## CS249: SPECIAL TOPICS MINING INFORMATION/SOCIAL NETWORKS

#### **Overview of Networks**

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#### **Overview of Information Network**

Analysis

Network Representation

Network Properties

Network Generative Models

Random Walk and Its Applications

#### **Networks Are Everywhere**



Aspirin





#### Yeast protein interaction network



rom H. Jeong et al Nature 411, 41 (2001)

## **Representation of a Network: Graph**

- G = < V, E >
  - $V = \{u_1, ..., u_n\}$ : node set
  - $E \subseteq V \times V$ : edge set
- Adjacency matrix
  - $A = \{a_{ij}\}, i, j = 1, ..., N$ 
    - $a_{ij} = 1, if < u_i, u_j > \in E$
    - $a_{ij} = 0$ , if  $\langle u_i, u_j \rangle \notin E$
- Network types
  - Undirected graph vs. Directed graph
    - $A = A^{\mathrm{T}} vs. A \neq A^{\mathrm{T}}$
  - Binary graph Vs. Weighted graph
    - Use W instead of A, where  $w_{ij}$  represents the weight of edge  $< u_i, u_j >$

#### Example





Adjacency matrix A

## **Degree of Nodes**

- Let a network G = (V, E)
- Undirected Network
  - Degree (or degree centrality) of a vertex: d(v<sub>i</sub>)
  - # of edges connected to it, e.g., d(A) = 4, d(H) = 2
- Directed network
  - In-degree of a vertex d<sub>in</sub>(v<sub>i</sub>):
    - # of edges pointing to v<sub>i</sub>
    - E.g., d<sub>in</sub>(A) = 3, d<sub>in</sub>(B) = 2
  - Out-degree of a vertex d<sub>out</sub>(v<sub>i</sub>):
    - # of edges from v<sub>i</sub>
    - E.g., d<sub>out</sub>(A) = 1, d<sub>out</sub>(B) = 2



 $\mathbf{H}$ 

в

D

# **Degree Distribution**

- Degree sequence of a graph: The list of degrees of the nodes sorted in non-increasing order
  - E.g., in G<sub>1</sub>, degree sequence: (4, 3, 2, 2, 1)
- Degree frequency distribution of a graph: Let N<sub>k</sub> denote the # of vertices with degree k
  - ( $N_0$ ,  $N_1$ , ...,  $N_t$ ), t is max degree for a node in G
  - E.g., in G<sub>1</sub>, degree frequency distribution: (0, 1, 2, 1, 1)
- Degree distribution of a graph:

Probability mass function f for random variable X

- (f(0), f(1), ..., f(t), where  $f(k) = P(X = k) = N_k/n$
- E.g., in G<sub>1</sub>, degree distrib.: (0, 0.2, 0.4, 0.2, 0.2)

Graph G<sub>1</sub>

## Path

- Path: A sequence of vertices that every consecutive pair of vertices in the sequence is connected by an edge in the network
- Length of a path: # of edges traversed along the path
- Total # of path of length 2 from *j* to *i*, via any vertex in  $N_{ii}^{(2)}$  is

$$N_{ij}^{(2)} = \sum_{k=1}^{n} A_{ik} A_{kj} = [A^2]_{ij}$$

Generalizing to path of arbitrary length, we have:

$$N_{ij}^{(r)} = [A^r]_{ij}$$

## **Radius and Diameter**

- Eccentricity: The eccentricity of a node v<sub>i</sub> is the maximum distance from v<sub>i</sub> to any other nodes in the graph
  - $e(v_i) = max_j \{d(v_{i_i}, v_j)\}$
  - E.g., e(A) = 1, e(F) = e(B) = e(D) = e(H) = 2
- Radius of a connected graph G: the min eccentricity of any node in G
  - $r(G) = \min_{i} \{e(v_i)\} = \min_{i} \{\max_{j} \{d(v_{i_j}, v_j)\}\}$
  - E.g.,  $r(G_1) = 1$
- **Diameter** of a connected graph G: the max eccentricity of any node in G
  - $d(G) = \max_{i} \{e(v_i)\} = \max_{i, j} \{d(v_{i, j})\}$
  - E.g., d(G<sub>1</sub>) = 2
- Diameter is sensitive to outliers. Effective diameter: min # of hops for which a large fraction, typically 90%, of all connected pairs of nodes can reach each other

