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A SIMPLE METHOD FOR DETERMINING STATIC PARAMETERS OF LARGE SIGNAL SEMICONDUCTOR DIODE AND TRANSISTOR MODELS

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Abstract—A simple on-line interactive computing methodology for determining static parameters of large-signal semiconductor models is described. The procedure makes use of: (1) an automatic data collection scheme, (2) a single search, three-parameter optimization method for computing diode parameters, and (3) a partitioning scheme to separate the defining transistor equations into two single search, three-parameter problems which are solved by the diode optimization method described in (2).

The method is described by means of the CIRCUS diode and transistor models, and is compared experimentally with Sokal's method[1, 2]. The new approach is shown to be superior.

1. INTRODUCTION

THE ASSUMPTIONS under which the charge control, Ebers–Moll, and Linvill models[3–5] have been derived are valid only for moderate current conditions. For high currents however, no a priori guarantee of the soundness of the analogs exists, even when a variable current gain, Beta, is included in tabular form, as is done in the modified charge-control model used in the CIRCUS program[6]. Additional resistances are also added at the terminals to ensure a first order correction on the model characteristics. For this reason, as indicated by Lindholm[7], model parameter values, for device operations over a wide range of currents, obtained by the state of art methods should be viewed with a great deal of uncertainty.

Probably one of the first significant attempts in overcoming this difficulty was made by Sokal[1, 2]. In Sokal's method just enough experimental data points are taken so that the defining model equations can be solved explicitly. Although the method has been applied successfully to diode and transistor models alike, experiments have proved that a small error in a data point can be reflected as large errors in the model parameters[8].

Another approach to the problem has been presented by Bailey[9] who recommends a 'cut and try' method of fitting experimental data to the voltage–current characteristics of a given device. Even though a large number of data points are taken, thus encompassing more fully the performance of the device, the method is time consuming and is subject to human error.

More recently two highly sophisticated approaches have been presented by Parker and Wether[10], who advocate a classical pseudo-inverse technique, and by Rohrer *et al.*[11], who utilize the adjoint network method to implement a least square fit of experimental data. Both methods are computationally elaborate. Neither method realizes the possibility of maximizing the information which is obtained from the measurements by using a judiciously chosen data collection scheme.

It is the purpose of this paper to describe a simpler technique which is applicable to the CIRCUS class of models. The basic procedure is designed for the diode case and consists of a special data collection scheme and an optimization method that employs a single search, three parameter algorithm and sensitivity functions to determine step sizes. This procedure is then extended to include the transistor case for which the model equations are separated into two sets so that the basic diode case algorithms can be employed. The last part of the paper compares the new method to Sokal's method with experimental data obtained for both the diode and transistor cases.

2. THE BASIC METHOD: THE CIRCUS DIODE MODEL

The optimization procedure is best described by example. Consider the CIRCUS diode model shown in Table 1, where the defining circuit equation can be expressed by

$$I = I_s\{\exp[(V-RI)\theta]-1\}. \quad (1)$$

The parameters I_s, θ, and R are to be determined by fitting the known (measured), but judiciously collected, quantities of V and I.

*Data collection.** Recall the nonlinear diode characteristics whose current values are plotted on a logarithmic scale against voltage values on a linear scale. The region of interest is bounded by $I_{\min}$ and $I_{\max}$. In the lower part of the V–I characteristics, the curve is linear indicating that the voltage drop due to the resistance R and the I_s term in equation (1) can be neglected. With these assumptions, the diode voltage equation reduces to

$$V = \frac{1}{\theta}(\ln I - \ln I_s), \quad (2)$$

a straight line (on the log scale) with slope $1/\theta$ and intercept I_s. If equal increments of ΔV are taken, ΔI between consecutive points is a constant, i.e.,

$$\Delta I \approx \Delta V \cdot \theta I, \quad (3)$$

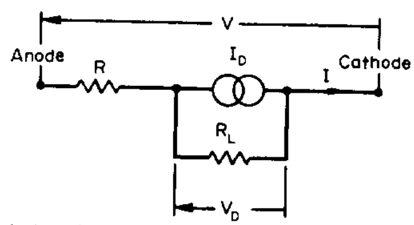
assuming ΔV is small enough. In other words in the linear (lower) portion of the diode characteristics, the current data should be collected according to a geometric progression.

