A Formal Approach to the Definition and the Design of Conceptual Schemata for Database Systems

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A formal approach is proposed to the definition and the design of conceptual database diagrams to be used as conceptual schemata in a system featuring a multilevel schema architecture, and as an aid for the design of other forms of schemata. We consider E-R (entity-relationship) diagrams, and we introduce a new representation called *CAZ-graphs*. A rigorous connection is established between these diagrams and some formal constraints used to describe relationships in the framework of the relational data model. These include functional and multivalued dependencies of database relations. The basis for our schemata is a combined representation for two fundamental structures underlying every relation: the first defined by its minimal atomic decompositions, the second by its elementary functional dependencies.

The interaction between these two structures is explored, and we show that, jointly, they can represent a wide spectrum of database relationships, of which the well-known one-to-one, one-to-many, and many-to-many associations constitute only a small subset. It is suggested that a main objective in conceptual schema design is to ensure a complete representation of these two structures. A procedure is presented to design schemata which obtain this objective while eliminating redundancy. A simple correspondence between the topological properties of these schemata and the structure of multivalued dependencies of the original relation is established. Various applications are discussed and a number of illustrative examples are given.

Categories and Subject Descriptors: H.2.1 [**Database Management**]: Logical Design—*data models; schema and subschema*

General Terms: Design

1. INTRODUCTION

Conceptual schemata in database management systems (DBMSs) are expected to yield benefits in three domains:

(a) *Data Independence.* A three schema level architecture, such as the one proposed by the ANSI/X3/SPARC [31], provides a high degree of physical and logical data independence. In this architecture the conceptual schema occupies a central position between the multiple views seen by the users

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(external schemata) and the internal schema which describes the physical organization of the database. The conceptual schema establishes the logical and functional connections between the internal and external levels.

(b) *Design Aid.* In a top-down methodology a conceptual schema supplies a logical representation from which a physical implementation is designed to maximize performance on typical queries and/or updates.

(c) *Liaison to the Enterprise World.* A conceptual model, also called an enterprise schema, is often advocated as a means for successful communication between the data processing department and the rest of the enterprise.

In order to realize the benefits just considered, a conceptual schema must describe the intrinsic logical relationships among external data independent of the underlying implementation (by external data we denote information of interest to the enterprise as opposed to data introduced by the data processing department for the purpose of storing and processing this information). To obtain this objective, a conceptual schema must satisfy two requirements. The first is that the schema must be capable of characterizing completely and unambiguously the various relationships among external data; following GUIDE and SHARE [19], we refer to this as the *complete relatability requirement.* The second is that the schema must be expressive and concise.

The second requirement is most naturally met by using graphical representations and diagrams. Early forms of graphical schemata such as hierarchies and networks are generally considered not suited for a conceptual representation since they combine information about logical structure of data with access path description, leaving some ambiguity as to the exact nature of the former. Thus new forms of graphical schemata have been proposed to express the logical structure of external data in an implementation-independent fashion. Among the many which were proposed are those presented in [9, 25, 27]. Of these, we only discuss E-R (entity-relationship) diagrams [9], which have gained considerable acceptance in the field.

A common thrust of these works consists in analyzing and characterizing by means of examples the various relationships commonly encountered in a DBMS, and in proposing graphical representations for it. The designer who is cognizant of the semantics of his data is then expected to recognize similar relationships for the database at hand and produce a schema which reflects his understanding of the structure of his data. While these approaches have contributed to the understanding of data semantics, they have failed to achieve the degree of definiteness and mathematical rigor which by contrast is available in the framework of the relational approach to database systems. This paper presents a rigorous approach to the definition and the design of graphical schemata in the framework of the relational data model. This approach uses and applies various concepts and analytical tools which were presented in a companion paper [33].

2. BASIC CONCEPTS

We assume that the reader is familiar with the most common concepts pertaining to relational database theory, including the definition of relations, projections,

joins, functional dependencies, and multivalued dependencies. Date's book [13] provides the reader with an introduction to these topics, whereas Beeri et al. [5] supply a sophisticate's review and a list of more advanced readings. The notation and terminology used here is the same as that used in [33]. Thus, if $R(\Omega)$ is a relation with attribute set Ω, and Γ and Δ are two subsets of Ω, we write $\Gamma \rightarrow \Delta$ or $\Gamma \twoheadrightarrow \Delta$ to denote, respectively, that Δ is functionally dependent (FD) on Γ or that Δ is multivalued dependent (MD) or Γ. We may write $\Gamma \not\rightarrow \Delta$ or $\Gamma \not\twoheadrightarrow \Delta$ to denote that those dependencies do not hold. If the FD "$f: \Gamma \rightarrow \Delta$" holds in R, then the MD "$g: \Gamma \twoheadrightarrow \Delta$" also holds in R; g will be called the MD *counterpart* of f. The Δ-projection of $R(\Omega)(\Delta \subseteq \Omega)$ will be denoted $\Pi R(\Delta)$. The natural join of relations $R(\Omega)$ and $S(\Delta)$ is denoted $R(\Omega) \cdot S(\Delta)$. When Ω and Δ are disjoint (i.e., their intersection is empty) this natural join reduces to the Cartesian product $R(\Omega) \times S(\Delta)$. Natural joins are commutative and associative. Thus we can refer to the natural join of a set of two or more relations without specifying the order in which this join is computed. By convention, R is the natural join of the singleton set $\{R\}$.

2.1 The Decomposition Approach

We follow the decomposition approach to relational schema design [11, 15, 33]. In this approach, the design is developed through a sequence of successive refinement steps as follows:

(i) Generate a first gross description of the database as a set of relations.
(ii) Derive the set of functional and multivalued dependencies which hold for these relations at every instant in time.
(iii) Use these dependencies to recast the initial set of relations into the desired schema.

This design discipline is certainly laborious, but it deserves our attention because it is rigorous and it improves our understanding of the structure of conceptual schemata.

For concreteness, let us consider an example. Assume that we need some information regarding departments of a given organization. More precisely, we are interested in the following relationships:

(1) the employees working in a department (an employee works in only one department);
(2) the salary of each employee (an employee only has one salary);
(3) the name of the department manager (a department only has one manager);
(4) the location of a department (we will assume that a department can only have one location).

As the first step, we can represent this information in a single table, say DEPT, with the following attributes:

DN: unique names identifying department;
SAL: annual salaries in dollars;
E#: unique numbers identifying employees;

MGN: names of managers;
LOC: geographical locations (at which departments are located).

At this point, the designer should consider typical samples of his database content. In our case, for instance, we could have

DEPT	(DN,	E#,	SAL,	MGN,	LOC)	
	SHOES	4441	20K	SMITH	LA	
	SHOES	3321	25K	SMITH	LA	(2.1)
	TOYS	1121	30K	BROWN	SF	
	TOYS	7321	10K	BROWN	SF	

This materialization of a sample of database content will prompt the designer to recognize possible ambiguities or oversights in his previous statement of intension. In our example, for instance, the designer would be prompted to ask such questions as: "Can two departments have managers with the same name?" Thus he will be forced to resolve these ambiguities and perhaps to clarify or modify the initial statement of intension. Other basic issues will also be resolved. For instance, if the designer wants to represent two distinct locations, say, location of a department and location at which an employee lives, the obvious inadequacy of (2.1) to hold these two distinct pieces of information under the single column LOC will emerge. The designer may therefore be forced to revise his initial selection of relations. In the situation just discussed, he may either add one more column denoting locations to DEPT or he may add a completely new relation. Thus, this materialization ensures that the initial choice of relations is compatible with the stated intension. Thus an application of syntactically correct FD and MD rules produces semantically correct results within each relation. For a further discussion of this subtle but important problem, see [8, 15, 33].

After completing step (i), the designer will proceed with step (ii) and formalize the time-independent constraints on these relations using FDs and MDs.

This operation was discussed at some length in [33], where the concepts of *elementary FD* and *multiple elementary MD* were introduced. An FD of R is called elementary if it has the form $\Gamma \rightarrow A$, where $A \notin \Gamma$ and R contains no $\Gamma' \rightarrow A$ where $\Gamma' \subseteq \Gamma$. An MD of R, $\Gamma \twoheadrightarrow \Delta$, is called elementary if Δ is nonempty and disjoint from Γ and R does not contain another MD, say, $\Gamma' \twoheadrightarrow \Delta'$, where $\Gamma' \subseteq \Gamma$ and $\Delta' \subseteq \Delta$. Therefore, both elementary FDs and elementary MDs have minimum left side and minimum right side. The right side of an elementary FD always reduces to a single attribute, but that of an elementary MD contains one or more attributes. The MD counterpart of an elementary FD is elementary. An elementary MD of R is called *multiple* if R contains other elementary MDs with the same left side; otherwise, it is called *single*. Elementary FDs and multiple elementary MDs have a simple structure appealing to intuition and certain formal properties which may help the designer in carrying out step (ii) [33]. Moreover, only these dependencies are actually used by the schema design procedures discussed later in this paper. Thus, we assume that our designer performs step (ii) by simply listing the elementary FDs and the multiple elementary MDs which hold in a time-independent manner in the given relations. This discipline, applied

to relation (2.1), yields the following list:

$$\begin{aligned}
&\text{D1: } E\# \rightarrow \text{SAL}\\
&\text{D2: } E\# \rightarrow \text{DN}\\
&\text{D3: } E\# \rightarrow \text{MGN}\\
&\text{D4: } E\# \rightarrow \text{LOC}\\
&\text{D5: DN} \rightarrow \text{MGN}\\
&\text{D6: DN} \rightarrow \text{LOC}\\
&\text{D7: DN} \rightarrow\!\!\!\!\rightarrow \{E\#, \text{SAL}\}
\end{aligned} \tag{2.2}$$

The multiple elementary MDs for a relation are given as follows: in general, we list the elementary FDs and the multiple elementary MDs of a relation. For simplicity, however, we do not actually list the MD counterpart of an elementary FD. Thus, in (2.2), the MD counterpart of D1 is not explicitly given. However, since one knows that D1 is an elementary FD, he also knows that its MD counterpart "$E\# \rightarrow\!\!\!\!\rightarrow$ SAL" is elementary. Some other elementary dependencies with left-hand side E# are also listed in (2.2); thus one knows that this elementary MD is also multiple. Each FD (MD) of a relation can be constructed from its elementary FDs (MDs) using the rules of reflexivity, additivity, and augmentation [33].

At the end of step (ii), a set of relations is obtained, each defined by its attribute set and by its elementary FDs and multiple elementary MDs. Next, the designer will proceed with step (iii) as described in the following sections. We will always use the term "relation" in the sense of "description of a relation." To refer to the extension of a relation, we use terms such as "content of a relation" or "instance of a relation."

2.2 Schema Design

The approach to relational schema definition proposed by Codd is based on the concept of relational keys and normal forms [11]. A key for a relation is an attribute combination which nonredundantly identifies each row of the relation (i.e., Γ is a key for $R(\Omega)$ iff $\Gamma \rightarrow \Omega$ and for each $\Lambda \subseteq \Gamma$, $\Lambda \not\rightarrow \Omega$). Every relation contains one or more keys which are customarily identified by suitable underlining of the attributes comprising these keys. Thus, a relational schema in the sense of [11] and [8] consists of a set of relations and for each relation the specification of one or more keys. If Γ is a key for R, and A an attribute of R not in Γ, then the FD "$f: \Gamma \rightarrow A$" must hold in R. Following [8], we will say that f is embodied in R. Thus keys can be regarded as a device to specify the existence of certain FDs in a relation. The set of FDs specified consists of those directly embodied plus those derived from them by the FD inference rules [6].

However, keys are not always sufficient to characterize completely the dependencies of a relation. For our example DEPT, for instance, there is only one key: {E#}. Thus, D1, D2, D3, and D4 from set (2.2) are embodied in this schema. D5, D6, and D7, however, are neither embodied nor derivable from the FDs embodied using the FD inference rules. Thus, a first problem with this schema is that it

fails the complete relatability requirement. A second problem is the presence of the update anomalies discussed in [11].

