{k} \sum_{p \in C_i} (d(p, c_i))^2$$

- Given k, find a partition of k clusters that optimizes the chosen partitioning criterion
  - Global optimal: exhaustively enumerate all partitions
  - Heuristic methods: *k-means* and *k-medoids* algorithms
  - <u>k-means</u> (MacQueen'67, Lloyd'57/'82): Each cluster is represented by the center of the cluster
  - <u>k-medoids</u> or PAM (Partition around medoids) (Kaufman & Rousseeuw'87): Each cluster is represented by one of the objects in the cluster

#### The K-Means Clustering Method

- Given k, the k-means algorithm is implemented in four steps:
  - Step 0: Partition objects into k nonempty subsets
  - Step 1: Compute seed points as the centroids of the clusters of the current partitioning (the centroid is the center, i.e., *mean point*, of the cluster)
  - Step 2: Assign each object to the cluster with the nearest seed point
  - Step 3: Go back to Step 1, stop when the assignment does not change

# An Example of K-Means Clustering



Until no change

# **Theory Behind K-Means**

Objective function

$$\bullet J = \sum_{j=1}^{k} \sum_{C(i)=j} ||x_i - c_j||^2$$

- Total within-cluster variance
- Re-arrange the objective function

$$\bullet J = \sum_{j=1}^{k} \sum_{i} w_{ij} ||x_i - c_j||^2$$

- $w_{ij} \in \{0,1\}$
- $w_{ij} = 1$ , if  $x_i$  belongs to cluster j;  $w_{ij} = 0$ , otherwise
- Looking for:
  - The best assignment  $w_{ij}$
  - The best center  $c_j$

# **Solution of K-Means**

Iterations

- $J = \sum_{j=1}^{k} \sum_{i} w_{ij} ||x_i c_j||^2$
- Step 1: Fix centers  $c_j$ , find assignment  $w_{ij}$  that minimizes J
  - =>  $w_{ij} = 1$ ,  $if ||x_i c_j||^2$  is the smallest
- Step 2: Fix assignment  $w_{ij}$ , find centers that minimize J
  - => first derivative of J = 0

• => 
$$\frac{\partial J}{\partial c_j}$$
 =  $-2\sum_i w_{ij}(x_i - c_j) = 0$ 

- => $c_j = \frac{\sum_i w_{ij} x_i}{\sum_i w_{ij}}$ 
  - Note  $\sum_i w_{ij}$  is the total number of objects in cluster j

#### Comments on the K-Means Method

- Strength: Efficient: O(tkn), where n is # objects, k is # clusters, and t is # iterations. Normally, k, t << n.</li>
- Comment: Often terminates at a local optimal
- Weakness
  - Applicable only to objects in a continuous n-dimensional space
    - Using the k-modes method for categorical data
    - In comparison, k-medoids can be applied to a wide range of data
  - Need to specify *k*, the *number* of clusters, in advance (there are ways to automatically determine the best k (see Hastie et al., 2009)
  - Sensitive to noisy data and *outliers*
  - Not suitable to discover clusters with *non-convex shapes*

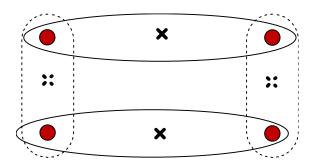
#### Variations of the K-Means Method

- Most of the variants of the k-means which differ in
  - Selection of the initial k means
  - Dissimilarity calculations
  - Strategies to calculate cluster means



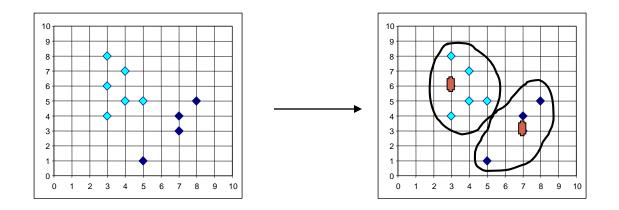


- Using new dissimilarity measures to deal with categorical objects
- Using a <u>frequency</u>-based method to update modes of clusters
- A mixture of categorical and numerical data: k-prototype method