D

Graph G<sub>1</sub>

F

## **Clustering Coefficient**

- Real networks are sparse: Corresponding to a complete graph
- Clustering coefficient of a node v<sub>i</sub>: A measure of the density of edges in the neighborhood of v<sub>i</sub>
- Let G<sub>i</sub> = (V<sub>i</sub>, E<sub>i</sub>) be the subgraph induced by the neighbors of vertex v<sub>i</sub>, |V<sub>i</sub>| = n<sub>i</sub> (# of neighbors of v<sub>i</sub>), and |E<sub>i</sub>| = m<sub>i</sub> (# of edges among the neighbors of v<sub>i</sub>)
- Clustering coefficient of v<sub>i</sub> for undirected network is

• C

$$C(v_i) = \frac{\# \ edges \ in \ G_i}{max \ \# \ edges \ in \ G_i} = \frac{m_i}{\binom{n_i}{2}} = \frac{2 \times m_i}{n_i(n_i - 1)}$$
  
For directed network,  
$$C(v_i) = \frac{\# \ edges \ in \ G_i}{max \ \# \ edges \ in \ G_i} = \frac{m_i}{n_i(n_i - 1)}$$
  
ustering coefficient of a graph G:  
$$C(G) = \frac{1}{n} \sum C(v_i)$$

Averaging the local clustering coefficient of all the vertices (Watts & Strogatz)

#### Overview of Information Network Analysis

Network Representation



Network Generative Models

Random Walk and Its Applications

## **More Than a Graph**

- A typical network has the following common properties:
  - Few connected components:
    - often only 1 or a small number, independent of network size
  - Small diameter:
    - often a constant independent of network size (like 6)
    - growing only logarithmically with network size or even shrink?
  - A *high* degree of clustering:
    - considerably more so than for a random network
  - A *heavy-tailed* degree distribution:
    - a small but reliable number of high-degree vertices
    - often of *power law* form



- For complete Graph
  - Average degree: N
- For real-world network
  - Average degree:  $\langle k \rangle = 2E/N \ll N$

## **Small World Property**

- Small world phenomenon (Six degrees of separation)
  - Stanley Milgram's experiments (1960s)
  - Microsoft Instant Messaging (IM) experiment: J. Leskovec & E. Horvitz (WWW'08)
    - 240 M active user accounts: Est. avg. distance 6.6 & est. mean median 7
- Why small world?

• 
$$N(d) \approx 1 + \langle k \rangle + \langle k \rangle^2 + \dots + \langle k \rangle^d = \frac{\langle k \rangle^{d+1} - 1}{\langle k \rangle - 1} \approx \langle k \rangle^d$$

• E.g., 
$$d \approx \frac{\ln N}{\ln \langle k \rangle} \approx \frac{\ln (7 \times 10^9)}{\ln (10^3)} \approx 3.28$$

## **Degree Distribution: Power Law**



# **High Clustering Coefficient**

- Clustering effect: a high clustering coefficient for graph G
  - Friends' friends are likely friends.
  - A lot of triangles
  - C(k): avg clustering coefficient for nodes with degree
     k

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## **Network Generative Models**

- All of the network generation models we will study are *probabilistic* or *statistical* in nature
- They can generate networks of any size
- They often have various *parameters* that can be set:
  - size of network generated
  - average degree of a vertex
  - fraction of long-distance connections
- The models generate a *distribution* over networks
- Statements are always *statistical* in nature:
  - with high probability, diameter is small
  - on average, degree distribution has heavy tail

## **Examples**

#### • Erdös-Rényi Random graph model:

- Gives few components and small diameter
- does not give high clustering and heavy-tailed degree distributions
- is the mathematically most well-studied and understood model
- Watts-Strogatz small world graph model:
  - gives few components, small diameter and high clustering
  - does not give heavy-tailed degree distributions
- Barabási-Albert Scale-free model:
  - gives few components, small diameter and heavy-tailed distribution
  - does not give high clustering
- Stochastic Block Model