A solution to both previous problems is supplied by a transformation called *decomposition*. A relation $R(\Omega)$ will be said to be decomposable into the pair of projections $\Pi R(\Omega_1)$ and $\Pi R(\Omega_2)$ when for every instance $\bar{R}(\Omega)$ of $R(\Omega)$ the following is true:

$$\bar{R}(\Omega) = \Pi\bar{R}(\Omega_1) \cdot \Pi\bar{R}(\Omega_2).$$

The following well-known property holds.

PROPOSITION 2.1. $\Gamma \twoheadrightarrow \Delta$ *in* $R(\Omega)$ *iff* $R(\Omega)$ *is decomposable into the pair* $\Pi R(\Omega_1)$ *and* $R(\Omega_2)$, *where* $\Omega_1 = \Gamma \cup \Delta$ *and* $\Omega_2 = \Gamma \cup (\Omega - \Delta)$.

The MD "$\Gamma \twoheadrightarrow \Delta$" is called *trivial* when $\Omega_1 = \Omega$ or $\Omega_2 = \Omega$.

Thus, the presence of a nontrivial MD is sufficient and necessary for a relation to be decomposable into a pair of proper subprojections, without losing information on the content of the relation. A relation containing only trivial MDs is called *atomic*. It immediately follows that a relation is atomic if, and only if, it contains no multiple elementary MD. When a relation $R(\Omega)$ containing an MD "g: $\Gamma \twoheadrightarrow \Delta$" is replaced by the pair of relations $\Pi R(\Omega_1)$ and $\Pi R(\Omega_2)$, where $\Omega_1 = \Gamma \cup \Lambda$ and $\Omega_2 = \Gamma \cup (\Omega - \Delta)$, then we will say that R was decomposed according to g. DEPT, for instance, is decomposable according to DN $\rightarrow$ {MGN, LOC} (or, more precisely, according to its MD counterpart).

As a result of such decomposition, we obtain

ΠDEPT	(E#,	DN,	SAL)	
	4441	SHOES	20K	
	3321	SHOES	25K	(2.3)
	1121	TOYS	30K	
	7321	TOYS	10K	

ΠDEPT	(DN,	MGN,	LOC)	
	SHOES	SMITH	LA	(2.4)
	TOYS	BROWN	SF	

The natural join of (2.3) and (2.4) returns the original (2.1). This is true not only for the particular instance shown above, but most important, it holds for every instance of DEPT as well, since DN $\twoheadrightarrow$ {MGN, LOC} expresses a time-independent constraint. Thus, this decomposition ensures preservation of content. It also implements the principle of complete relatability since the integrity constraints and the relationships of interest are now completely represented by schema (2.3), (2.4). This property is demonstrated by the fact that the various dependencies in (2.2) are either embodied in the schema or derived from these using the inference rules for FDs and MDs [6]. Thus, we will say that the above decomposition has preserved the dependency structure of the original relation. To illustrate the importance of structure preservation in schema design, one

could consider decomposing DEPT according to E# → {DN, SAL}. The pair

$$\Pi\text{DEPT}(\underline{\text{E\#}}, \text{DN}, \text{SAL}) \qquad \Pi\text{DEPT}(\underline{\text{E\#}}, \text{MGN}, \text{LOC})$$

so obtained preserves the content of the original DEPT but not its structure (e.g., DN → LOC is not inferable from this second schema). As a result, this schema does not correctly define the relationships of interest, and it is affected by obvious update anomalies.[1] Therefore, preservation of both content and structure is needed for a successful schema design through decomposition. The fundamental role that preservation of content and structure plays in the design of normal form schemata is also discussed in [5], where it is viewed as a realization of the representation principle.

In this paper, we present a decomposition approach to schema design whose objective is the preservation of both content and structure with minimum redundancy. However, the style of the schemata discussed here is not the usual one first proposed in [11] (third normal form) and later refined in [12] (Boyce-Codd normal form) and [14] (fourth normal form). The normal form approach relies on the concept of key; it also assumes that unnecessary proliferation of relations should be avoided to eliminate complexity and possibly redundancy and to improve the conciseness and the intuitive appeal of a schema. For these purposes concepts such as optimal third normal form [11] and minimum relational count [8] were introduced. In our example, for instance, further decomposition of (2.3) and (2.4) will not cause any loss of content or structure, nor will it introduce update anomalies. Yet in the usual approach, this decomposition will not take place, since (2.3), (2.4) already constitute a "good" schema; thus one should not decompose any further.

In this paper, we discuss an approach to schema definition and design where decomposition is pushed to the limit, that is, to the point at which atomic relations are obtained. Moreover, we do not use keys to define FDs. Instead, we combine the representation of atomic subprojections with the representation of elementary FDs into a graphical form which is both concise and expressive.

The desirability of using schemata built upon semantic units which are as small as possible has inspired a number of previous works [16, 18, 24] (just to mention a few). A main motivation for our research has been the awareness that the database intension evolves and changes with time. A DBMS is expected to absorb and minimize the effects of changes (i.e., the need for conversion and translation). Now atomic relations seem less sensitive to mutation in the intension than normal form relations. With respect to the example proposed, for instance, decomposition of (2.3) and (2.4) into atomic subrelations yields

$$\begin{aligned} &1.\ \Pi\text{DEPT} \quad (\text{E\#}, \text{DN}) \\ &2.\ \Pi\text{DEPT} \quad (\text{E\#}, \text{SAL}) \\ &3.\ \Pi\text{DEPT} \quad (\text{DN}, \text{MGN}) \\ &4.\ \Pi\text{DEPT} \quad (\text{DN}, \text{LOC}). \end{aligned} \tag{2.5}$$

[1] These notions also underlie the concept of "independent components" of a relation [24].

Now, assume that the intension of our relation DEPT changes such that more than one location (LOC) can be associated with a given department. Then the MD "DN $\rightarrow\!\!\!\rightarrow$ LOC" will hold in (2.4), which is not third normal form any longer and must be decomposed. Thus, a relationship changing from many-to-one into many-to-many may force a modification in normal-form schemata. By contrast, DEPT can be decomposed into (2.5), with complete preservation of content and structure, even after the previous change has taken place.

3. ELEMENTARY DEPENDENCIES AND DECOMPOSITION

In this section, we define some basic concepts and lay the technical foundation for the development of the following sections. Specifically, we introduce the notion of atomic decomposition and atomic component of a relation. Then, we explore the useful relationships between the structure of a decomposition and the MD structure of the original relation.

3.1 Atomic Decompositions

The decomposition process is recursive in nature. Once a relation is decomposed into two subprojections, these will have to be decomposed in turn according to some valid MD. One concise way to summarize the decomposition of a relation is to use an unordered binary tree. For instance, the tree of Figure 1 denotes a decomposition leading to the set of atomic subprojections (2.5). The internal nodes of the tree denote a decomposition step and are labeled by the dependency used in the decomposition. The bottom nodes of the tree denote the atomic subrelations obtained as the end result of the decomposition.

To each internal node of a tree there corresponds a unique projection to the given relation. The attribute set of this projection is defined as the union of the attribute set of its two successors. Thus the root of the tree corresponds to the initial relation. Each internal node is labeled by the MD holding in the corresponding projection and used for its decomposition. In Figure 1, for instance, the first decomposition step takes place according to D7.

Two projections result from this first step. One, with attribute set {E#, DN, SAL}, is further decomposed according to E# $\rightarrow$ DN. The other, with attribute set {DN, LOC, MGN}, is decomposed using, say, DN $\rightarrow$ MGN. In general, if such a binary tree has n internal nodes (decomposition steps) it has $n + 1$ terminal nodes (atomic projections). After dealing with a few examples, one realizes that usually there are many decomposition trees leading to the same set of projections, even when only multiple elementary MDs are used. Figure 2, for instance, shows a second decomposition producing the same end result.

Therefore, we need a means to characterize succinctly and uniquely the end product of a decomposition independent of the particular decomposition sequence which led to it. For this purpose we now introduce the concept of minimal atomic decomposition. If Σ denotes a set of projections of a relation $R(\Omega)$, then

(i) Σ is called a *decomposition* of $R(\Omega)$ if for *every instance* of $R(\Omega)$ the natural join of the projections in Σ is equal to this instance of $R(\Omega)$.
(ii) A decomposition Σ of $R(\Omega)$ is called *minimal* when no proper subset of Σ is a decomposition of $R(\Omega)$.

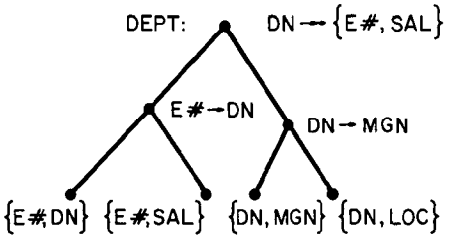


Fig. 1. A first decompositon for DEPT.

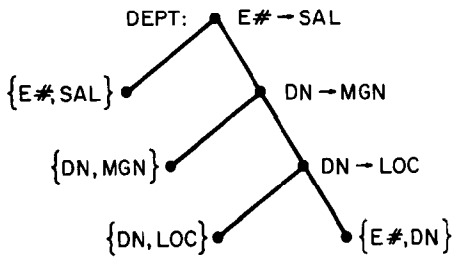


Fig. 2. A second decomposition for DEPT.

(iii) A decomposition of Σ is called *atomic* if every member of Σ is an atomic relation.

Thus a *minimal atomic decomposition* is one which satisfies all three of the previous properties; that is, it is a set of projections which have a well-defined structure (point iii) and which also preserve the content information of the original relation (point i) in a nonredundant way (point ii). If Σ is a decomposition of a given relation, then it has the lossless join property discussed in [1]. An algorithm to decide whether a given set of projections has this property is described in the referenced work.

Every element of a minimal atomic decomposition of R is called an *atomic component* of R. Atomic components can be regarded as the minimal independent granules by which data can be stored or represented. In a logical view of data, two atomic components could be treated separately and represented as two separate view relations, or they could be represented jointly in one relation. Also, two atomic components can be stored separately in two distinct record types, or they can be joined and stored as one record type. In no case, however, can the designer further refine atomic components into smaller granules which he can treat as self-contained objects.

In a multilevel schema architecture [31] the data representation chosen and the design criteria used must be a function of the schema level considered. This paper focuses on the problem of designing a *conceptual schema*, built upon atomic components, to ensure complete relatability with minimum redundancy. At the internal level, these atomic relations may then be joined together and rearranged, under various storage organizations, to ensure, say, a better performance on typical transactions. Likewise, at the external level, the end-user view of data may also be different and, perhaps, consists of normalized relations. The problem of designing internal and external schemata in this context, which has been the subject of previous research [20, 32], is outside the scope of this paper.

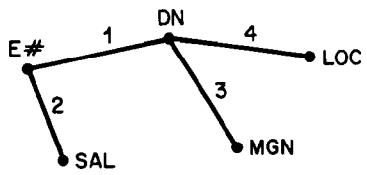


Fig. 3. A graph representation for an atomic decomposition of DEPT.

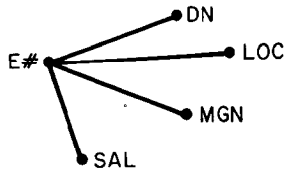


Fig. 4. A second atomic decomposition for DEPT.

3.2 Structure and Properties of Decompositions

A set of binary relations is most naturally represented by an undirected graph. Figure 3, for instance, represents atomic decomposition (2.5) for our example DEPT.

The vertices of the graph correspond to the attributes of the original relation; the edges correspond to the relations produced by the decomposition. Each edge is denoted by a unique name, usually positive integers as in Figure 3. Actually, when the graph is simple (i.e., there is at the most one edge between any two vertices), then explicit names for the edges are not necessary and can be dropped (as, for example, in Figure 4).

This kind of graphical representation is very useful in visualizing the alternative decompositions of a relation. (The problem of choosing among alternative decompositions is discussed in Section 4.) Figure 4, for instance, describes an alternative atomic decomposition of our DEPT (obtainable using the first four FDs in (2.2).)