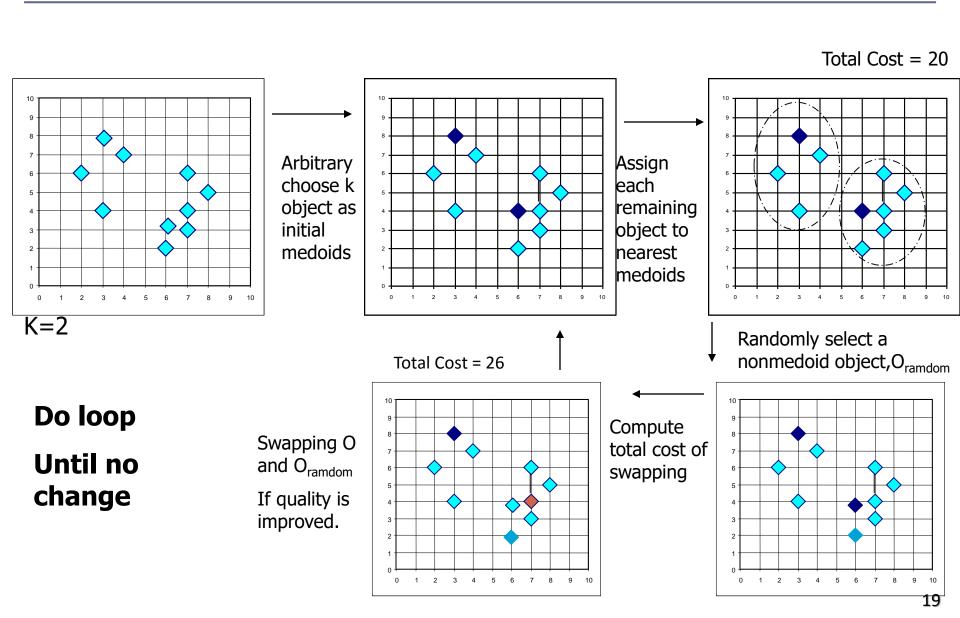


#### What Is the Problem of the K-Means Method?

- The k-means algorithm is sensitive to outliers!
  - Since an object with an extremely large value may substantially distort the distribution of the data
- K-Medoids: Instead of taking the mean value of the object in a cluster as a reference point, medoids can be used, which is the most centrally located object in a cluster



#### PAM: A Typical K-Medoids Algorithm



#### **The K-Medoid Clustering Method**

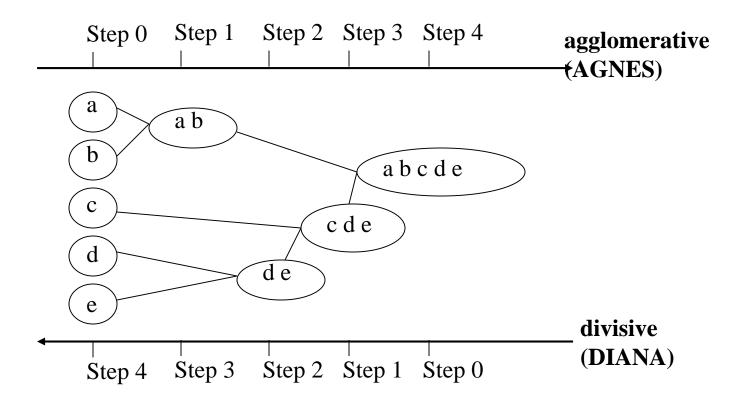
- K-Medoids Clustering: Find representative objects (medoids) in clusters
  - *PAM* (Partitioning Around Medoids, Kaufmann & Rousseeuw 1987)
    - Starts from an initial set of medoids and iteratively replaces one of the medoids by one of the non-medoids if it improves the total distance of the resulting clustering
    - PAM works effectively for small data sets, but does not scale well for large data sets (due to the computational complexity)
- Efficiency improvement on PAM
  - CLARA (Kaufmann & Rousseeuw, 1990): PAM on samples
  - CLARANS (Ng & Han, 1994): Randomized re-sampling

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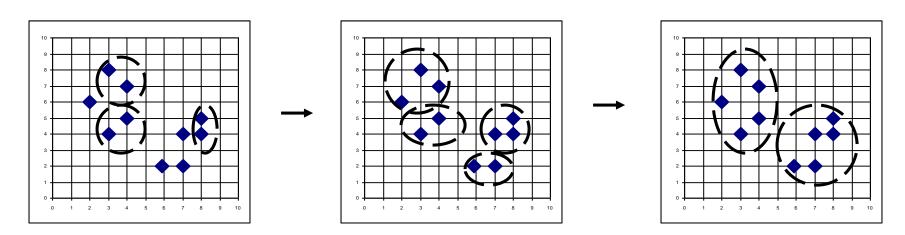
# **Hierarchical Clustering**

• Use distance matrix as clustering criteria. This method does not require the number of clusters k as an input, but needs a termination condition