At high current levels (the upper part of the diode

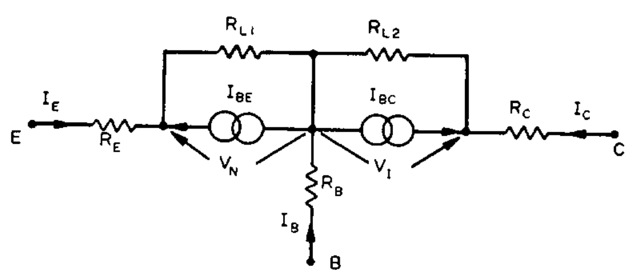
*A programmable test is available. Therefore, it is preferred to use a judicious distribution of the data set to insure the best estimate of the parameter value.

Table 1. *CIRCUS models*

1. *D.C. diode model*
   $I_D = I_S[\exp(\theta V_D)-1]$
   R = Diode ohmic series resistance
   R_L = Junction ohmic leakage resistance
   $\theta = M(q/KT)$
   M = Emission constant for diode junction
   I_S = Saturation current
   V_D = Junction voltage



2. *D.C. npn transistor model*
   $I_{BE} = (1/\beta_N+1)I_N - I_I$
   $I_{BC} = -I_N + (1/\beta_I+1)I_I$
   $I_N = I_{ES}[\exp(\theta_N V_N)-1]$
   $I_I = I_{CS}[\exp(\theta_I V_I)-1]$
   R_{L1} = Emitter–base junction ohmic leakage resistance
   R_{L2} = Collector–base junction ohmic leakage resistance
   R_E = Extrinsic emitter resistance
   R_B = Extrinsic base resistance
   R_C = Extrinsic collector resistance
   $\theta_N = M_N(q/KT)$
   M_N = Emission constant for emitter–base diode
   $\theta_I = M_I(q/KT)$
   M_I = Emission constant for collector–base diode
   β_N = D.C. current gain under normal operation
   β_I = Inverse d.c. current gain
   I_{ES} = Emitter saturation current
   I_{CS} = Collector saturation current



characteristics), the series resistance is dominant; therefore according to equation (1) a constant interval ΔI between successive values should exist between measurements.

The conflicting requirements for ΔI between the linear and nonlinear portions of the characteristics have to be correlated. Suppose $N+1$ data points I_j of current are taken between I_{min} and I_{max}, where $I_0 = I_{min} \ll I_N = I_{max}$. Defining $[N/3]$ as the smallest integer $\geq N/3$, and ΔI_j the interval between consecutive points, the current values are given by $I_{j+1} = I_j + \Delta I_j$. The upper $[N/3]$ points are measured according to an arithmetic progression, while the remaining points are taken according to the geometric progression specified by equation (3). The interval is therefore a function of the current such as shown in Fig. 1.

Once the current interval limits I_{min} and I_{max}, and the number of data points, indicated by $N+1$, are specified, the experiments can be conducted starting with I_{min} and run through to I_{max} in increments of

$$\Delta I_j = K \cdot I_j \qquad \text{for} \qquad 0 \leq j < N - [N/3]$$

and

$$\Delta I_j = K \cdot I_{N-[N/3]} \qquad \text{for} \qquad N - [N/3] \leq j \leq N$$

Noting that

$$I_{N-[N/3]} = I_{min}(1+K)^{(N-[N/3])} = I_{max} - [N/3] \cdot K I_{(N-[N/3])} \tag{4}$$