The main limitation of undirected graphs is that they can only represent binary relations. In databases we are interested in relations of assorted degree. Even atomic relations can have three or more attributes. For instance, consider the following example taken from [33]. We have a relation

$$\text{RANK (CT, C\#, SN, P)} \tag{3.1}$$

which describes the rank order of students at the completion of their courses. A student is identified by his name (SN). A course is identified by a title (CT) and also a number (C#); thus C#$\leftrightarrow$CT. The column P denotes the position obtained by a student in the course he completed. Assuming that two different students cannot have the same position in one course (no ties), we find that the elementary FDs and the multiple elementary MDs of this relation are

$$\begin{aligned}
&1.\ \text{CT} \rightarrow \text{C\#} \\
&2.\ \text{CT} \rightarrow\!\!\!\rightarrow \{\text{SN, P}\} \\
&3.\ \text{C\#} \rightarrow \text{CT} \\
&4.\ \text{C\#} \rightarrow\!\!\!\rightarrow \{\text{SN, P}\} \\
&5.\ \{\text{CT, SN}\} \rightarrow \text{P} \\
&6.\ \{\text{C\#, SN}\} \rightarrow \text{P} \\
&7.\ \{\text{CT, P}\} \rightarrow \text{SN} \\
&8.\ \{\text{C\#, P}\} \rightarrow \text{SN}
\end{aligned} \tag{3.2}$$

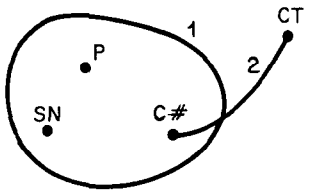


Fig. 5. The hypergraph of a first atomic decomposition of RANK.

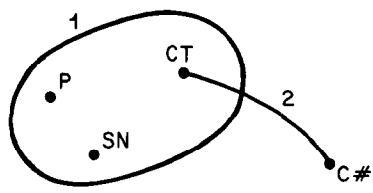


Fig. 6. The hypergraph of a second atomic decomposition of RANK.

Decomposing according to the multiple elementary MDs listed above, we obtain two atomic decompositions:

$$\text{I.} \begin{cases} 1.\ \text{RANK}(C\#, SN, P) \\ 2.\ \text{RANK}(CT, C\#) \end{cases} \qquad \text{II.} \begin{cases} 1.\ \text{RANK}(CT, SN, P) \\ 2.\ \text{RANK}(CT, C\#) \end{cases}$$

To represent decompositions such as these, where we have subrelations of degree higher than 2, we use the graphical conventions currently adopted to represent *hypergraphs*. Hypergraphs supply a natural generalization to the concept of undirected graphs. In this paper we review only a few very basic and very simple notions on hypergraphs. These notions will enable us to derive some important results on the structure of minimal atomic decompositions and its relationship to the MD structure of the original relation. The interested reader is referred to [7] for a more complete and general treatment of the properties of hypergraphs.

A hypergraph consists of vertices and edges just like an ordinary graph, but the edges of a hypergraph can contain one or more vertices. Edges with two vertices are represented by a line connecting these vertices just as in a conventional graph. An edge with three or more vertices is represented by a contour encircling its vertices. An edge with only one vertex is represented by a line which begins and ends at this vertex (a loop). The hypergraph of a set of relations is defined as one where each relation is represented by a distinct edge having as vertices the attributes of this relation. Thus the vertex set of the hypergraph of a set of relations Σ is, by definition, equal to the union of the attribute-sets of all relations in Σ. Figures 3 and 4 represent the hypergraph of two decompositions of relation DEPT (when all edges contain exactly two vertices, the representations of hypergraphs and conventional graphs coincide). Figures 5 and 6 show the hypergraphs of the two atomic decompositions of RANK previously given.

Next, we state two simple propositions, whose applications are discussed later.

PROPOSITION 3.1. *The vertex set of the hypergraph of a decomposition of* $R(\Omega)$ *is* Ω.

Indeed, if we have a set of projections of $R(\Omega)$ such that the union of their attribute set is $\Omega' \subset \Omega$, then by taking their natural join we would obtain a relation $R'(\Omega')$ which is not equal to $R(\Omega)$.

In a hypergraph an edge is said to be *minimal* if it does not contain any other edge. In the hypergraph of a decomposition Σ, an edge contained in another edge denotes that a member of Σ is a subprojection of another member in Σ. Thus, we have the following proposition.

PROPOSITION 3.2. *The hypergraph of a minimal decomposition of a relation contains only minimal edges.*

We next discuss the very useful notion of connectivity for hypergraphs. Two edges of a hypergraph are said to be *adjacent* if they have one or more vertices in common. The *connectivity relationship* between the edges of a hypergraph is defined as follows:

1. An edge is connected to itself.
2. Two adjacent edges are connected.
3. Two edges connected to a common edge are connected.

Thus connectivity is an equivalence relation which partitions the original hypergraph into groups of connected edges; each group is called a *connected component* of the hypergraph. Two vertices of a hypergraph are connected if they belong to the same connected component of the hypergraph. A hypergraph which consists of one connected component is said to be connected. The hypergraphs considered in Figures 3–6 are connected. We have the following property:

PROPOSITION 3.3. *If the hypergraph of a decomposition of* $R(\Omega)$ *consists of* r *connected components with vertex sets* $\Omega_1, \ldots, \Omega_r$, *then* $R(\Omega) = \Pi R(\Omega_1) \times \cdots \times \Pi R(\Omega_r)$.

Indeed, one can construct $R(\Omega)$ by joining the subrelations in each connected component first, and then take the Cartesian product of these joins.

Let H be a hypergraph with vertex set Ω. Given $\Gamma \subset \Omega$ we construct H' from H, as follows:

1. Remove every edge whose vertices all belong to Γ.
2. Replace every other edge i with vertices Δ_i by an edge i with vertices Δ_i', where $\Delta_i' = \Delta_i - \Gamma$.

The hypergraph H' so constructed will be called the *subhypergraph of H after the removal of* Γ.

We can now prove an important theorem.

PROPOSITION 3.4. *Let H be the hypergraph of a decomposition of* $R(\Omega)$ *and* H_1 *be any connected component of the subhypergraph of H after the removal of* Γ, *where* $\Gamma \subset \Omega$. *If* Δ_1 *denotes the vertex set of* H_1, *then* $\Gamma \rightarrow\!\!\!\rightarrow \Delta_1$ *in* $R(\Omega)$.

PROOF. The proposition is trivially true if H_1 is the only connected component of our subhypergraph. Otherwise, let Σ be a decomposition of $R(\Omega)$. Let Σ_1 be the subset of Σ corresponding to the edges of H_1 (i.e., the set of those $\Pi R(\Delta_i)$ of Σ such that $\Delta_i' = \Delta_i - \Gamma$ is an edge of H_1). If $R_1(\Omega_1)$ and $R_2(\Omega_2)$ denote the natural join of the projections in Σ_1 and in $\Sigma_2 = \Sigma - \Sigma_1$, respectively, then $R_1(\Omega_1) \cdot R_2(\Omega_2) = R(\Omega)$. Thus $(\Omega_1 \cap \Omega_2) \rightarrow\!\!\!\rightarrow \Omega_1$ in $R(\Omega)$. Clearly, Δ_1 and Γ are disjoint and $\Delta_1 \subseteq \Omega_1$. Moreover, for every $\Pi R(\Delta_i)$ in Σ_1, $\Delta_i \subseteq \Delta_1 \cup \Gamma$. Thus $\Omega_1 \subseteq \Delta_1 \cup \Gamma$. Thus $\Omega_1 - \Gamma = \Delta_1$. Also, every $\Pi R(\Delta_j)$ in Σ_2 is disjoint from Δ_1. Thus Ω_2 and Δ_1 are disjoint. Thus $\Omega_1 \cap \Omega_2 \subseteq \Gamma$. Therefore, since $(\Omega_1 \cap \Omega_2) \rightarrow\!\!\!\rightarrow \Omega_1$, then $\Gamma \rightarrow\!\!\!\rightarrow \Omega_1$. From this, $\Gamma \rightarrow\!\!\!\rightarrow (\Omega_1 - \Gamma)$. Q.E.D.

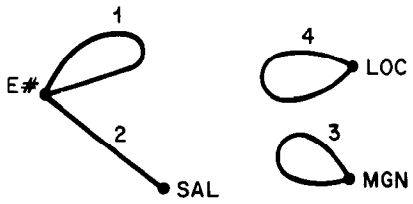


Fig. 7. The hypergraph of Figure 3 after removal of vertex DN.

When Γ is the empty set, Proposition 3.4 reduces to Proposition 3.3. Propositions 3.3 and 3.4 establish important relationships between the connectivity properties of a decomposition hypergraph of a relation and its MD structure.

If the hypergraph H with vertex set Ω has k connected components and the subhypergraph of H after the removal of Γ, where $\Gamma \subset \Omega$, has more than k connected components, then Γ is called an *articulation set* for H. If Γ contains only one vertex, this is called an *articulation vertex*.

Thus, by inspecting the articulation set of a decomposition hypergraph, one immediately infers that certain MDs hold in the original relation. Take Figure 3, for instance. The attribute DN is an articulation vertex whose removal breaks up the original hypergraph into three connected components, as shown in Figure 7. (As we described previously, loops are used to represent edges with only one vertex.) In Figure 7, we have three connected components with vertices {E#, SAL}, {LOC}, and {MGN}.

Thus by Proposition 3.4 the following MDs hold in DEPT: DN $\rightarrow\!\!\!\rightarrow$ {E#, SAL}, DN $\rightarrow\!\!\!\rightarrow$ LOC, DN $\rightarrow\!\!\!\rightarrow$ MGN.

Let us now consider the application of these formal results to the problem of determining the minimality of a decomposition. If Σ is a decomposition of $R(\Omega)$, then a projection $\Pi R(\Delta_i)$ in Σ will be called *superfluous* if $\Sigma - \{\Pi R(\Delta_i)\}$ is still a decomposition of $R(\Omega)$. An edge of a decomposition hypergraph will be called superfluous when it represents a superfluous subprojection. For instance, if we decompose RANK using first {CT, SN} $\rightarrow$ P and then C# $\rightarrow$ CT, we obtain the decomposition hypergraph

$$\{1: \{CT, SN, P\}, 2: \{C\#, CT\}, 3: \{C\#, SN\}\}.$$

Here edge 3 is superfluous, since edges 1 and 2 above already define a decomposition for RANK (see Figure 6). Thus a decomposition is minimal if it does not contain any superfluous subprojection (i.e., its hypergraph contains no superfluous edge). Now Proposition 3.1 prescribes that every edge of a decomposition hypergraph which contains some vertex not contained in any other edge cannot be superfluous. For example, in Figure 5, CT is only contained in edge 2; P is only contained in edge 1. Thus there is no superfluous edge, and we have a minimal atomic decomposition. By the same reasoning, Figure 6 also describes a minimal atomic decomposition for RANK.

Let us discuss the applications of Proposition 3.3. This states that given a relation R which is not the Cartesian product of two subprojections (i.e., there is no MD with an empty left side), then (1) the hypergraph of a decomposition of R must be connected, and (2) every edge whose removal will break the hypergraph in two or more connected components cannot be superfluous. For instance, the hypergraph of Figure 3 represents a decomposition for DEPT which is known to

contain no MD with an empty left side. Now edges 2, 3, and 4 each contain some vertex not contained by other edges; thus they are not superfluous. If we remove edge 1, the hypergraph is broken into two components; thus 1 cannot be superfluous. Figure 3 thus represents a minimal atomic decomposition of DEPT. Therefore one finds that each edge in Figures 3–6 represents an atomic component for its relation. In passing, we note that these atomic components will survive even after some many-to-one relationships change into many-to-many. In DEPT, for instance, we have previously observed that if a department is associated with more than one location, then the FD, "DN $\rightarrow$ LOC" will not be valid any longer. The MD "DN $\rightarrow\!\!\!\rightarrow$ LOC" will hold instead. Even then, Figure 3 defines a minimal atomic decomposition and ΠDEPT(DN, LOC) remains a valid atomic component for DEPT.