#### Erdös-Rényi (ER) Random Graph Model

- Every possible edge occurs independently with probability p
  - G(N, p): a network of N nodes, each node pair is connected with probability of p
    - Paul Erdős and Alfréd Rényi: "On Random Graphs" (1959)
    - E. N. Gilbert: "Random Graphs" (1959) (proposed independently)
  - Usually, N is large and p ~ 1/N
    - Choices: p = 1/2N, p = 1/N, p = 2/N, p = 10/N, p = log(N)/N, etc.

## **Degree Distribution**

 The degree distribution of a random (small) network follows binomial distribution

• 
$$p_k = \begin{pmatrix} N-1 \\ k \end{pmatrix} p^k (1-p)^{N-1-k}$$

• When N is large and Np is fixed, approximated by Poisson distributi  $p_k = e^{-\langle k \rangle} \frac{\langle k \rangle^k}{k!}$ 



From Barabasi 2016

## Watts-Strogatz small world model

- Interpolates between regular lattice and a random network to generate graphs with
  - Small-world: short average path lengths



#### Barabási-Albert Model: Preferential Attachment

- Major limitation of the Watts-Strogatz model
  - It produces graphs that are homogeneous in degree
  - Real networks are often inhomogeneous in degree, having hubs and a scale-free degree distribution (*scale-free networks*)
- Scale-free networks are better described by the preferential attachment family of models, e.g., the Barabási–Albert (BA) model
  - *"rich-get-richer":* New edges are more likely to link to nodes with higher degrees
  - **Preferential attachment:** The probability of connecting to a node is proportional to the current degree of that node
- This leads to the proposal of a new model: scale-free network, a network whose degree distribution follows a power law, at least asymptotically

#### Overview of Information Network Analysis

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- Network Generative Models
- Random Walk and Its Applications

## **The History of PageRank**

- PageRank was developed by Larry Page (hence the name Page-Rank) and Sergey Brin.
- It is first as part of a research project about a new kind of search engine. That project started in 1995 and led to a functional prototype in 1998.
- Shortly after, Page and Brin founded Google.

## **Ranking web pages**

- Web pages are not equally "important"
  - <u>www.cnn.com</u> vs. a personal webpage
- Inlinks as votes
  - The more inlinks, the more important
- Are all inlinks equal?
  - Higher ranked inlink should play a more important role
  - Recursive question!

## **Simple recursive formulation**

- Each link's vote is proportional to the importance of its source page
- If page P with importance x has n outlinks, each link gets x/n votes
- Page P's own importance is the sum of the votes on its inlinks



## **Matrix formulation**

- Matrix M has one row and one column for each web y a m
- Suppose page j has n outlinks
  - If  $j \rightarrow i$ , then  $M_{ij}=1/n$
  - Else M<sub>ij</sub>=0



- M is a column stochastic matrix
  - Columns sum to 1
- Suppose r is a vector with one entry per web page
  - r<sub>i</sub> is the importance score of page i
  - Call it the rank vector
  - $|\mathbf{r}| = 1$  (i.e.,  $r_1 + r_2 + \dots + r_N = 1$ )

## **Eigenvector formulation**

The flow equations can be written

*r* = *Mr* 

- So the rank vector is an eigenvector of the stochastic web matrix
  - In fact, its first or principal eigenvector, with corresponding eigenvalue 1

### Example



$$\begin{array}{ccccccc} y & a & m \\ y & 1/2 & 1/2 & 0 \\ a & 1/2 & 0 & 1 \\ m & 0 & 1/2 & 0 \end{array}$$

$$\mathbf{r} = \mathbf{M} * \mathbf{r}$$

$$\begin{bmatrix} \mathbf{y} \\ \mathbf{a} \\ \mathbf{m} \end{bmatrix} = \begin{bmatrix} 1/2 \ 1/2 \ 0 \\ 1/2 \ 0 \ 1 \\ 0 \ 1/2 \ 0 \end{bmatrix} \begin{bmatrix} \mathbf{y} \\ \mathbf{a} \\ \mathbf{m} \end{bmatrix}$$