the value of K can be solved for implicitly from the expression,

$$\frac{I_{max}}{I_{min}} = (1 + k[N/3])(1+k)^{(N-[N/3])} \tag{5}$$

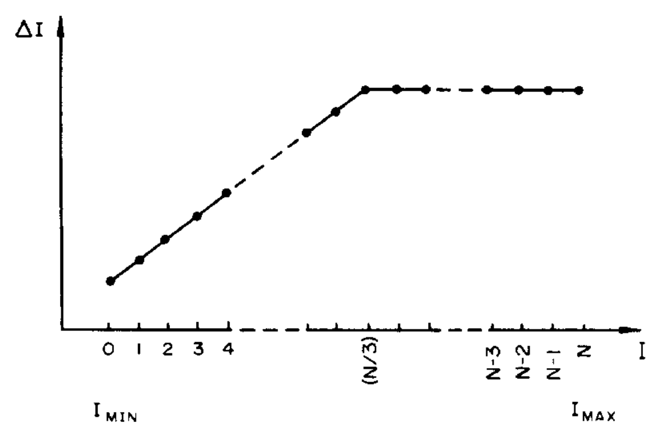


Fig. 1. Distribution of the values of current intervals to be used in the measurements.

quite conveniently by a computer program. The current settings necessary to conduct the experiments can now be specified.

In order to assume that the measurements on the device are taken at constant temperature, pulses whose duration are at least four or five times longer than the electric and much smaller than the thermal time constants of the device are employed. For the experiments conducted in the present research, a Fairchild 600 tester proved to be excellent.

Problem formulation. In the treatment of the experimental data, i.e., the pairs, (I_j, V_j), $j = 0, 1, \ldots, N$, the current I_j is considered the independent variable, while V_j is considered the dependent variable. Consequently (1) is rewritten as

$$V(I) = \frac{1}{\theta}\ln\left(\frac{I}{I_s}+1\right) + RI \tag{6}$$

so that the voltage variable is a function of I. Then defining the point-by-point error as

$$E_j = V_j - V(I_j) \qquad j = 0, 1, \ldots, N \tag{7}$$

an error function can be formulated. It was found that excellent results, speedy convergence and computational simplicity could be achieved with the error function,

$$E_c(\theta, I_s, R) = {}^{c}\sqrt{\left(\frac{\sum_{i=0}^{N} |E_j|^c}{N+1}\right)} \tag{8}$$

for $c = 1$. (Different values of c between 1 and 2 were tested and essentially the same results were obtained. So $c = 1$ was chosen for computational simplicity.) Finding the minimum value of E_c while θ, I_s and R are treated as parameters constitutes the optimization problem.

Search procedure. Among the various search procedures amenable to digital computer implementation, a single-parameter search procedure seemed best for the problem at hand, since it offers the best compromise between computation time and ease of programming. A three-parameter search problem is substituted by a single-search problem attacked iteratively, a situation readily suited for on-line computation. The single-search algorithm is described in the appendix.

The selection of good starting values and increments of the parameters are crucial to the success of the algorithm. In fact, convergence to an absolute minimum will definitely not occur unless the initial starting point is taken close enough to the minimum such that a monotonically decreasing path between the two points exists. Likewise if improper increments are chosen, the minimum can be bypassed without really knowing it.

These difficulties are easily remedied however. A good estimate to the starting values is found by Sokal's method which computes a first order approximation to the diode parameters. It has proved to be highly successful in making the initial guess.

The most critical factor for the success of the parameter search procedure rests with the proper choice of parameter increments. Too long a step can lead to divergence while too short of one will increase the time needed to locate the minimum. This difficulty can be overcome by making use of sensitivity functions. Therefore, a step size inversely proportional to the sensitivity of the diode characteristics to each parameter should then be employed in the search. Defining the sensitivity function as,

$$S_x{}^y = \frac{(\Delta y/y)}{(\Delta x/x)} \approx \frac{\mathrm{d}\ln y}{\mathrm{d}\ln x}, \tag{9}$$

for the diode case expressed by equation (1), the sensitivities of V to θ, I_s and R are respectively:

$$S_\theta{}^V = -\left(1 - \frac{RI}{V}\right)$$

$$S_{I_s}^V = -\frac{1}{\theta V}$$

$$S_R{}^V = \frac{IR}{V}.$$

For a silicon diode, assuming that the voltage varies between 0.5 and 1.5 V, while the ohmic drop due to R varies between 0 and 0.5 V, the sensitivities varies according to

$$-1 \leqslant S_\theta{}^V \leqslant -0.7, 0.33 \leqslant S_{I_s}^V \leqslant 0.1,$$

and

$$0 \leqslant S_R{}^V \leqslant 0.33.$$

Using the sensitivity coefficients obtained above, the size of the steps which ensure the fastest convergence are given in the table below where the stepsize range is expressed as a percentage of the parameter value.