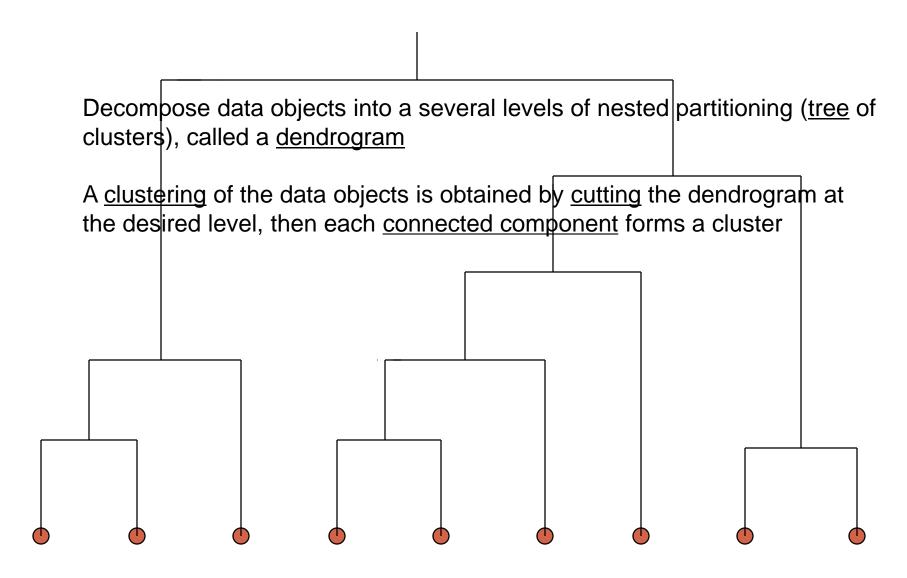


#### **AGNES (Agglomerative Nesting)**

- Introduced in Kaufmann and Rousseeuw (1990)
- Implemented in statistical packages, e.g., Splus
- Use the single-link method and the dissimilarity matrix
- Merge nodes that have the least dissimilarity
- Go on in a non-descending fashion
- Eventually all nodes belong to the same cluster

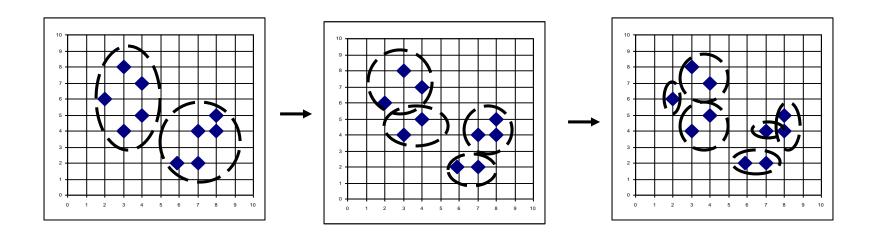


#### **Dendrogram: Shows How Clusters are Merged**

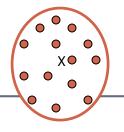


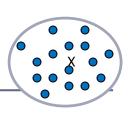
#### **DIANA (Divisive Analysis)**

- Introduced in Kaufmann and Rousseeuw (1990)
- Implemented in statistical analysis packages, e.g., Splus
- Inverse order of AGNES
- Eventually each node forms a cluster on its own



#### Distance between Clusters





- Single link: smallest distance between an element in one cluster and an element in the other, i.e.,  $dist(K_i, K_i) = min(t_{ip}, t_{iq})$
- Complete link: largest distance between an element in one cluster and an element in the other, i.e.,  $dist(K_i, K_j) = max(t_{ip}, t_{jq})$
- Average: avg distance between an element in one cluster and an element in the other, i.e.,  $dist(K_i, K_i) = avg(t_{ip}, t_{iq})$
- Centroid: distance between the centroids of two clusters, i.e., dist(K<sub>i</sub>, K<sub>j</sub>) = dist(C<sub>i</sub>, C<sub>j</sub>)
- Medoid: distance between the medoids of two clusters, i.e., dist(K<sub>i</sub>, K<sub>j</sub>) = dist(M<sub>i</sub>, M<sub>j</sub>)
  - Medoid: a chosen, centrally located object in the cluster

# Centroid, Radius and Diameter of a Cluster (for numerical data sets)

Centroid: the "middle" of a cluster

$$C_{m} = \frac{\sum_{i=1}^{N} (t_{ip})}{N}$$

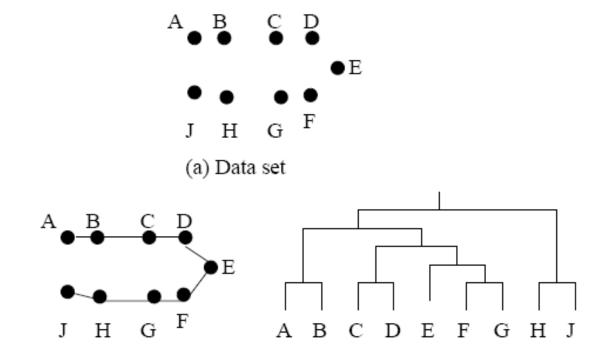
• Radius: square root of average distance from any point of the cluster to its centroid  $\sum_{n=0}^{\infty} N_n (t_n) = \frac{2}{n}$ 

$$R_m = \sqrt{\frac{\sum_{i=1}^{N} (t_{ip} - c_m)^2}{N}}$$

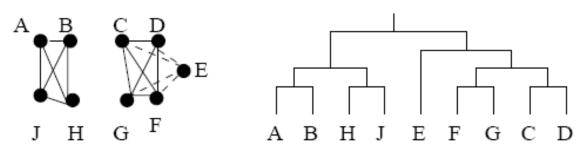
 Diameter: square root of average mean squared distance between all pairs of points in the cluster

$$D_{m} = \sqrt{\frac{\sum_{i=1}^{N} \sum_{i=1}^{N} (t_{ip} - t_{iq})^{2}}{N(N-1)}}$$