In conclusion, minimality of an atomic decomposition can usually be determined by a simple visual inspection of its decomposition hypergraph. This seems to be true even when more complex decompositions are involved, such as those which we describe in the next section. These will require application of Proposition 4.4.

Hypergraph models for a set of relations find other interesting applications, for example, in connection with the lossless join problem [32]. These are outside the scope of this paper.

4. SCHEMA DEFINITION AND DESIGN

We now present a novel approach to the definition and the design of graphical schemata for relational databases. In this approach we characterize the logical structure of the database in terms of elementary FDs and of atomic components of database relations. In conformity with the terminology used in [32] and [33], we refer to the atomic component structure and to the elementary FD structure of a relation as its A-structure and its Z-structure, respectively.

Our discussion will evolve as follows. In Section 4.1 it is shown that the combined definition of elementary FDs and atomic components provides an unambiguous characterization of a wide spectrum of logical relationships between database attributes. We also examine the rules which constrain the A-structure of a relation to its Z-structure. In Section 4.2 we present a simple and suggestive graph representation of the combined A- and Z-structure. In Section 4.3 we propose an algorithm to design the proper schema using the functional and multivalued dependencies of a relation. In Section 4.4 we discuss the various limitations of the proposed algorithm.

4.1 Atomic Components and Elementary FDs

We now examine the various configurations under which elementary FDs and atomic components combine and interact in a relation. Moreover, we show that by describing these configurations we can in fact characterize a wide spectrum of semantic relationships between database attributes, including the well-known one-to-one, one-to-many, and many-to-many archetypes. Instrumental in our developments is the concept of *scope* for an elementary FD [32]. If $\Gamma \rightarrow A$ is an elementary FD, then $\Gamma \cup \{A\}$ is said to be its scope. Thus the scope of an elementary FD includes all the attributes pertaining to this FD.

Let us now consider the relationships described by atomic components and the elementary FDs contained therein. Note that an elementary FD can be contained in an atomic relation only if it has as scope the whole attribute set of this atomic relation. In a relation R, the presence of a binary relationship between its two attributes A and B is denoted by the presence of an atomic component ΠR (A, B) and by the set of elementary FDs with scope $\{A, B\}$. If there are two FDs with scope $\{A, B\}$, namely, $A \rightarrow B$ and $B \rightarrow A$, then this is a one-to-one relationship, also called a one-to-one correspondence. If there is only one such FD, say, $A \rightarrow B$, then this is a many-to-one relationship from A to B. If no such FD exists, then this a many-to-many relationship. In our example RANK, for instance, we have a one-to-one correspondence between CT and C# (one course number per each course, and vice versa). Thus we find an atomic component ΠRANK(CT, C#) containing the two elementary FDs: CT → C# and C# → CT. In DEPT we find several many-to-one relationships. For instance, many employees are associated with a given department; however, only one department is associated with any given employee. This relationship is described by the presence of an atomic component ΠDEPT(E#, DN) containing one FD: E# → DN. For an example of a many-to-many relationship one could assume that, in DEPT, one department has more than one location (i.e., DN →→ LOC). As previously discussed, then DEPT still has ΠDEPT(DN, LOC) as an atomic component. Now, however, there exists no elementary FD with scope {DN, LOC}. In summary, the combined definition of atomic components and elementary FDs is sufficient to describe the three basic types of relationships which occur between two attributes.

Let us consider now how atomic components and elementary FDs describe associations involving three or more attributes. For instance, consider the relationship between the three attributes C#, SN, and P in relation RANK. This relationship is denoted by the presence of the atomic component ΠRANK (C#, SN, P) with FDs {C#, SN} → P and {C#, P} → SN. This kind of relationship (i.e., one involving three or more attributes) cannot be classified under one of the three basic binary types previously discussed. At best, it can be viewed as a structured collection of binary subrelations obtained by holding the remaining attributes constant. Thus, for a ternary relationship one can set each of three attributes constant and then describe the relationships between the remaining two attributes. For instance, our relationship between C#, SN, and P can be described as follows:

1. For a given C# there is a one-to-one correspondence between SN and P.
2. For a given SN there is a many-to-one relationship between C# and P.
3. For a given P there is a many-to-one relationship between C# and SN.

This way of looking at our relationship reminds us of the orthographic projection technique which allows a three-dimensional object to be represented by three two-dimensional views. Unfortunately, as the order of a relationship grows, the number of bidimensional views required by this approach grows exponentially. However, these two-dimensional views can easily be derived given the elementary FDs (of an atomic component) whose number never exceeds the number of attributes in the atomic component.

Time \ Day	MO	TU	WD	TH	FR
8 AM	G1		G1		
9 AM	G1		G1		
10 AM	G2	G2	G2	G2	G4
11 AM		G3		G3	G4
12 AM	G1	G3	G1	G3	

Fig. 8. Weekly schedule of occupancy of a conference room.

Thus it appears that by supplying both the atomic components and the elementary FDs of a relation, one can represent unambiguously and concisely a wide spectrum of logical relationships between two or more attributes. Moreover, as we study further the possible configurations in which elementary FDs and atomic components combine and interact, we find that they are capable of describing relationships whose complex nature is hard to grasp by sheer intuition. We have just discussed the case in which the scope of an elementary FD and the attribute set of an atomic component coincide. Now we will analyze the configurations where one properly contains the other. An elementary FD cannot have scope properly contained in an atomic component. However, there may exist atomic components with attribute sets properly contained in an elementary FD. To illustrate this situation with an example, consider the relation

WS(DAY, TIME, GROUP)

which describes the weekly schedule of occupancy of a conference room, where various groups meet. Typically, the content of such a relation is represented by a table such as that shown in Figure 8, where the time and day are respectively rows and columns, while the entries of the matrix are the names of the groups.

Only one group meets at any given day and time in the room. Thus {DAY, TIME} $\rightarrow$ GROUP. Assume also that a group must follow the same schedule for any day in which it uses the room. Under these conditions the set of hours assigned to a group does not depend upon the particular day in which the group uses the room. Thus GROUP $\rightarrow\!\!\!\rightarrow$ TIME and GROUP $\rightarrow\!\!\!\rightarrow$ DAY. Indeed, the natural joint of ΠWS(DAY, GROUP) and ΠWS(TIME, GROUP) shown below returns the instance of relation (4.1) filled in as in Figure 8.

ΠWS(DAY, GROUP)		ΠWS(TIME, GROUP)	
MO	G1	8 AM	G1
MO	G2	9 AM	G1
TU	G2	12 AM	G1
TU	G3	10 AM	G2
WD	G1	11 AM	G3
WD	G2	12 AM	G3
TH	G2	10 AM	G4
TH	G3	11 AM	G4
FR	G4		

Thus, if one looks at relation WS, from the viewpoint of the elementary FD "{DAY, TIME} → GROUP" he may regard it as an indivisible logical association between these three attributes. According to its atomic components this is not an indivisible binary association, but rather it consists of two binary relationships. Other interesting examples of relations having a similar kind of twofold logical structure are given in [3, 32]. These present examples of relations characterized by elementary FDs of nested scope.

Since we have an atomic component $\Pi R(\Delta)$ where Δ is not the scope of any elementary FD, and we can also have an elementary FD with scope Δ where $\Pi R(\Delta)$ is not an atomic component, then it is clear that the atomic component structure of a relation (the A-structure) and its elementary FD structure (the Z-structure) are in many respects distinct and independent. However, their independence is restricted by important limitations, which result from the following theorems:

PROPOSITION 4.1. *The left side of any nontrivial MD of $R(\Omega)$ contains the right side attribute of any elementary FD of scope Ω.*

PROOF. If $(\Omega - \{A\}) \rightarrow A$ is the elementary FD and $\Gamma \twoheadrightarrow \Delta$ is the nontrivial MD, then we need to prove that $A \in \Gamma$. Assume by contradiction that $A \notin \Gamma$. Then either $A \in \Delta$ or $A \in \Lambda = \Omega - (\Gamma \cup \Delta)$. If $A \in \Delta$ by complementation and augmentation, one obtains: $(\Gamma \cup (\Delta - \{A\})) \twoheadrightarrow A$. But since $\Gamma \twoheadrightarrow \Delta$ is not trivial, $(\Gamma \cup (\Delta - \{A\})) = ((\Gamma \cup \Delta) - \{A\}) \subset (\Omega - \{A\})$. Thus $(\Omega - \{A\}) \twoheadrightarrow A$ is not elementary, and that is a contradiction since we know that the counterpart of an elementary FD is elementary [33]. By the same reasoning, we find that A cannot belong to Λ. Thus, $A \in \Gamma$. Q.E.D.

In our previous relation WS, for instance, we had {DAY, TIME} → GROUP while GROUP ↠ DAY and GROUP ↠ TIME.

PROPOSITION 4.2. *An n-ary relation $R(\Omega)$ which contains at least $n - 1$ elementary FDs with scope Ω is atomic.*

PROOF. If $\Gamma \twoheadrightarrow \Delta$ is a MD of R, then according to Proposition 4.1 $|\Gamma| \geq n - 1$. Thus, $\Gamma \twoheadrightarrow \Delta$ is trivial.

The following theorem establishes an interesting relationship between elementary FDs and atomic components (defined in Section 3.1 as the members of some minimal atomic decomposition).

PROPOSITION 4.3. *If $R(\Omega)$ contains some elementary FD with scope Δ and $\Pi R(\Delta)$ is atomic, then $\Pi R(\Delta)$ is an atomic component for R.*

PROOF. If $(\Delta - \{A\}) \rightarrow A$ is one of the elementary FDs with scope Δ, then R is decomposable into the pair $\Pi R(\Delta)$ and $\Pi R(\Omega - \{A\})$. Moreover, $\Pi R(\Omega - \{A\})$ may be further decomposable, say, in a set of atomic relations Σ. $\Pi R(\Delta)$ is atomic by assumption. Thus $\Sigma \cup \{\Pi R(\Delta)\}$ is an atomic decomposition of $R(\Omega)$, although it need not be minimal. A minimal atomic decomposition for $R(\Omega)$ can be obtained by removing all superfluous relations from $\Sigma \cup \{\Pi R(\Delta)\}$. However, this operation cannot remove $\Pi R(\Delta)$ since no relation in Σ contains attribute A. Thus $\Pi R(\Delta)$ remains as a component of the resulting minimal atomic decomposition. Q.E.D.

Thus we have an important corollary which supplies the formal basis to our combined representations of A- and Z-structures:

PROPOSITION 4.4. *If* $R(\Omega)$ *contains* n *elementary FDs with common scope* $\Delta \subseteq \Omega$ *and* $n \geq |\Delta| - 1$, *then* $\Pi R(\Delta)$ *is an atomic component of* $R(\Omega)$.

Thus if R has an elementary FD with scope $\{A, B\}$, then it also has an atomic component $\Pi R(A, B)$; if R has at least two elementary FDs with scope $\{A, B, C\}$, then it has an atomic component $\Pi R(A, B, C)$, etc.

4.2 *CAZ*-Graphs

We propose that atomic components and elementary FDs should be used as the primitives for schema definition. Thus a schema for a relation R consists of the pair (*ACOVER*, *ZCOVER*), where

ACOVER denotes a set of atomic components of R, which constitutes a minimal decomposition for this relation, and

ZCOVER denotes a set of elementary FDs of R which constitutes a minimal cover for the FDs of this relation.

These two sets, however, cannot be selected independently. Instead, they must satisfy the mutual constraint, dictated by the *admissibility condition* [33]. This ensures that atomic components and elementary FDs are properly combined to represent the various relationships among attributes, as per the examples in the previous section. This condition also supplies the basis for the graphical representation of schemata and for the schema design algorithm which will be discussed in this and in the following sections. A pair (*ACOVER*, *ZCOVER*) will be called *admissible* when it has the following properties:

(i) If *ZCOVER* contains an elementary FD with scope Δ, then it also includes every elementary FD of R with scope Δ. Moreover, if $\Pi R(\Delta)$ is atomic, then Δ is also contained in *ACOVER* (by Proposition 4.3 this is an atomic component for R).