Range	θ	I_s	R
Δ% min	1	7	4
Δ% max	3	24	12

The sensitivity coefficients are also useful to evaluate the consistency of the results of parameter determination methods by comparing the dispersion of the outcomes for each parameter against its sensitivity coefficient.

2.1 *A separation technique: the transistor case*

The three-parameter search algorithm can be extended to include the transistor case. Consider the CIRCUS transistor model shown in Table 1. The quantities measured or directly derived from the measurements are the terminal voltages and currents V_{BE}, V_{BC}, I_E, and I_C, and the current gains β_I and β_N. The unknown parameters are I_{ES}, I_{CS}, θ_N, θ_I, R_E, R_B, R_C. The leakage resistances, R_{L1} R_{L2}, are neglected.

For this case a seven parameter optimization problem is defined. Fortunately not all parameters need be considered at one time, since the original problem can be recast into two four parameter problems, one involving the base–emitter diode equation, the other the base–collector diode equation. One of the parameters is common to both equations and can be determined by a separate measurement. As will be shown below, by manipulating the equations, the four parameter problem is solved by means of the three parameter search algorithm, applied to each case, and iterating between the two.

First consider the base–emitter diode equation,

$$I_N = I_{ES}\{\exp\left[\theta_N(V_{BE} + R_E I_E - R_B I_B)\right] - 1\}. \tag{10}$$

Noting that $R_E I_E - R_B I_B$ in the argument of the exponential function can be rewritten as

$$R_E I_E - R_B I_B = -R_E \frac{\beta_N}{\beta_N + 1} I_C - R_B \frac{I_C}{\beta_N}$$
$$= -I_C\left(R_E \frac{\beta_N}{\beta_N + 1} + \frac{R_B}{\beta_N}\right), \tag{11}$$

and assuming that $I_C \approx I_N \gg I_I$, the collector current can be expressed as

$$I_C = I_{ES}\left\{\exp\left[\theta_N\left(V_{BE} - I_C\left(R_E\frac{\beta_N}{\beta_N+1} + \frac{R_B}{\beta_N}\right)\right)\right] - 1\right\}. \quad (12)$$

Similarly, the emitter current, for the inverse characteristics,

$$I_E = I_{CS}\{\exp\left[\theta_I(V_{BC} + R_C I_C - R_B I_B)\right] - 1\} \quad (13)$$

can be manipulated to

$$I_E = I_{CS}\left\{\exp\left[\theta_I\left(V_{BC} - I_E\left(R_C\frac{\beta_I}{\beta_I+1} + \frac{R_B}{\beta_I}\right)\right)\right] - 1\right\} \quad (14)$$

assuming $I_E \approx I_I \gg I_N$.

The current equations (12) and (14) have the identical form as (1), for the diode, except that a more complicated resistance expression appears in each equation. These resistances can be lumped as a single equivalent value given by

$$R_{N\text{eq}} = R_E\frac{\beta_N}{\beta_N+1} + \frac{R_B}{\beta_N} \quad (15)$$

$$R_{I\text{eq}} = R_C\frac{\beta_I}{\beta_I+1} + \frac{R_B}{\beta_I} \quad (16)$$

which are correct only when β_N and β_I are constant. On a practical basis, when β_N and β_I are treated as variables, experience has shown that any effort to determine both values of R_E and R_B in (15), via data obtained in the forward bias condition proved to be unsuccessful. This is due to the fact that a locus of minima is found rather than a unique one. Similar comments can be made concerning the resistances R_C and R_B in (16). It is for this reason that only three parameters are determined from the forward active characteristics and three from the inverse mode of operation. Three measurements, $I_{C_{\text{sat}}}$, $I_{B_{\text{sat}}}$, and $V_{C_{\text{sat}}}$, are taken in the saturation region to determine the last remaining parameter. This also has the effect of improving the performance of the model operating there. These data are substituted into (10) and (13), which are rearranged as:

$$V_{C_{\text{sat}}} = -R_E I_{E_{\text{sat}}} + R_C I_{C_{\text{sat}}} + \frac{1}{\theta_N}\ln\left(\frac{I_N}{I_{ES}}+1\right) - \frac{1}{\theta_I}\ln\left(\frac{I_I}{I_{CS}}+1\right). \quad (17)$$

Combining two three-parameter searches with (17) produces a sufficient number of relations to uniquely determine the seven parameters. Since no explicit formula can be given to compute them, a simple iteration routine is employed instead.