### **Example: Single Link vs. Complete Link**



(b) Clustering using single linkage



(c) Clustering using complete linkage

### **Extensions to Hierarchical Clustering**

- Major weakness of agglomerative clustering methods
  - Can never undo what was done previously
  - <u>Do not scale</u> well: time complexity of at least  $O(n^2)$ , where n is the number of total objects
- Integration of hierarchical & distance-based clustering
  - \*BIRCH (1996): uses CF-tree and incrementally adjusts the quality of sub-clusters
  - \*CHAMELEON (1999): hierarchical clustering using dynamic modeling

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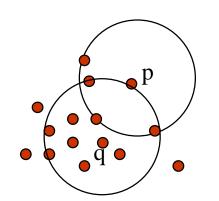
### **Density-Based Clustering Methods**

- Clustering based on density (local cluster criterion), such as density-connected points
- Major features:
  - Discover clusters of arbitrary shape
  - Handle noise
  - One scan
  - Need density parameters as termination condition
- Several interesting studies:
  - <u>DBSCAN:</u> Ester, et al. (KDD'96)
  - OPTICS: Ankerst, et al (SIGMOD'99).
  - <u>DENCLUE</u>: Hinneburg & D. Keim (KDD'98)
  - <u>CLIQUE</u>: Agrawal, et al. (SIGMOD'98) (more grid-based)

#### **DBSCAN: Basic Concepts**

- Two parameters:
  - *Eps*: Maximum radius of the neighborhood
  - *MinPts*: Minimum number of points in an Epsneighborhood of that point
- $N_{Eps}(q)$ : {p belongs to D | dist(p,q)  $\leq$  Eps}
- Directly density-reachable: A point *p* is directly density-reachable from a point *q* w.r.t. *Eps, MinPts* if
  - p belongs to  $N_{Eps}(q)$
  - core point condition:

$$|N_{Eps}(q)| \ge MinPts$$



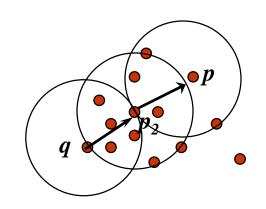
MinPts = 5

Eps = 1 cm

#### **Density-Reachable and Density-Connected**

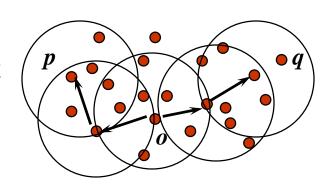
#### Density-reachable:

• A point p is density-reachable from a point q w.r.t. Eps, MinPts if there is a chain of points  $p_1, \ldots, p_n, p_1 = q, p_n = p$  such that  $p_{i+1}$  is directly density-reachable from  $p_i$ 



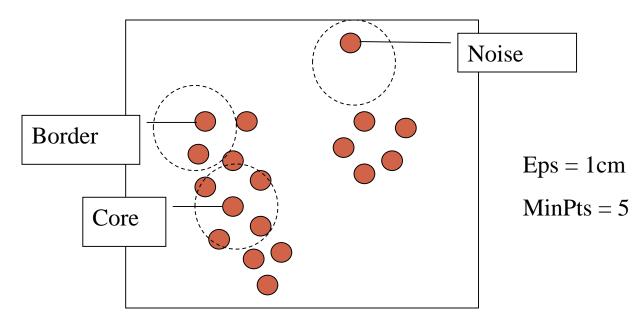
#### Density-connected

• A point *p* is density-connected to a point *q* w.r.t. *Eps, MinPts* if there is a point *o* such that both, *p* and *q* are density-reachable from *o* w.r.t. *Eps* and *MinPts* 



# DBSCAN: Density-Based Spatial Clustering of Applications with Noise

- Relies on a density-based notion of cluster: A cluster is defined as a maximal set of density-connected points
- Noise: object not contained in any cluster is noise
- Discovers clusters of arbitrary shape in spatial databases with noise



#### **DBSCAN: The Algorithm**

```
(1)
     mark all objects as unvisited;
(2)
     do
(3)
          randomly select an unvisited object p;
          \max p as visited;
(5)
           if the \epsilon-neighborhood of p has at least MinPts objects
(6)
                create a new cluster C, and add p to C;
(7)
                let N be the set of objects in the \epsilon-neighborhood of p;
(8)
                for each point p' in N
(9)
                      if p' is unvisited
(10)
                            mark p' as visited;
(11)
                            if the \epsilon-neighborhood of p' has at least MinPts points,
                            add those points to N;
(12)
                      if p' is not yet a member of any cluster, add p' to C;
(13)
                end for
(14)
                output C;
(15)
          else mark p as noise;
     until no object is unvisited;
```

• If a spatial index is used, the computational complexity of DBSCAN is O(nlogn), where n is the number of database objects. Otherwise, the complexity is  $O(n^2)$ 