(ii) If, moreover, *ACOVER* contains Δ, then every elementary FD of R with scope Δ is included in *ZCOVER*.

We suggest that the sets *ACOVER*, *ZCOVER*, combined under the admissibility condition, can be used as a conceptual database schema. Figure 9 gives such a schema for our relation RANK (in the next section we show that this is in fact a good schema for our relation).

For convenience we have introduced an index (called LABEL) which uniquely identifies subsets of attributes; the subset of attributes identified by label i will be denoted Δ_i.

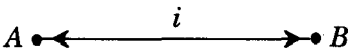
We now introduce a graph representation of our schema. This will be called a combined A- and Z-graph or, for short, a *CAZ*-graph. The *CAZ*-graph, which represents the schema of a relation, has as vertices the attributes of this relation. Every arc of the graph has a label attached. An arc can either be of type one-to-one or type many-to-one, or type many-to-many. The graph representation of a schema (*ACOVER*, *ZCOVER*) is constructed as follows (*Direct Mapping*):

1. *One-to-one arcs*: There is a one-to-one arc labeled i between vertices A and B if *ZCOVER* contains two FDs with common scope Δ_i and with right sides A

LABEL	*ACOVER*	*ZCOVER*
1	{C#, P, SN}	{C#, SN} → P {C#, P} → SN
2	{C#, CT}	C# → CT CT → C#

Fig. 9. A schema for RANK.

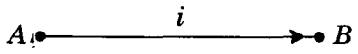
and B, respectively. This arc will be represented as follows:



2. *Many-to-one-arcs*: There is a many-to-one arc labeled i between A and B if the following conditions are both satisfied:

(a) *ZCOVER* contains an FD of scope Δ_i whose left side contains A and whose right side is B. (Thus Δ_i contains both A and B.)
(b) There is no one-to-one edge with label i between A and B.

This arc will be represented as follows:

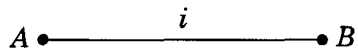


We shall say that this arc is leading into B.

3. *Many-to-many arcs*: There is a many-to-many arc labeled i between A and B if the following conditions are both satisfied:

(a) *ACOVER* contains Δ_i which contains both A and B.
(b) There is no one-to-one or one-to-many arc with label i between A and B.

This arc is represented by a plain line as follows:



The *CAZ*-graph corresponding to the pair (*ACOVER*, *ZCOVER*) of Figure 9 is given in Figure 10.

We now give the rules to derive the elementary FDs and the atomic components defined by a *CAZ*-graph G. If i is a label of some arc of G, then G_i will denote the subgraph of G defined by i; G_i is obtained by first removing from G all arcs not labeled i and then removing all the isolated vertices (i.e., those to which no arc is attached). For instance, if G is the graph of Figure 10, then G_2 reduces to the one-to-one arc labeled 2 and the two end vertices of this arc. If in G_i there is an arc between every two vertices, then G_i is called a *clique*. Thus the direction of the arcs is immaterial in the definition of the cliques of a graph. Then the set of elementary FDs, $\overline{ZCOVER}$, and the set of atomic components, $\overline{ACOVER}$, defined by the *CAZ*-graph G can be constructed as follows.

Inverse Mapping. For every label i of G consider G_i, with vertex set Δ_i, and

(a) if G_i is a clique, then enter Δ_i in $\overline{ACOVER}$;
(b) if B is a vertex of G_i connected to every other vertex of G_i by a one-to-one arc or by a many-to-one-arc leading into B, then enter $(\Delta_i - \{B\}) \rightarrow B$ in $\overline{ZCOVER}$.

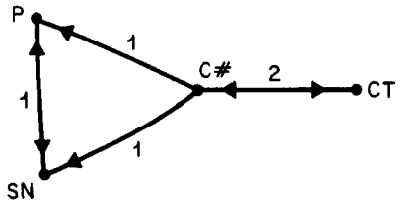


Fig. 10. A *CAZ*-graph for relation RANK.

For instance, if G is the graph of Figure 10, then in G_1 we find a vertex SN connected to P by a one-to-one arc and connected to C# by a one-to-many arc (leading into SN). Thus {P, C#} → SN. (Note the importance of labels in defining these FDs. Without labels we could have interpreted the arrows leading into SN as two distinct FDs: P → SN and C# → SN.) Also, G_1 defines {C#, SN} → P. In G_2 we find C# → CT and CT → C#. G has two cliques: G_1 with vertices {P, SN, C#}, and G_2 with vertices {C#, CT}. Therefore, from Figure 10 we have reconstructed the original pair (*ACOVER*, *ZCOVER*) of Figure 9.

Let us now prove that the *CAZ*-graph representation is unambiguous and correct. We show that there is a one-to-one correspondence between a pair (*ACOVER*, *ZCOVER*) satisfying the admissibility conditions and its *CAZ*-graph G, constructed by the direct mapping rules 1, 2, and 3. If $\overline{ACOVER}$, $\overline{ZCOVER}$ denote the pair constructed from G by the inverse mapping rules (a) and (b), then we need to prove that $\overline{ACOVER} = ACOVER$ and $\overline{ZCOVER} = ZCOVER$. The second equality is an immediate consequence of the definitions. To prove the first equality, observe that $\overline{ACOVER} \supseteq ACOVER$ follows immediately from the way in which we constructed the many-to-many edges of G. To prove that $\overline{ACOVER} \supseteq ACOVER$, we must prove that if, for some label i, there is a clique G_i with vertex set Δ_i, then $\Pi R(\Delta_i)$ is an atomic component of R. Again, if G_i has some many-to-many edge, the conclusion follows directly from the definition. Otherwise, let G_i be a clique where every arc is either one-to-one or many-to-one. We prove that R contains at least $|\Delta_i| - 1$ elementary FDs with scope Δ_i. Indeed, if R has $|\Delta_i| - 2$ or fewer such FDs, there are two attributes in Δ_i, neither of which is the right side of an elementary FD with scope Δ_i. Thus these two attributes are not connected in G_i by a one-to-one or a many-to-one arc; this is a contradiction.

Now since R contains at least $|\Delta_i| - 1$ elementary FDs of scope Δ_i, it follows from Proposition 4.4 that $\Pi R(\Delta_i)$ is an atomic component for R. Moreover, since *ZCOVER* contains some elementary FD with scope Δ_i, by the admissibility condition *ACOVER* must contain $\Pi R(\Delta_i)$. This concludes our proof.

CAZ-graphs exhibit relationships between attributes in a simple fashion. Binary relationships are represented by an arc whose type defines the nature of the relationship. An edge with two opposite arrowheads denotes a one-to-one relationship (e.g., in Figure 10 C# ↔ CN), an edge with one arrowhead denotes a many-to-one relationship, and an edge with no arrowhead denotes a many-to-many relationship. Relationships among three or more attributes are represented by the orthographic projection technique previously discussed. Each arc denotes a relationship holding between two attributes when the remaining attributes are held constant. In Figure 10, for instance, the arc between P and SN denotes that

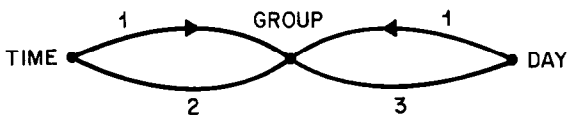


Fig. 11. A *CAZ*-graph representing relation WS.

a one-to-one correspondence exists between P and SN, when C# is held constant. The arc between C# and P denotes a many-to-one relationship with SN constant, and the arc between C# and SN denotes a many-to-one relationship with P constant.

CAZ-graphs allow us to represent unambiguously complex relationships such as that involving elementary FDs of nonatomic scope. For instance, a *CAZ*-graph representing the structure of our relation WS is given in Figure 11.

4.3 Schema Design

We now present a formal procedure for schema design using the functional and multivalued dependencies of a relation. The objective of this procedure is to produce an admissible pair (*ACOVER*, *ZCOVER*) which preserves the information of the original relation in terms of both content and structure, and represents it with minimal redundancy. This procedure consists of two steps:

1. a *decomposition step* which produces the admissible (*ACOVER*, *ZCOVER*) where *ZCOVER* is the minimal cover for the FDs of the given relation, and
2. a *verification step* to verify the minimality of the atomic decomposition *ACOVER*.

Next, we discuss the decomposition step. The verification step will be discussed later. The algorithm to perform the decomposition step was presented in [33]. This algorithm basically consists of a recursive procedure which operates as follows. Say that $\Pi R(\Omega)$ is the relation at hand with a set $\bar{F}$ of elementary FDs and a set $\bar{G}_m$ of multiple elementary MDs; also let F_0 be the set of elementary FDs in $\bar{F}$ having Ω as scope. Then F_0 is added to *ZCOVER* and, if $\Pi R(\Omega)$ is atomic (i.e., $\bar{G}_m$ is empty), then $\Pi R(\Omega)$ is entered in *ACOVER* and the procedure is completed. If $\Pi R(\Omega)$ is not atomic, then $\bar{G}_m$ is searched for an elementary MD which ensures preservation of structural information. Once such an MD, say, $\Gamma \twoheadrightarrow \Delta$, is found, then $\Pi R(\Omega)$ is decomposed into $\Pi R(\Gamma \cup \Delta)$ and $\Pi R\ (\Omega - \Delta)$. Next these two subrelations are decomposed in turn.

As discussed in [33], the MD "$\Gamma \twoheadrightarrow \Delta$" ensures preservation of structural information if the following two conditions are satisfied:

$$(F_1 \cup F_2)^+ \supseteq (\bar{F} - F_0) \tag{4.1}$$

$$(\bar{G}_F \cup G_{11} \cup G_{22})^+ \supseteq \bar{G}_m \tag{4.2}$$

where

F_1 and F_2 denote the FDs in $\bar{F}$ whose scope is contained in $(\Gamma \cup \Delta)$ and $(\Omega - \Delta)$, respectively.

$\bar{G}_F$ denotes the MD counterparts of the FDs of $\bar{F}$,

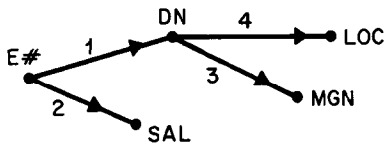


Fig. 12. The *CAZ*-graph for DEPT.

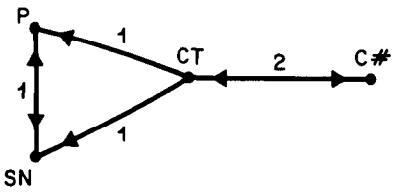


Fig. 13. A second *CAZ*-graph for RANK.

G_{11} and G_{22} denote the MDs of $\bar{G}_m$ having right sides disjoint from Γ, and left sides contained, respectively, in $(\Gamma \cup \Delta)$ and $(\Omega - \Delta)$.

Conditions (4.1) and (4.2) ensure that the FDs and the MDs of the initial relation are derivable from the dependencies of the resulting decomposition [33].

Before this recursive procedure is applied to the given relation, the sets *ACOVER* and *ZCOVER* are set to empty. Once the procedure completes, *ACOVER* and *ZCOVER* contain, respectively, a set of atomic subprojections of a given relation and a set of elementary FDs which satisfy the admissibility condition. In [33] we prove that *ZCOVER* so obtained constitutes a *minimal cover* for the FDs of R. While ZCOVER alone preserves and minimally represents the Z-structure of the initial relation, its A-structure is captured by *ACOVER* and *ZCOVER* combined.

Efficient algorithms to verify conditions (4.1) and (4.2) can be found in [4] and [3], respectively. An algorithm for deriving the multiple elementary MDs in projections of a given relation was presented in [33].

Application of this decomposition algorithm to our example DEPT produces the *CAZ*-graph of Figure 12.