The initial values for the three-parameters searches are obtained by applying Sokal's diode method to the (12) and (14). Data are also collected just as if two separate diode problems at hand. A guess for one of the resistance parameters in (15) or (16) must be given at the start however. The logical candidate R_B, which is common to both, is initially set to zero.

The entire procedure therefore consists of the following steps:

$T0$: Set R_B to zero

$T1$: θ_N, I_{ES}, R_E are computed by the three parameter search algorithm operating on the forward active characteristic data.

$T2$: θ_I, I_{CS}, R_C are computed by the three parameter search algorithm operating on the inverse active characteristic data.

$T3$: Recompute the values of R_E, R_B, R_C to satisfy (17).

$T4$: Stop if convergence is met; otherwise go to step $T1$.

At $T3$ the updating of the unknown parameters R_E, R_B, R_C to new values R'_E, R'_B, R'_C takes place. All the other parameters remain constant at this point. However by rewriting (17) as

$$-R'_E I_{E\text{sat}} + R'_C I_{C\text{sat}} = V_{C\text{sat}} - \frac{1}{\theta_N}\ln\left(\frac{I_N}{I_{ES}}+1\right) + \frac{1}{\theta_I}\ln\left(\frac{I_I}{I_{CS}}+1\right) \quad (18)$$

it can be seen that a non unique set of values of R'_E and R'_C can satisfy the equation. This dilemma can be eliminated by ensuring that the previous values of $R_{N\text{eq}}$ and $R_{I\text{eq}}$ given by (15) and (16), are maintained. Thus

$$R'_E\frac{\beta_N}{\beta_N+1} + \frac{R'_B}{\beta_N} = R_E\frac{\beta_N}{\beta_N+1} + \frac{R_B}{\beta_N} \quad (19)$$

$$R'_B\frac{1}{\beta_I} + R'_C\frac{\beta_I}{\beta_I+1} = R_B\frac{1}{\beta_I} + R_C\frac{\beta_I}{\beta_I+1}. \quad (20)$$

Equations (18), (19), and (20) form a system of three linear equations in three unknowns R'_E, R'_B, and R'_C, which is easily solved by Kramer's method. The procedure then proceeds to step $T4$ to check convergence.

As a computational detail, it is observed in (18) that the values of I_N and I_I must be known before the equations can be solved. These values can be obtained by setting the external currents $I_{E_{sat}}$ and $I_{C_{sat}}$ in the equations (see Table 1),

$$I_E = -(1/\beta_N + 1)I_N + I_I \tag{21}$$

$$I_C = I_N - (1/\beta_I + 1)I_I \tag{22}$$

and solving for I_I and I_N. That is,

$$I_I = \frac{I_{E_{sat}} + (1 + 1/\beta_N)I_{C_{sat}}}{1 - (1 + 1/\beta_I)(1 + 1/\beta_N)} \tag{23}$$

$$I_N = \frac{I_{C_{sat}} + (1 + 1/\beta_I)I_{E_{sat}}}{1 - (1 + 1/\beta_I)(1 + 1/\beta_N)}. \tag{24}$$

Note that

$$\beta_N = \beta_N(I_N) \tag{25}$$

$$\beta_I = \beta_I(I_I) \tag{26}$$

in (23) and (24) are functions of I_N and I_I and are known only experimentally. Values for I_N, I_I, β_N, and β_I can be determined by iteratively solving (23)–(26). Note that a linear interpolation scheme is required to represent (25) and (26).

2.2 *Experimental results and model modification*

Experiments were conducted to compare the new method with Sokal's. The Fairchild FD77 diode and a Fairchild 2N3009 transistor were employed in both sets of experiments. The results are shown in Table 2.