## **Power Iteration method**

- Simple iterative scheme
- Suppose there are N web pages
  - Initialize:  $\mathbf{r}^{0} = [1/N, ..., 1/N]^{T}$
  - Iterate:  $\mathbf{r}^{k+1} = \mathbf{M}\mathbf{r}^k$
  - Stop when  $|\mathbf{r}^{k+1} \mathbf{r}^k|_1 < \epsilon$ 
    - $|\mathbf{x}|_1 = \sum_{1 \le i \le N} |\mathbf{x}_i|$  is the L<sub>1</sub> norm
    - Can use any other vector norm e.g., Euclidean

#### **Power Iteration Example**



 $r_0$  $\boldsymbol{r}_1$ **r**<sub>3</sub>  $r_2$ ....

## **Random Walk Interpretation**

- Imagine a random web surfer
  - At any time t, surfer is on some page P
  - At time t+1, the surfer follows an outlink from P uniformly at random
  - Ends up on some page Q linked from P
  - Process repeats indefinitely
- Let p(t) be a vector whose i<sup>th</sup> component is the probability that the surfer is at page i at time t
  - **p**(t) is a probability distribution on pages

## The stationary distribution

- Where is the surfer at time t+1?
  - Follows a link uniformly at random
  - p(t+1) = Mp(t)
- Suppose the random walk reaches a state such that p(t+1) = Mp(t) = p(t)
  - Then p(t) is called a stationary distribution for the random walk
- Our rank vector r satisfies r = Mr
  - So it is a stationary distribution for the random surfer

## **Existence and Uniqueness**

A central result from the theory of random walks (aka Markov processes):

For graphs that satisfy certain conditions, the stationary distribution is unique and eventually will be reached no matter what the initial probability distribution at time t = 0.

## **Spider traps**

- A group of pages is a spider trap if there are no links from within the group to outside the group
  - Random surfer gets trapped
- Spider traps violate the conditions needed for the random walk theorem

#### **Microsoft becomes a spider trap**



## **Random teleports**

- The Google solution for spider traps
- At each time step, the random surfer has two options:
  - With probability  $\beta$ , follow a link at random
  - With probability 1-β, jump to some page uniformly at random
  - Common values for  $\beta$  are in the range 0.8 to 0.9
- Surfer will teleport out of spider trap within a few time steps

## Random teleports ( $\beta = 0.8$ )



····→ : teleport links from "Yahoo"

## Random teleports ( $\beta = 0.8$ )



## **Matrix formulation**

- Suppose there are N pages
  - Consider a page j, with set of outlinks O(j)
  - We have  $M_{ij} = 1/|O(j)|$  when j->i and  $M_{ij} = 0$  otherwise
  - The random teleport is equivalent to
    - adding a teleport link from j to every other page with probability  $(1-\beta)/N$
    - reducing the probability of following each outlink from 1/|O(j)| to  $\beta/|O(j)|$
    - Equivalent: tax each page a fraction (1- $\beta$ ) of its score and redistribute evenly

## PageRank

Construct the N-by-N matrix A as follows

•  $A_{ij} = \beta M_{ij} + (1-\beta)/N$ 

- Verify that A is a stochastic matrix
- The page rank vector r is the principal eigenvector of this matrix

satisfying r = Ar

 Equivalently, r is the stationary distribution of the random walk with teleports

#### **Dead ends**

- Pages with no outlinks are "dead ends" for the random surfer
  - Nowhere to go on next step

#### **Microsoft becomes a dead end**



## **Dealing with dead-ends**

#### Teleport

- Follow random teleport links with probability 1.0 from dead-ends
- Adjust matrix accordingly
- Prune and propagate
  - Preprocess the graph to eliminate dead-ends
    - Might require multiple passes
  - Compute page rank on reduced graph
  - Approximate values for deadends by propagating values from reduced graph

## **Dealing dead end: teleport**



## **Dealing dead end: reduce graph**



# **Computing PageRank**

- Key step is matrix-vector multiplication
  - r<sup>new</sup> = Ar<sup>old</sup>
- Easy if we have enough main memory to hold
   A, r<sup>old</sup>, r<sup>new</sup>
- Say N = 1 billion pages
  - We need 4 bytes for each entry (say)
  - 2 billion entries for vectors, approx 8GB
  - Matrix A has N<sup>2</sup> entries
    - 10<sup>18</sup> is a large number!