As one can easily verify, the result of the decomposition of DEPT does not depend on the order in which the multiple elementary MDs in (2.2) are tested for compliance with conditions (4.1) and (4.2) and then used in the decomposition. However, if one decomposes RANK, he will obtain the graph of Figure 10 or that of Figure 13, depending on whether CT $\rightarrow$ C# or C# $\rightarrow$ CT is considered first. The fact that there exist two solutions is a logical consequence of the one-to-one correspondence existing between CT and C#. Clearly, either graph supplies an acceptable schema and both contain the same information.

The purpose of the second step of our design procedure is to verify the minimality of the atomic decomposition, *ACOVER*, provided by the first step. The use of only multiple elementary MDs in the decomposition supplies an extremely effective heuristic toward ensuring this minimality. To convince oneself of this, one could decompose RANK using a single elementary MD instead, for example, the counterparts of the elementary FDs 5, 6, and 7 in (3.2), and obtain a clearly nonminimal decomposition. Nevertheless, there is no formal assurance that the decomposition step produces a minimal atomic decomposition. Actually, we know of a counterexample where a decomposition obtained using multiple elementary MDs is not minimal. This example, however, involves subrelations which are losslessly decomposable into three projections but not into two projections. As we know, the semantic constraints obeyed by these relations cannot be

modeled by MDs [23]. Whether minimality can be violated for decomposition of relations which only obey constraints expressible in terms of FDs and MDs is still an open problem.

Say, therefore, that our conservative designer wants to verify the minimality of the atomic decomposition Σ produced by the first step of the design procedure. Σ is minimal if it does not contain any superfluous relation. We know that the following members of Σ cannot be superfluous: (1) those which contain some attribute not appearing in any other member of Σ and, in the absence of MDs, with an empty left side; (2) those whose removal would break the connectivity of the associated hypergraph. For the examples we have considered so far these two rules were sufficient to infer minimality. In more complex examples, for example, the one which we consider next, some subrelation which could not be decided to be nonsuperfluous by the previous two rules could remain. Then for each such subrelation, say, $\Pi R(\Delta)$ in Σ, one can use the algorithm presented in [1] to decide whether $\Sigma - \{R(\Delta)\}$ has the lossless join property. In the presence of MDs, however, this algorithm may run in exponential time. A second approach is to use Proposition 3.4, as we are now going to illustrate with the help of an example taken from [33]. The attributes of interest are as follows:

LIC: license numbers of motor vehicles;
MAKE: manufacturers of motor vehicles;
MODEL: models of vehicles;
YEAR: the year in which the vehicle was manufactured;
VALUE: the current value of the vehicle;
OWNER: the unique identifier of a person (for simplicity, we give only the family name; in reality, a composite ID which includes the first name, birth date, and location, etc., may be needed);
DRVL: the driving license numbers;
VIOL: the code numbers for traffic violations:
DATE: month, day, and year of violation.

The following information is required:

1. the make, model, and year of any licensed vehicle;
2. the current (blue book) value of a given type of vehicle;
3. the legal owner of a given vehicle;
4. the driving license number of a given person (and the person having a certain license number);
5. the traffic violation history of any driver; the records consist of pairs: violation code and date of warrant.

Then we have a relation:

DMV (LIC, MAKE, MODEL, YEAR, VALUE, OWNER, DRVL, VIOL, DATE),

with elementary FDs and multiple elementary MDs:

D1: LIC $\rightarrow$ MAKE
D2: LIC $\rightarrow$ YEAR
D3: LIC $\rightarrow$ MODEL
D4: LIC $\rightarrow$ VALUE

LABEL	*ACOVER*	*ZCOVER*
1	{VALUE, MAKE, MODEL, YEAR}	{MAKE, YEAR, MODEL} → VALUE
2	{LIC, MAKE}	LIC → MAKE
3	{LIC, YEAR}	LIC → YEAR
4	{LIC, MODEL}	LIC → MODEL
5	{OWNER, DRVL}	OWNER → DRVL DRVL → OWNER
6	{OWNER, VIOL, DATE}	None
7	{LIC, OWNER}	LIC → OWNER

Fig. 14. A schema for relation DVM.

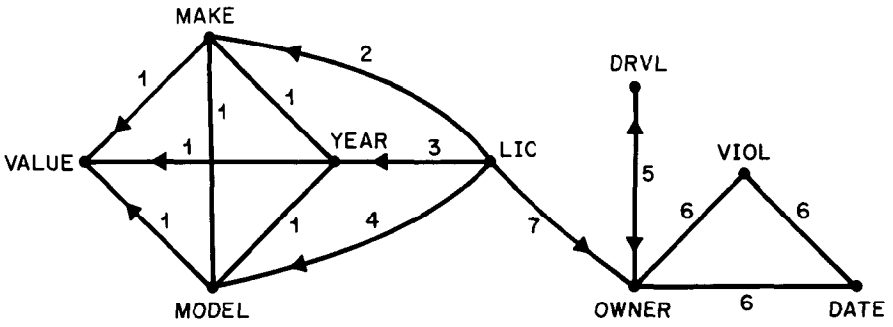


Fig. 15. *CAZ*-cover graph for relation DMV.

D5: LIC → OWNER
D6: LIC → DRVL
D7: LIC →→ {VIOL, DATE}
D8: {MAKE, YEAR, MODEL} → VALUE
D9: {MAKE, YEAR, MODEL} →→ {LIC, OWNER, DRVL, VIOL, DATE}
D10: OWNER → DRVL
D11: OWNER →→ {VIOL, DATE}
D12: OWNER→→ {LIC, MAKE, YEAR, MODEL, VALUE}
D13: DRVL → {OWNER}
D14: DRVL → {VIOL, DATE}
D15: DRVL →→ {LIC, MAKE, YEAR, MODEL, VALUE}

By applying the decomposition algorithm previously discussed we obtain the schema of Figure 14 [33]. This is represented by the *CAZ*-graph of Figure 15.

If G is a *CAZ*-graph defining the set of atomic subrelations $\overline{ACOVER}$, then the hypergraph representation of $\overline{ACOVER}$ will be called the hypergraph associated with the cliques of G, denoted $H(G)$. Given a *CAZ*-graph, it is easy to visualize the hypergraph associated with its cliques. For instance, let G be the graph of Figure 15; $H(G)$ has an edge corresponding to G_1 with vertices {MAKE, YEAR, MODEL, VALUE} and an edge corresponding to G_6 with vertices {OWNER, VIOL, DATE}. Moreover, $H(G)$ has some binary edges corresponding to the arcs labeled 2, 3, 4, 5, and 7 of Figure 15.

For convenience, we let the label of each clique of G become the name of the corresponding edge of $H(G)$. We can now observe that edges 1, 5, and 6 of our

$H(G)$ are not superfluous since they contain some vertex not included in any other edge. Edge 7 cannot be superfluous since its removal would break the connectivity of the hypergraph. The question whether edges 2, 3, and 4 are superfluous can be resolved using Proposition 3.4. Assume that we remove edge 2. Then {YEAR, MODEL} becomes an articulation set for the hypergraph so obtained; this is broken by the removal of YEAR and MODEL into two connected components with vertices {MAKE, VALUE} and {LIC, OWNER, DRVL, VIOL, DATE}. Thus if ΠDMV(LIC, MAKE) is superfluous in our decomposition, then, by Proposition 3.4, {YEAR, MODEL} $\rightarrow\!\!\!\!\rightarrow$ {MAKE, VALUE} in DMV. But since such an MD does not hold in DMV, ΠDMV(LIC, MAKE) cannot be superfluous. By analogy, one derives the fact that edges 3 and 4 cannot be superfluous. Therefore, even in this more complex example we were able to verify the minimality of our atomic decomposition by direct inspection of the connectivity properties of the associated hypergraph; we found no need to resort to the algorithm presented in [1].

Let us see how the dependency structure of a relation is preserved and represented by its *CAZ*-graph. For FDs the situation is clear. The set $\overline{ZCOVER}$ defined by the graph supplies a minimal cover for the FDs of the whole relation. For MDs the situation is more complex; it involves both sets $\overline{ACOVER}$ and $\overline{ZCOVER}$ defined by the graph. A first group of MDs consists of those derivable as the counterparts of FDs in *ZCOVER*. A second group of MDs is derivable from the connectivity properties of the graph, using Proposition 3.4. In the case of DMV, for instance, the two elementary MDs

D11: OWNER $\rightarrow\!\!\!\!\rightarrow$ {VIOL, DATE}
D12: OWNER $\rightarrow\!\!\!\!\rightarrow$ {LIC, MAKE, YEAR, MODEL, VALUE}

can be derived from the graph of Figure 15 by simply observing that the removal of OWNER breaks the hypergraph associated with its cliques in three connected components, with vertices {VIOL, DATE}, {LIC, MAKE, YEAR, MODEL, VALUE}, and {DRVL}. Thus, Proposition 3.4 produces the desired conclusion. A third group of MDs is derivable from the MDs of the first two groups by using the MD inference rules [6].

D7: LIC $\rightarrow\!\!\!\!\rightarrow$ {VIOL, DATE} and
D14: DRVL $\rightarrow\!\!\!\!\rightarrow$ {VIOL, DATE}

are derivable by composing, respectively, LIC $\rightarrow\!\!\!\!\rightarrow$ OWNER and DRVL $\rightarrow\!\!\!\!\rightarrow$ OWNER with D11.

4.4 Discussion

A recent survey [5] has exposed three basic principles supplying a common conceptual basis to the formal approaches to schema design proposed so far. These are as follows:

1. the principle of separation;
2. the principle of representation;
3. the principle of minimal redundancy.

In spite of this common conceptual basis, different works on schema design have used different formulation of these principles and pursued them to a different

degree. Thus we now assess the performance of our approach in the framework of these principles.

The principle of separation presides at the choice of "good" primitives for schemata. Certain relationships should not be represented together by a "big" relation but should be *separated* and represented independently by smaller relations. However, while everyone agrees that "small is good," the issue "How small is good enough?" is somewhat controversial. A measure of this controversy is supplied by the large number of normal forms which are at present roaming the field. In our approach we have pursued the principle of separation to an extreme degree by ensuring that independent relationships are always represented by separate (atomic) relations. Thus, we decompose relations into finer granules than the normal form approach would: while atomic components are always fourth normal form, the reverse is not true. The admissibility condition is instrumental in implementing this separation principle. To illustrate this point, assume that we represent our previous example RANK by a schema which does not satisfy the admissibility condition: assume that in *ZCOVER* of Figure 9 we replace {C#, SN} → P by {CT, SN} → P. No loss of information has thus occurred since our old *ZCOVER* and the new *ZCOVER* have the same FD closure. After such a change, however, the schema represents the relationship between {C#, P, SN} in a less natural and less direct fashion. Indeed, to understand that for a constant C# there is a one-to-one correspondence between SN and P, one needs to look at a fourth attribute, CT, and, after finding which FDs hold in this larger context, derive the desired conclusion by transitivity. Thus the admissibility condition ensures that the representation of certain basic relationships is complete and self-contained, so that they can be treated independently from the context in which they appear. The benefits accrued by using the admissibility condition in the design of third normal form relations are discussed in [33].

In the framework of the decomposition approach the principle of *representation* prescribes that the final schema represents the same information as the original relations did. Since we have assumed that these relations with their dependencies completely describe the logical relationships of interest, the principle of representation becomes an immediate consequence of the complete relatability principle discussed in the introduction.

We have applied the representation principle with rigor by requiring that both content and structure be preserved in the decomposition. Content preservation has been ensured as per the lossless join concept described in [1]. Preservation of structural information has been ensured with respect to both FDs and MDs—thus in a more general context than the one discussed in [24] and [2]. As a result of this, the elementary FDs of the given relation are either represented by the resulting *CAZ*-graph or inferable from it by the FD inference rules. The MDs of the original relation are instead defined by the graph, either by being the counterpart of an FD, or by the connectivity property of the graph, or indirectly through the MD inference rules. Thus it is quite legitimate and natural to speak of the *CAZ*-graphs generated by our design procedure as a cover graph for the given relations.

