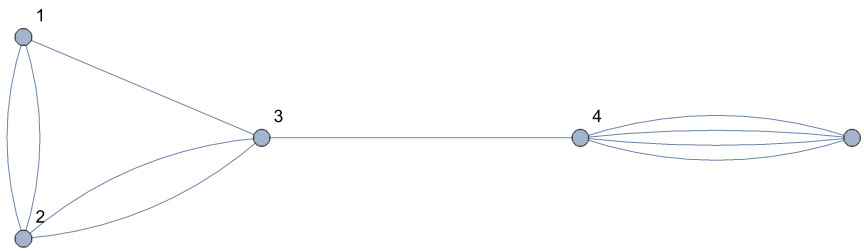
Example of Modularity Maximization

Bs = λDs

In[2]:= G =



;

In[15]:=
```
(A = Normal@AdjacencyMatrix@G) // MatrixForm
```

Out[15]//MatrixForm=

$$\begin{pmatrix} 0 & 2 & 1 & 0 & 0 \\ 2 & 0 & 2 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 4 & 0 \end{pmatrix}$$

In[5]:=
```
k = VertexDegree@G
```

Out[5]= {3, 4, 4, 5, 4}

Use K instead of D for that D is a build - in function name

In[16]:=
```
(K = DiagonalMatrix@k) // MatrixForm
```

Out[16]//MatrixForm=

$$\begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{pmatrix}$$

In[7]:=
```
m = Total@k / 2
```

Out[7]= 10

In[17]:=
```
(B = Table[Table[A[[i, j]] - k[[i]] k[[j]]/(2 m), {i, 5}], {j, 5}]) // MatrixForm
```

Out[17]//MatrixForm=

$$\begin{pmatrix} -\frac{9}{20} & \frac{7}{5} & \frac{2}{5} & -\frac{3}{4} & -\frac{3}{5} \\ \frac{7}{5} & -\frac{4}{5} & \frac{6}{5} & -1 & -\frac{4}{5} \\ \frac{2}{5} & \frac{6}{5} & -\frac{4}{5} & 0 & -\frac{4}{5} \\ -\frac{3}{4} & -1 & 0 & -\frac{5}{4} & 3 \\ -\frac{3}{5} & -\frac{4}{5} & -\frac{4}{5} & 3 & -\frac{4}{5} \end{pmatrix}$$

In[24]:=
```
N[Eigensystem[{B, K}], 5] // Grid
```

Out[24]=

$-0.93286 + 0.\times 10^{-6} i$	$0.83355 + 0.\times 10^{-6} i$	$-0.62737 + 0.\times 10^{-6} i$	$-0.27332 + 0.\times 10^{-6} i$	0
$\{0.014637 + 0.\times 10^{-7} i,$	$\{-0.88933 + 0.\times 10^{-6} i,$	$\{-4.1885 + 0.\times 10^{-5} i,$	$\{3.3965 + 0.\times 10^{-5} i,$	$\{1.0000,$
$-0.19606 + 0.\times 10^{-6} i,$	$-0.84896 + 0.\times 10^{-6} i,$	$4.9576 + 0.\times 10^{-5} i,$	$0.42078 + 0.\times 10^{-6} i,$	$1.0000,$
$0.35116 + 0.\times 10^{-6} i,$	$-0.52598 + 0.\times 10^{-6} i,$	$-2.0320 + 0.\times 10^{-5} i,$	$-3.6265 + 0.\times 10^{-5} i,$	$1.0000,$
$-0.93286 + 0.\times 10^{-6} i,$	$0.83355 + 0.\times 10^{-6} i,$	$-0.62737 + 0.\times 10^{-6} i,$	$-0.27332 + 0.\times 10^{-6} i,$	$1.0000,$
$1.0000\}$	$1.0000\}$	$1.0000\}$	$1.0000\}$	$1.0000\}$

In[25]:=
```
Timing[N[Eigensystem[{B, K}], 5];]
```

Out[25]= {10.3358, Null}

As = λDs

In[26]:=
```
N[Eigensystem[{A, K}], 5] // Grid
```

Out[26]=

1.0000	$-0.93286 + 0.\times 10^{-6} i$	$0.83355 + 0.\times 10^{-6} i$	$-0.62737 + 0.\times 10^{-6} i$	$-0.27332 + 0.\times 10^{-6} i$
$\{1.0000,$	$\{0.014637 + 0.\times 10^{-7} i,$	$\{-0.88933 + 0.\times 10^{-6} i,$	$\{-4.1885 + 0.\times 10^{-5} i,$	$\{3.3965 + 0.\times 10^{-5} i,$
$1.0000,$	$-0.19606 + 0.\times 10^{-6} i,$	$-0.84896 + 0.\times 10^{-6} i,$	$4.9576 + 0.\times 10^{-5} i,$	$0.42078 + 0.\times 10^{-6} i,$
$1.0000,$	$0.35116 + 0.\times 10^{-6} i,$	$-0.52598 + 0.\times 10^{-6} i,$	$-2.0320 + 0.\times 10^{-5} i,$	$-3.6265 + 0.\times 10^{-5} i,$
$1.0000,$	$-0.93286 + 0.\times 10^{-6} i,$	$0.83355 + 0.\times 10^{-6} i,$	$-0.62737 + 0.\times 10^{-6} i,$	$-0.27332 + 0.\times 10^{-6} i,$
$1.0000\}$	$1.0000\}$	$1.0000\}$	$1.0000\}$	$1.0000\}$

Lu = λu

In[20]:=
```
L = (Sqrt@Inverse@K).A.(Sqrt@Inverse@K)
```

Out[20]=
$$\left\{\left\{0, \frac{1}{\sqrt{3}}, \frac{1}{2\sqrt{3}}, 0, 0\right\}, \left\{\frac{1}{\sqrt{3}}, 0, \frac{1}{2}, 0, 0\right\}, \left\{\frac{1}{2\sqrt{3}}, \frac{1}{2}, 0, \frac{1}{2\sqrt{5}}, 0\right\}, \left\{0, 0, \frac{1}{2\sqrt{5}}, 0, \frac{2}{\sqrt{5}}\right\}, \left\{0, 0, 0, \frac{2}{\sqrt{5}}, 0\right\}\right\}$$

In[41]:=
```
N@Eigensystem[L] // Grid
```

Out[41]=

1.	-0.932863	0.83355	-0.627371	-0.273316
{0.866025, 1., 1., 1.11803, 1.}	{0.0126762, -0.196064, 0.351165, -1.04297, 1.}	{-0.770183, -0.848963, -0.525976, 0.931937, 1.}	{-3.62731, 4.95759, -2.03203, -0.701422, 1.}	{2.94144, 0.420775, -3.62649, -0.305577, 1.}

Result

In[42]:=
```
CommunityGraphPlot[G, {{1, 2, 3}, {4, 5}}]
```

Out[42]=