The average error for the diode case, shown in the last column was computed using (8). The maximum deviation from the average is tabulated for each parameter and appears in the last row. Note that the average error is reduced by an order of magnitude with the new method and that the maximum deviation is 16–30 times smaller than the corresponding deviations obtained by Sokal's method.

Observation of the values calculated by Sokal's transistor method reveals a wide dispersion, thereby indicating a large inconsistency of parameter values. With the new method, the results are contained within an acceptable margin (10 per cent at worst but usually much less in general).

In order to fully test the reliability of the method, different data of the active characteristics were employed first; then this data was kept constant while the saturation characteristics were measured at different points. As can be seen by the two sets of results at the bottom of Table 2, the dispersion of parameter values obtained is reasonable in both instances. Moreover, the average error is, likewise, consistently small.

The value obtained for R_B in Table 2 is negative. This should not be surprising since the terminal resistances of the model constitute a mathematical rather than a physical analog. In fact such a value ensures a first order correction on the exponential characteristics of the model, and accounts not only for the extrinsic resistance of the device regions, but also for all the complex phenomena which occurs under high current rates. Moreover, notice that the equivalent impendance which is seen at the terminals [see (15) and (16)] remains positive under normal operation conditions. When the particular configuration in which either the collector or the emitter is left open, an equivalent negative resistance is seen at the terminals, and the validity of the derived mode under these conditions becomes questionable. This means the model is reliable only when a sound method for determining its parameter values from measurements covering the intended region of operations of the device is used. The results of Table 2 which are based upon the device being driven from low to high current rates, indicate that, with the above mentioned limitations, the CIRCUS model can provide excellent analogs for large signal semiconductor devices.

Consistent results of the new method have also shown that for the diode, the calculated value of I_s is much larger than the reverse leakage current I_0 measured directly from the reverse characteristics (see Table 1). Whenever I_s is forced to equal I_0 a deterioration of model performance in the active region occurs. For this reason, it is proposed by the authors that two different values of I_s be employed, one for the negative function voltage, the other for positive function voltage.

This small change can easily be implemented in most prestored models of the currently available computer circuit analysis programs. If such a modification can not be reasonably done, an alternate

Table 2. Comparative results

(a) *Results obtained for the diode case using Sokal method*

Exper.	θ	I_S (A)	R (Ω)	Av error (V)
I	16.4	13.0E-9	2.49	0.227E-1
II	16.4	11.0E-9	2.51	0.444E-1
III	16.4	8.92E-9	2.49	0.282E-1
IV	18.3	3.59E-9	2.9	0.168E-1
Maximum deviation from average	9%	65%	12%	

(b) *Results obtained for the same diode using the optimization method*

N(1)	θ	I_S (nA)	R (Ω)	Av error (V)
13	16.65	7.358	2.527	0.328E-2
19	16.65	7.505	2.534	0.302E-2
25	16.62	7.690	2.541	0.369E-2
28	16.64	7.774	2.544	0.378E-2
34	16.57	7.844	2.515	0.316E-2
	0.17%	3%	0.6%	

(c) *Results obtained for the transistor case using Sokal method*

Exper.	θ_N	θ_I	I_{SN}	I_{SI}	R_E	R_B	R_C
I	26.7	16.7	2.69E-11	2.13E-8	0.329	3.02	0.981
II	19.2	20.0	4.18E-7	3.34E-7	1.0E-3	0.199	2.16
III	23.6	18.1	3.08E-10	8.74E-9	0.345	2.08	0.801
IV	25.5	17.6	5.73E-11	1.208E-8	0.403	1.19	0.75

(d) *Results for the same transistor for different active characteristic measurements, using the proposed method*

N (1)	I_{csat}	$\frac{I_{csat}}{I_{bsat}}$	θ_N	θ_I	I_{SN}	I_{SI}	R_E	R_B	R_C	Av error fwd char.	Av error rev char.
13	0.04	10	37.02	26.18	0.263E-13	0.319E-9	0.481	−2.031	0.754	0.886E-3	0.559E-2
16	0.04	10	36.80	26.69	0.256E-13	0.208E-9	0.468	−1.834	0.713	0.949E-3	0.666E-2
19	0.04	10	36.43	26.69	0.289E-13	0.179E-9	0.506	−1.718	0.681	0.136E-2	0.282E-2
Maximum deviation from average			0.7%	1.3%	2.5%	3.5%	4.3%	10%	5%		