The principle of minimal redundancy has been applied here with rigor also Our schema consists of a set of elementary FDs, *ZCOVER*, and of a set of atomic

components, *ACOVER*. The *ZCOVER* generated by our decomposition algorithm was proved minimal in [33]; thus no smaller subset of it is a cover for the FDs of the initial relation. Moreover, the second step in the design procedure also guarantees the minimality of *ACOVER*: no proper subset of it can be joined back into the original relation. It must be noted here that since this minimality condition is applied to atomic relations, it becomes a more stringent requirement than it would be otherwise. To illustrate this point, we can consider a decomposition of DMV containing some nonatomic subrelations. Say, for instance, that we take the decomposition defined by the graph of Figure 15 and we replace ΠDMV(LIC, MAKE), ΠDMV(LIC, YEAR), and ΠDMV(LIC, MODEL) by the pair

$$\Pi\text{DMV(LIC, MAKE, YEAR)} \tag{4.3}$$
$$\Pi\text{DMV(LIC, YEAR, MODEL)}. \tag{4.4}$$

This new decomposition, consisting of the two projections above and of those defined by the cliques labeled 1, 7, 5, and 6 of Figure 15, is still minimal, as one can easily verify using Proposition 3.4. Yet in this minimal decomposition there is a significant amount of redundancy since the binary relationship between LIC and YEAR is represented in both (4.3) and (4.4).

Thus our design procedure realizes a very high standard for all three basic principles which are generally considered important in schema design. As a result of this high standard, the schemata produced by this procedure are "good" schemata in every respect. In all examples we have considered the natural relationships of interest were captured and represented in a rather natural, expressive, and nonambiguous fashion. Of course, a price had to be paid for pursuing such high standards. There exist relations for which all the previous objectives cannot be met and either the decomposition algorithm or the verification step will fail. The decomposition algorithm could fail because there exists no multiple elementary MD for which the complete relatability conditions (4.1) or (4.2) are satisfied. Then the designer would be forced to intervene and decide how to resolve the situation. In the large majority of cases we have considered, the design procedure we have presented completes automatically without designer intervention. Therefore, we had available for analysis only a few examples of plausible semantics for which the design procedure would fail. In these examples the source of the problem was not the decomposition algorithm itself, but rather the objective impossibility of finding a solution satisfying all the strict standards which had been set for separation, preservation, and minimal redundancy. One way to ensure that the procedure completes automatically in every case would be to relax some of these standards. For instance, some workers argue that decomposition into Boyce–Codd normal form or fourth normal form should be performed even when preservation of structural information cannot be obtained [13, 15]. On the contrary, a weaker statement of minimal redundancy is accepted in [8] for the purpose of obtaining third normal form relations. Here, instead, we decided to follow a different approach: the basic principles for a "good" design were strictly enforced, and we accepted the fact that on occasion all these principles cannot be met and a more direct intervention is required by the

designer. This discipline offers two significant advantages. First, it ensures that schemata automatically generated by the design procedure are sound and well behaved by the strictest standards of good design principles. Our experience suggests that the great majority of real-life examples are still handled automatically. A second advantage is that those infrequent situations which are recalcitrant to the application of these "good" design principles are singled out, and the designer's attention is drawn to them. As a result of a deeper look, the designer may recognize some peculiarities with the relation at hand. Our experience shows that these may be caused by some particular constraints between the relationships represented or possibly by a poor choice of the initial relation which was intended to represent them. Thus a halt in the decomposition procedure will supply some useful feedback to the designer and stimulate needed corrections. To illustrate this point, one can consider a relation DICT(F, G, E) which supplies a dictionary of corresponding technical terms or concepts in French, German, and English [33]. Assuming that a concept can be described by multiple synonyms but no terms describe more than one concept, then every attribute in DICT is MD on every other attribute. It is easy to see that no decomposition which satisfies (4.2) exists, and the only good solution to this problem is to introduce a new column, say, C for concept, to identify the concept corresponding to a set of synonyms. Then the decomposition procedure operates successfully on the new augmented relation (yielding the schema of [33], Figure 2).

On the contrary, if the designer feels that his initial schema was well suited for his statement of intension, then he may proceed in the decomposition in the way he deems most appropriate for the case at hand. For instance, if condition (4.1) is not satisfied, he may decide to disregard some elementary FD and then choose a minimal decomposition which preserves the remaining ones, or he may decide to accept redundancy and decompose the relation into larger subprojections or into more than two subprojections to ensure that a cover for the FDs of the relation is preserved. In either case, however, it is clear that there will be some side constraints not embodied in the schema of which the designer better be aware.

In conclusion, we suggest that our approach supplies a reasonable compromise between the desire to have a formal algorithm for automatic schema design and the requirement that this algorithm in fact deliver a good schema for the application at hand according to some well-defined design criterion. Nevertheless, this is a topic which deserves further investigation. For instance, various extensions and improvements to the algorithm are proposed in [28].

When thinking of graphical schemata for databases, one normally envisions diagrams describing the whole database or at least substantial portions of it. Clearly, it is possible to describe a whole database by one *CAZ*-graph. To design this graph one could start from a "universal relation" which includes all the attributes and all the relationships of interest. While the "universal relation" is a useful analytical tool [5], representing every relationship of interest by one relation is certainly cumbersome and possibly unattainable [32]. Thus, a few relations of a reasonable size supply a more practical starting point. Then at the end of the design the *CAZ*-graphs of the various relations will be pasted together

into one graph. If we assume that no interrelational dependency or redundancy occurs among the initial relations, then the *CAZ*-graph so constructed supplies a good schema according to the three design principles discussed in this section.

5. APPLICATIONS

5.1 E-R Diagrams

A useful correspondence can be established between *CAZ*-minimal cover graphs and the entity-relationship (E-R) diagrams which were developed by Chen as models for database definition and design [9, 10]. E-R diagrams use two types of nodes: entity nodes depicted by rectangular boxes and relationship nodes depicted by diamond-shaped boxes. The correspondence between an E-R diagram and a *CAZ*-graph, G, can be established as follows: the entity nodes correspond to the vertices of G while the relationship nodes correspond to its labels. To each partial subgraph G_j, where j is a label of G, there corresponds in the E-R diagram a set of arcs connecting the relationship node j to the various vertices of G_j. If A is one of these nodes, then the arc connecting A with j in the E-R diagram is tagged "one" if G_j has either a one-to-one or a one-to-many arc leading into A (i.e., if the arrowheads lead into it); this arc is tagged "many" otherwise. In the diagrams, "one" is actually shown as "1" while "many" is shown as either "*N*" or "*M*" or "*P*". The transformation of a *CAZ*-graph into an E-R diagram, and vice versa, is indeed very intuitive and easily understood from an example. The E-R diagram corresponding to the cover graph of Figure 15 is given in Figure 16. In this E-R diagram, each single attribute constitutes a separate entity. Thus, each *diamond-shaped box* defines an *atomic component* of our relation DMV. In other words, an atomic relationship exists beween the entities adjacent to this diamond. For instance, the diamond marked "1" denotes an atomic relationship between MAKE, YEAR, MODEL, and VALUE. Moreover, the tags of the arcs radiating from a diamond define the elementary FDs having as scope the set of attributes of the atomic relation denoted by the diamond. The tag "1" at an arc establishes that the entity at the end of this arc is FD on the remaining entities of the atomic relationship; moreover, this FD is elementary. For instance, the leftmost diamond in Figure 16 denotes an atomic relationship between the four attributes MAKE, YEAR, MODEL, and VALUE. The tag "1" at the end of the leftmost arc shows that there exists an elementary FD from the first three attributes to the fourth one. (By analogy with the notation used to denote FDs, therefore, one could use an arrowhead on the arc to denote an arc of type "one"; then arcs of type "many" would be left without arrowheads.) In summary, there exists a simple correspondence between E-R diagrams and *CAZ*-graphs, inasmuch as they use the same primitives to represent relationships. These primitives are atomic components and elementary FDs, which, in turn, reexpress the structure of FDs and MDs of a relation. However, there are differences between *CAZ*-graphs and the E-R model which cannot be overlooked. These are discussed next.

A major aim of Chen's work is to produce a general description of the database called the enterprise schema. Thus certain conceptual objects of general interest to the whole enterprise are identified and catalogued. These objects are called entities, and E-R diagrams are used to give an overall representation of the

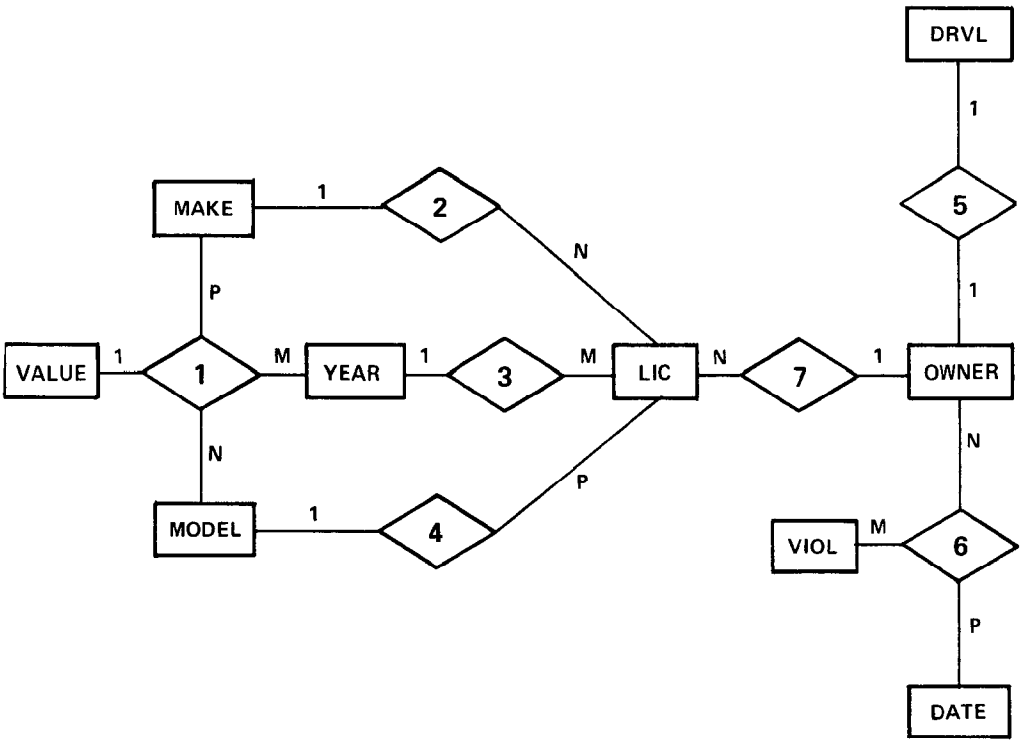


Fig. 16. The E-R diagram representation of the *CAZ*-graph of Figure 15.

relationships between the entities supported by the database system. The use of entities, by making the whole picture more succinct and suggestive, supplies a congenial interface to the management of the enterprise; in brief, it provides the means to achieve the main objectives of an enterprise schema. However, both the users in formulating their queries and data manipulation requests and the database implementor are concerned with a much finer level of detail: they are primarily concerned with attributes and their values. Thus the upper level entities are then redefined at a lower level as a collection of attributes.