#### **DBSCAN: Sensitive to Parameters**

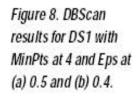
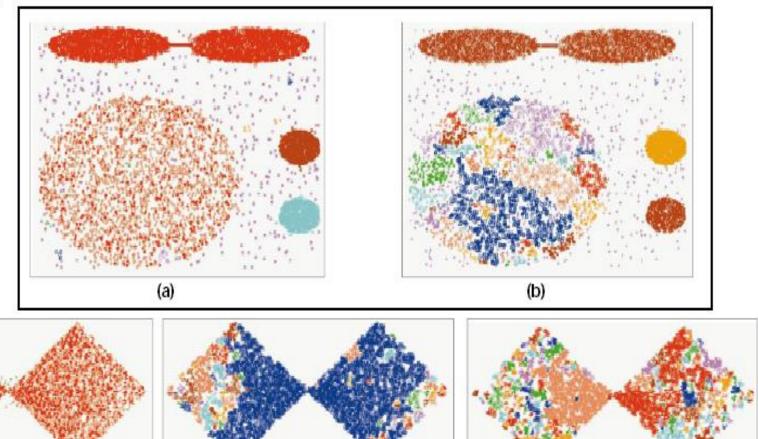
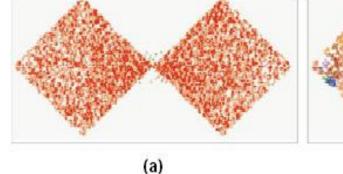
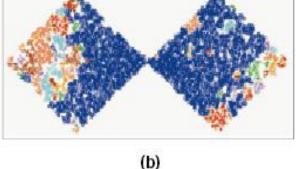
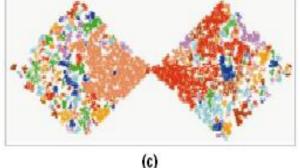


Figure 9. DBScan results for DS2 with MinPts at 4 and Eps at (a) 5.0, (b) 3.5, and (c) 3.0.









**DBSCAN online Demo:** 

## **Questions about Parameters**

- Fix Eps, increase MinPts, what will happen?
- Fix MinPts, decrease Eps, what will happen?

#### \*OPTICS: A Cluster-Ordering Method (1999)

- OPTICS: Ordering Points To Identify the Clustering Structure
  - Ankerst, Breunig, Kriegel, and Sander (SIGMOD'99)
  - Produces a special order of the database wrt its density-based clustering structure
  - This cluster-ordering contains info equiv to the density-based clusterings corresponding to a broad range of parameter settings
  - Good for both automatic and interactive cluster analysis, including finding intrinsic clustering structure
  - Can be represented graphically or using visualization techniques
  - Index-based time complexity: O(N\*logN)

#### **OPTICS: Some Extension from DBSCAN**

- Core Distance of an object p: the smallest value ε' such that the εneighborhood of p has at least MinPts objects
  - Let  $N_{\epsilon}(p)$ :  $\epsilon$ -neighborhood of p,  $\epsilon$  is a distance value;  $\operatorname{card}(N_{\epsilon}(p))$ : the size of set  $N_{\epsilon}(p)$
  - Let MinPts-distance(p): the distance from p to its MinPts' neighbor

$$\begin{aligned} &\text{Core-distance}_{\epsilon, \text{ MinPts}}(\textbf{p}) = & \begin{cases} \text{Undefined, if } \text{card}(\textbf{N}_{\epsilon}(\textbf{p})) \leq \text{MinPts} \\ \text{MinPts-distance}(\textbf{p}), \text{ otherwise} \end{cases} \end{aligned}$$

- Reachability Distance of object p from core object q is the min radius value that makes p density-reachable from q
  - Let distance(q,p) be the Euclidean distance between q and p

```
Reachability-distance<sub>\epsilon, MinPts</sub>(p, q) =
 \begin{cases}
    \text{Undefined, if q is not a core object} \\
    \text{max(core-distance(q), distance(q, p)), otherwise}
\end{cases}
```