(e) *Results for the same transistor for different saturated region measurements, using the proposed method*

N(1)	I_{csat}	$\frac{I_{csat}}{I_{bsat}}$	θ_N	θ_I	I_{SN}	I_{SI}	R_E	R_B	R_C	Av error fwd char.	Av error rev char.
13	0.04	5	37.02	26.1831	0.263E-13	0.3196E-9	0.480	−1.965	0.735	0.889E-3	0.559E-2
13	0.04	10	37.02	26.1831	0.263E-13	0.3196E-9	0.481	−2.031	0.754	0.886E-3	0.559E-2
13	0.10	5	37.05	26.1864	0.259E-13	0.3191E-9	0.480	−1.795	0.689	0.913E-3	0.558E-2
13	0.10	10	37.05	26.180	0.259E-13	0.319E-9	0.479	−1.746	0.674	0.916E-3	0.558E-2
Maximum deviation from average			0.05%	0.1%	1%	0.03%	0.2%	7%	3%		

1N is the number of points of the characteristics which were measured.

method making use of the original model, a dependent current source, and a switch will produce the same effect [8].

3. SUMMARY AND CONCLUSIONS

The new method provides a simple, inexpensive (about 30 sec. CPU on a Burroughs B5500 computer) on-line procedure to determine static parameters for the CIRCUS diode and transistor class of models. Particular features of the method are:

(a) The collection of experimental data and initialization of parameters in the search procedure have been automated.

(b) The single parameter search algorithm in conjunction with the separation technique is probably one of the most economical as well as most convenient method for handling such problems.

One particular disadvantage to the new method is that it is not applicable to the more complex models such as Gummel–Poon [13]. For these cases, the more sophisticated optimization methods are recommended.

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APPENDIX

The single parameter search algorithm for up to three parameters can be summarized using the notation of Knuth [13]. The following program variables are used:

X_j $j = 1, 2, 3$ — search variables (corresponding to θ, I_s and R respectively in the diode case).

X_{0j} $j = 1, 2, 3$ — variables storing the parameter values at the local minimum.

ΔX_j $j = 1, 2, 3$ — the initial step sizes, specified by the user as a percentage of parameter values.

$DIFF$ — the increment in the search step.

$FLAG$ — a flag indicating that a change in the search direction occurred in last operation.

E_1, E_2 — error values computed according to (8).

K — a parameter directly proportional to the precision desired at the local minimum.

$CONTRL$ — a parameter that takes on a value of three if the search algorithm reaches a minimum.

A single-search, three parameter algorithm

$S0$: Initialize X_j, $(j = 1, 2, 3)$, and set $j \leftarrow 1$, and $CONTRL \leftarrow 0$.

$S1$: Set $DIFF \leftarrow \Delta X_j$, $X0_j \leftarrow X_j$, $FLAG \leftarrow 0$; Compute E_1.

$S2$: Set $X_j \leftarrow X_j + DIFF$.

$S3$: Compute E_2; if $E_2 < E_1$ then set $FLAG \leftarrow 0, E_1 \leftarrow E_2$ and go to S_2, otherwise set $X_j \leftarrow X_j - DIFF$.

$S4$: If $FLAG = 0$ then set $DIFF \leftarrow -DIFF$, $FLAG \leftarrow 1$, go to $S2$, otherwise go to the next step.

$S5$: If $|DIFF| < |\Delta X_j/K|$ go to $S6$, otherwise set $DIFF \leftarrow DIFF/4$, $FLAG \leftarrow 0$, and go to $S2$.

$S6$: If $X_j \neq X0_j$ then set $CONTRL \leftarrow 0$, otherwise $CONTRL \leftarrow CONTRL + 1$.

$S7$: If $CONTRL = 3$ then go to $S8$, otherwise set $j \leftarrow j + 1$ mod $\equiv 3$, and go to $S1$.

$S8$: End of the search.