## **Rearranging the equation**

**r** = **Ar**, where  $A_{ii} = \beta M_{ii} + (1 - \beta)/N$  $r_i = \sum_{1 \le i \le N} A_{ii} r_i$  $r_i = \sum_{1 \le i \le N} [\beta M_{ii} + (1-\beta)/N] r_i$  $= \beta \sum_{1 \le i \le N} M_{ii} r_i + (1 - \beta) / N \sum_{1 \le i \le N} r_i$  $= \beta \sum_{1 \le i \le N} M_{ii} r_i + (1-\beta)/N$ , since  $|\mathbf{r}| = 1$  $\mathbf{r} = \beta \mathbf{M} \mathbf{r} + [(1-\beta)/N]_{N}$ 

where  $[x]_N$  is an N-vector with all entries x

## **Sparse matrix formulation**

- We can rearrange the page rank equation:
  - $\mathbf{r} = \beta \mathbf{Mr} + [(1-\beta)/N]_N$
  - $[(1-\beta)/N]_N$  is an N-vector with all entries  $(1-\beta)/N$
- M is a sparse matrix!
  - 10 links per node, approx 10N entries
- So in each iteration, we need to:
  - Compute  $\mathbf{r}^{\text{new}} = \beta \mathbf{M} \mathbf{r}^{\text{old}}$
  - Add a constant value  $(1-\beta)/N$  to each entry in  $\mathbf{r}^{new}$

## **Sparse matrix encoding**

- Encode sparse matrix using only nonzero entries
  - Space proportional roughly to number of links
  - say 10N, or 4\*10\*1 billion = 40GB
  - still won't fit in memory, but will fit on disk

source node	degree	destination nodes
0	3	1, 5, 7
1	5	17, 64, 113, 117, 245
2	2	13, 23

## **Basic Algorithm**

- Assume we have enough RAM to fit r<sup>new</sup>, plus some working memory
  - Store **r**<sup>old</sup> and matrix **M** on disk

#### **Basic Algorithm:**

- Initialize: r<sup>old</sup> = [1/N]<sub>N</sub>
- Iterate:
  - Update: Perform a sequential scan of M and r<sup>old</sup> to update
     r<sup>new</sup>
  - Write out **r**<sup>new</sup> to disk as **r**<sup>old</sup> for next iteration
  - Every few iterations, compute |r<sup>new</sup>-r<sup>old</sup>| and stop if it is below threshold
    - Need to read in both vectors into memory



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## Paper Sign-Up

- <u>https://docs.google.com/spreadsheets/d/1Sao</u>
   <u>PGP2SsYyaycX82T7mF\_efbiueOI53bnZtZS04Bt</u>
   <u>Q/edit?usp=sharing</u>
- If you are still on waiting list
  - Sign-up for Presenter 4 only

#### **Credits**

 This is 4-credit course, please change it if you are current enrolled with 2-credit

## **Course Project Examples**

- Citation graph summary
  - Find k papers that can tell the main structure evolution of a certain field
- Name disambiguation problem in DBLP
  - Different people may share the same name, e.g., distinguish "Wei Wang"'s;
  - Same person may have different forms of names, e.g., initials, middle names, typos

- User profile prediction in heterogeneous information networks
  - Suppose we only know small number of labels for people's ideology, profession, education, can we predict the remaining?
- Sentence embedding
  - Can we find the most similar sentences or S-V-O (subject-verb-object) triplets to the given one, by converting the text into a network?