### **Core Distance & Reachability Distance**

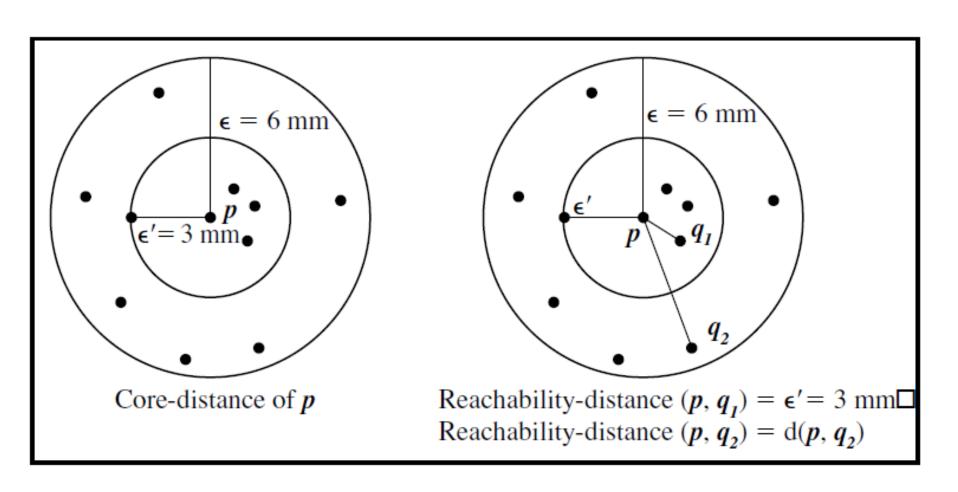
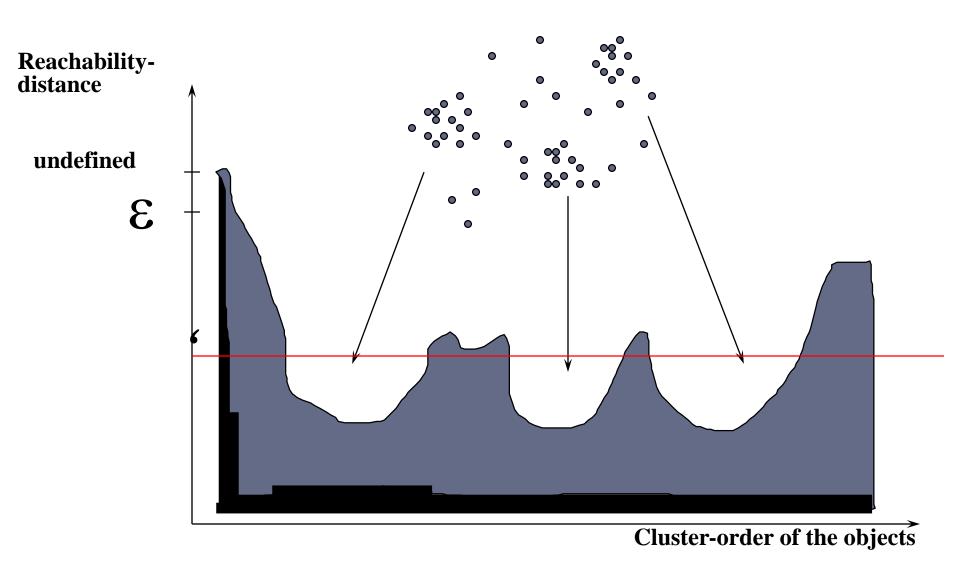


Figure 10.16: OPTICS terminology. Based on [ABKS99].

$$\varepsilon = 6mm, MinPts = 5$$

#### **Output of OPTICS: cluster-ordering**

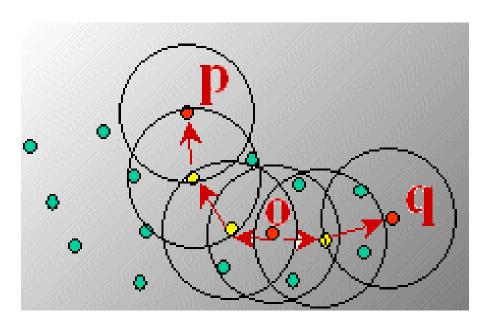


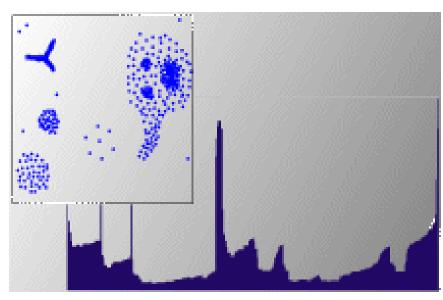
#### **Extract DBSCAN-Clusters**

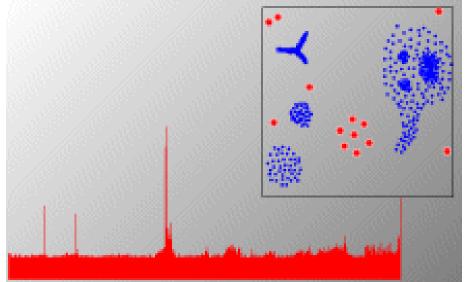
```
ExtractDBSCAN-Clustering (ClusterOrderedObjs,ε', MinPts)
// Precondition: ε' ≤ generating dist ε for ClusterOrderedObjs
  ClusterId := NOISE;
  FOR i FROM 1 TO ClusterOrderedObjs.size DO
    Object := ClusterOrderedObjs.get(i);
    IF Object.reachability_distance > ε' THEN
      // UNDEFINED > ε
      IF Object.core distance \leq \epsilon' THEN
        ClusterId := nextId(ClusterId);
        Object.clusterId := ClusterId;
      FLSE
        Object.clusterId := NOISE;
    ELSE // Object.reachability distance \leq \varepsilon
      Object.clusterId := ClusterId;
END; // ExtractDBSCAN-Clustering
```

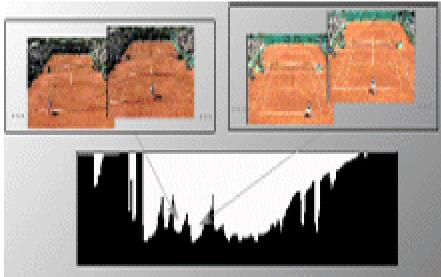
#### **Density-Based Clustering: OPTICS & Applications**

demo: http://www.dbs.informatik.uni-muenchen.de/Forschung/KDD/Clustering/OPTICS/Demo









#### \*DENCLUE: Using Statistical Density Functions

- DENsity-based CLUstEring by Hinneburg & Keim (KDD'98)
- Using statistical density functions:

$$f_{Gaussian}(x,y) = e^{-\frac{d(x,y)^2}{2\sigma^2}}$$
 influence of y on x

$$f_{Gaussian}^{D}(x) = \sum_{i=1}^{N} e^{-\frac{d(x,x_i)^2}{2\sigma^2}}$$

$$\nabla f_{Gaussian}^{D}(x, \underline{x_i}) = \sum_{i=1}^{N} (x_i - x) \cdot e^{\frac{-d(x, x_i)^2}{2\sigma^2}}$$

- Solid mathematical foundation
- Good for data sets with large amounts of noise
- Allows a compact mathematical description of arbitrarily shaped clusters in high-dimensional data sets
- Significant faster than existing algorithm (e.g., DBSCAN)
- But needs a large number of parameters

total influence

gradient of x in the direction of x<sub>i</sub>

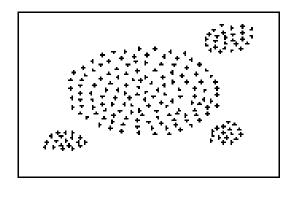
on x

### **Denclue: Technical Essence**

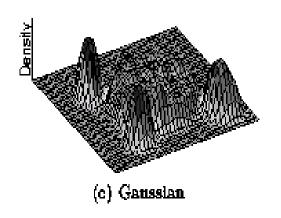
- Overall density of the data space can be calculated as the sum of the influence function of all data points
  - Influence function: describes the impact of a data point within its neighborhood
- Clusters can be determined mathematically by identifying density attractors
  - Density attractors are local maximal of the overall density function
  - Center defined clusters: assign to each density attractor the points density attracted to it
  - Arbitrary shaped cluster: merge density attractors that are connected through paths of high density (> threshold)

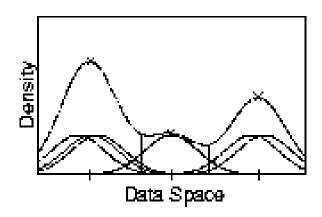
# **Density Attractor**

#### Can be detected by hill-climbing procedure of finding local maximums



(a) Data Set





#### **Noise Threshold**

- Noise Threshold  $\xi$ 
  - Avoid trivial local maximum points
  - A point can be a density attractor only if  $\hat{f}(x) \ge \xi$

# **Center-Defined and Arbitrary**

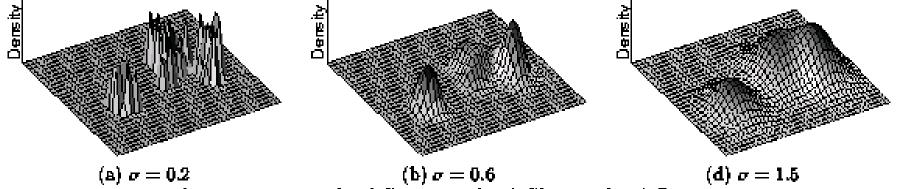


Figure 3: Example of Center-Defined Clusters for different  $\sigma$ 

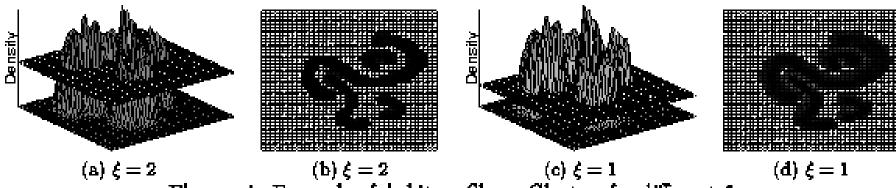


Figure 4: Example of Arbitray-Shape Clusters for different  $\xi$ 

# **Matrix Data: Clustering: Part 1**

- Cluster Analysis: Basic Concepts
- Partitioning Methods
- Hierarchical Methods
- Density-Based Methods
- Evaluation of Clustering
- Summary

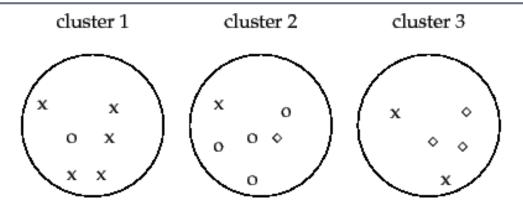
#### Measuring Clustering Quality

- Two methods: extrinsic vs. intrinsic
- Extrinsic: supervised, i.e., the ground truth is available
  - Compare a clustering against the ground truth using certain clustering quality measure
  - Ex. Purity, BCubed precision and recall metrics, normalized mutual information
- Intrinsic: unsupervised, i.e., the ground truth is unavailable
  - Evaluate the goodness of a clustering by considering how well the clusters are separated, and how compact the clusters are
  - Ex. Silhouette coefficient

# **Purity**

- Let  $C = \{c_1, ..., c_K\}$  be the output clustering result,  $\Omega = \{\omega_1, ..., \omega_K\}$  be the ground truth clustering result (ground truth class)
  - $c_k$  and  $w_k$  are sets of data points
  - $purity(C, \Omega) = \frac{1}{N} \sum_{k} \max_{j} |c_k \cap \omega_j|$

# **Example**



- ▶ Figure 16.1 Purity as an external evaluation criterion for cluster quality. Majority class and number of members of the majority class for the three clusters are: x, 5 (cluster 1); o, 4 (cluster 2); and  $\diamond$ , 3 (cluster 3). Purity is  $(1/17) \times (5+4+3) \approx 0.71$ .
- Clustering output: cluster 1, cluster 2, and cluster 3
- Ground truth clustering result: ×'s, ◊'s, and ○'s.

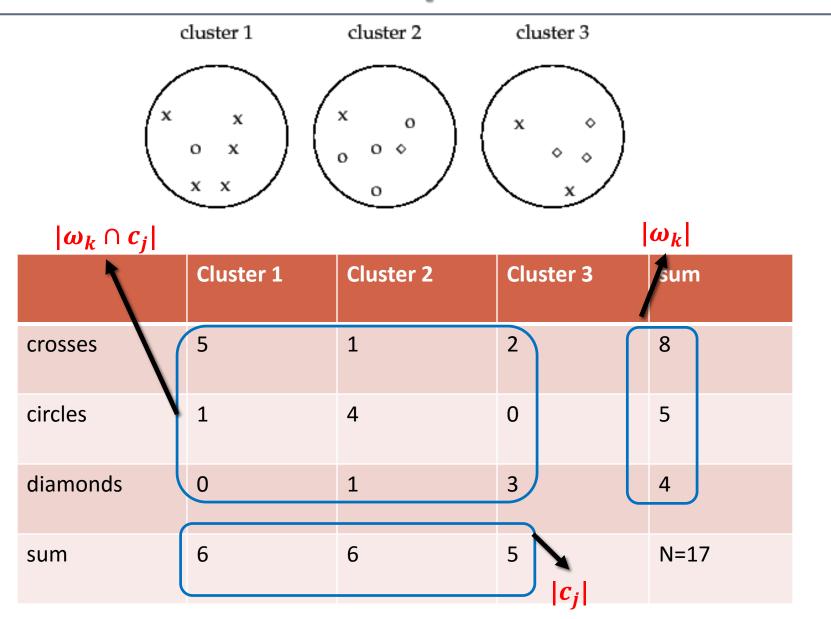
cluster 1 vs. ×'s, cluster 2 vs. ○'s, and cluster 3 vs. ◊'s

### **Normalized Mutual Information**

• NMI(
$$\Omega$$
,  $C$ ) =  $\frac{I(\Omega, C)}{\sqrt{H(\Omega)H(C)}}$   
•  $I(\Omega, C)$  =  $\sum_{k} \sum_{j} P(\omega_k \cap c_j) \log \frac{P(\omega_k \cap c_j)}{P(\omega_k)P(c_j)}$   
=  $\sum_{k} \sum_{j} \frac{|\omega_k \cap c_j|}{N} \log \frac{N|\omega_k \cap c_j|}{|\omega_k||c_j|}$ 

• 
$$H(\Omega) = -\sum_{k} \frac{|\omega_{k}|}{N} \log \frac{|\omega_{k}|}{N}$$

# **Example**



### **Precision and Recall**

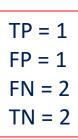
- $\bullet P = TP/(TP+FP)$
- $\cdot$  R = TP/(TP+FN)
- F-measure: 2P\*R/(P+R)

	Same cluster	Different clusters
Same class	TP	FN
Different classes	FP	TN

# **Example**

Index	Output clustering	Ground truth clustering (class)
1	1	2
2	1	2
3	2	2
4	2	1

- # pairs of data points: 6
  - (1,2): same class, same cluster
  - (1,3): same class, different cluster
  - (1,4): different class, different cluster
  - (2,3): same class, different cluster
  - (2,4): different class, different cluster
  - (3,4): different class, same cluster



# **Matrix Data: Clustering: Part 1**

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## **Summary**

- Cluster analysis groups objects based on their similarity and has wide applications; Measure of similarity can be computed for various types of data
- K-means and K-medoids algorithms are popular partitioningbased clustering algorithms
- AGNES and DIANA are interesting hierarchical clustering algorithms
- DBSCAN, OPTICS\*, and DENCLU\* are interesting density-based algorithms
- Clustering evaluation

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