Throughout this paper we have regarded each single attribute as a separate entity. The graph of Figure 16 is indeed the E-R diagram for our DMV study case where each attribute of interest is regarded as an entity. Nothing, however, restricts the designer from specifying that an entity correspond to a group of logically related attributes. For our DMV example, for instance, a designer might want to specify an entity called VEHICLE-TYPE consisting of four attributes; MAKE, YEAR, MODEL, and VALUE. The various concepts and results presented in the previous sections regarding the dependency structure of a relation, its decomposition, and its representation by means of graphic schemata remain valid and are applicable in this new context as well. In the case at hand, for instance, one need only replace the four attributes MAKE, YEAR, MODEL, and VALUE by VEHICLE-TYPE in relation DMV. The combinations of the values

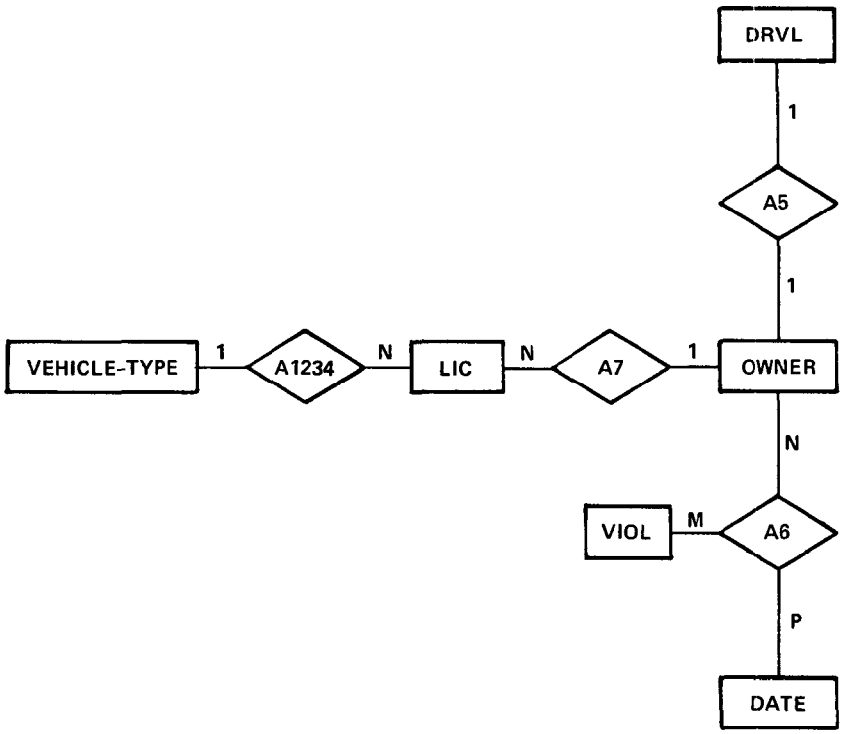


Fig. 17. The E-R diagram for relation DMV after combining VALUE, MAKE, MODEL, YEAR into VEHICLE-TYPE.

of these four attributes constitute the values of VEHICLE-TYPE. Thus, one can proceed to determine the dependencies of the relation so obtained and then to decompose it into a minimal cover schema using the procedure described in the last section. The result of this process is described by the E-R diagram in Figure 17. The relationships A5, A6, and A7 of Figure 17 correspond to the ones marked, respectively, 5, 6, and 7 in Figure 16. However, A1234 denotes a new relationship which has replaced those marked 1, 2, 3, and 4 in Figure 16.

A formal discipline is thus available for designing E-R models for databases: The designer starts from a database sample in tabular form. Next he must identify the entities of interest as aggregates of attributes of this relation and list the dependencies existing among these attributes. Finally, he will apply the design procedure given in the last section to obtain an E-R diagram representing the relationships between these entities. For instance, in [22] we discuss various choices of entities for our DMV example, and we give the corresponding E-R diagrams. This approach, therefore, allows the designer to select the aggregates he wants to use as entities. This is a reasonable policy since the question as to what constitutes an entity has a subjective answer. Some guidelines are, however, available to help the designer in his choice. For instance, in [17] it is suggested that entities should have a unique key attribute and at least one other attribute. Useful concepts on this issue can also be found in [29]. Finally, the work of [30]

suggests that this problem can be attached in the framework of FDs and MDs. This is the very framework within which we have developed our decomposition algorithm.

E-R diagrams are also useful as a database design aid. Indeed, Chen describes how they supply a unified basis for the design of network and hierarchical schemata as well as relational ones. Thus, we have now a discipline whereby, given the functional and multivalued dependencies of a relation, a network schema implementation for it can be derived. This is certainly an encouraging result for anyone who believes that a common theoretical foundation exists for the database field.[2] Because of their affinity with E-R diagrams, *CAZ*-graphs can be used as a design aid as well. In [33], we show how a *CAZ*-cover graph produced by the decomposition procedure of Section 4.3 can be turned into a 3NF schema using Bernstein's algorithm [8]; we also illustrate how this approach may in fact result in the design of better 3NF schemata.

The E-R diagrams were informally introduced by means of examples and advocated as being particularly congenial to nonspecialists. We now find that they are amenable to a much more rigorous definition: they define the atomic relationships of the database and the elementary functional dependencies having as scope these relationships. These relationships span conceptual objects called entities which are specified by the database administrator as combinations of elementary attributes. The E-R model does not display every atomic relationship, but only a minimal set which defines a cover in a sense completely analogous to the minimal cover concept for *CAZ*-graphs defined in the previous section. However, observe that, while the atomic relationships produced by the decomposition algorithm are inherently projections of the same relation, no such constraint needs to be assumed in an E-R diagram (or for that matter, in a *CAZ*-graph). Thus, for instance, in Figure 16, there could exist instances of OWNER which do not participate in some or all of relationships 5, 6, and 7. Then, the relationships of Figure 16 can be considered the projections of one relation only if this is allowed to contain null values [21].

The correspondence between *CAZ*-graphs and E-R diagrams exposes a problem area which is not discussed in Chen's paper. This problem is caused by elementary FDs of nonatomic scope which were discussed at length in Section 4. (See, for example, relation WS where we have {TIME, DAY} $\rightarrow$ GROUP while GROUP $\twoheadrightarrow$ DAY and GROUP $\twoheadrightarrow$ TIME.) In such cases it is not clear which relationships should be displayed in the E-R diagram. One solution is to ignore elementary FDs which are nonatomic in scope and to represent atomic relations only; vice versa, one could display the nonatomic elementary FDs by diamonds and neglect the finer relationships contained therein. Finally, one could display both the atomic relations and the nonatomic elementary FDs by diamonds in the E-R diagram. None of these three solutions is clearly superior to the others. Indeed, each presents some advantages and disadvantages and seems most reasonable, depending on individual cases [22]. Because of this problem and because E-R diagrams are not as close to the usual graphs as *CAZ*-graphs, we have used the latter in our formal development. Nevertheless, a practitioner could reasonably

[2] The use of dependencies in analyzing and improving E-R diagrams was also discussed in [26].

adopt a more pragmatic view by observing that elementary FDs of nonatomic scope are rare in practice and could thus be treated on an individual basis. With this proviso E-R diagrams would probably be considered as attractive alternates to *CAZ*-graphs.

5.2 Conceptual Schemata

The graphs developed in this paper can be used as a conceptual schema in a database system architecture such as that proposed by the ANSI/X3/SPARC study group [31]. The conceptual schema for such a system supplies the central frame of reference. It also establishes the functional connections linking the various external schemata that supply a convenient environment for user applications to the internal schema which is designed to optimize performance on typical queries and data manipulation requests. To provide data independence, the conceptual schema must define the logical structure of data independently of physical implementation. Current network or hierarchical schemata are designed to reflect not only the logical relationships among data, but also the expected usage of the data as seen by the database designer. Indeed, there are usually several possible hierarchical or network realizations of an E-R diagram. Moreover, the user may want to view the data as hierarchically organized in the external schema, although the data are actually organized into a plex structure. Thus the database systems should be capable of supporting multiple data models. A direct mapping between different models is a formidable problem, and it is even more difficult when there exists no one-to-one correspondence between the logical structure of data and the schema as in the previous cases. A more promising approach consists in using at the conceptual schema level a meta-model which is easily mapped into other models. Thus the mapping between the external and internal schema models is realized through an intermediate transformation into the metamodel. Because they are easy to transform into networks, hierarchies, and relations, E-R diagrams (or similar graphs) represent an obvious choice for the metamodel. However, since the system must translate data retrieval and manipulation of individual data fields, the conceptual schema requires a level of detail at which each attribute is visible. Thus diagrams such as that of Figure 16, which display the relationships between each single attribute, should be used. This solution suggests some obvious considerations regarding the relationships which should exist between the enterprise schema and the conceptual schema.

Since they are both used as a logical frame of reference for the enterprise, they should be, as far as possible, identical. Yet they must perform a different function. The former establishes the enterprise view of data, the latter links the external schema to the internal schema. The practical requirements dictated by their different roles could force undesirable differences upon them. On the other hand, the approach taken in this paper allows the two schemata to realize their individual goals along a very well-defined pattern. Both the enterprise schema and the conceptual schema are expressed by the same metamodel. The enterprise schema is only a summary of the conceptual schema obtained by grouping the attributes under categories called entities. Translation between the two schemata is accomplished by means of the shift operation between upper conceptual domain and lower conceptual domain described by Chen [10].

The database diagrams we have described represent a relational schema which is somewhat different from the one originally proposed by Codd. This is for a good reason: in Codd's approach a set of relations of assorted degree defines both the database schema and the database content. The schema is defined by the headings of the relations where the keys are underlined, and the database content is defined by the rows of the tables. This combined framework constitutes one of the most attractive features of the relational approach. Yet when we consider a multilevel schema architecture, such as that of [31], we find that database content need not be materialized at the conceptual schema level. Thus the congeniality of tables as vehicle for representing the database content becomes a moot quality for a conceptual schema. Unambiguous and complete definition of the logical structure of data (complete relatability) is the main requirement at this level, and it appears that graphical models attain this objective more completely and concisely than a set of normal form relations with keys. In particular, *CAZ*-graphs are capable of defining both atomic relationships and elementary FDs. These are not always inferable from the underlining of keys in normalized relations (see [5] for a discussion of the problems presented in this area by the various normal forms). Furthermore, a higher degree of data independence results from the use of small modules as the building blocks of the conceptual schema. In our diagrams we have used atomic relationships which are the smallest logical granules available. Thus, any change in one of these atomic relationships would only perturb the mapping to those external schemata which use that relationship. This is not true when third (or fourth) normal forms are used. Indeed, a change in any atomic relationship included in a relation (e.g., a change from many-to-one to many-to-many) would compromise the third normal form property and cause a change in the schema; then every relationship contained in a perturbed relation will also be affected. In conclusion, it appears that the diagrams discussed in this paper represent a reasonable usage of the relational model as the conceptual schema in an architectural framework such as that of [31]. This idea is explored even further in [32], where unnormalized relations are used at the external level, *CAZ*-graphs are used at the conceptual level, and fourth normal form type of relations are used at the internal level. It is also shown there how a query reformulation mechanism can be used to support the mapping from the external schema into the conceptual schema and thenceforth into the internal schema.

6. CONCLUSIONS

This paper has described a formal approach to the definition and the design of conceptual database diagrams which are useful in designing various forms of schemata and, most important, in the role of conceptual schemata for a DBMS architecture as that proposed by [31].

A number of previous authors have independently recognized and advocated graphical representations of database relationships as a powerful, convenient, and implementation-independent vehicle for conceptual data definition. The contribution of this paper consists in establishing a constructive and rigorous connection between certain types of database diagrams and some formal constraints which are currently used to describe database relationships in the framework of the relational data model. The database diagrams discussed here include E-R

diagrams and a newly introduced representation called a *CAZ*-graph. The relational constraints considered include both functional and multivalued dependencies.

The formal basis of our graphical schemata has been a combined representation of the two basic structures underlying every database relation: the *A*-structure defined by the atomic components of the relation, and the *Z*-structure defined by the elementary FDs of the given relation. This paper has contributed to a better understanding of the properties and the mutual relationships of these two structures. We have analyzed the various configurations under which atomic components and elementary FDs can combine in a relation and shown how their joined representation can describe a wide spectrum of database relationships, of which the well-known one-to-one, one-to-many, and many-to-many constitute only a small subset. The basic properties of the minimal atomic decompositions of a relation have also been investigated; a simple correspondence was established between the topological properties of these decompositions and the MD structure of the relation. This correspondence was shown useful in verifying the minimality of the resulting decompositions.

The proper schema for a given relation must supply a complete but nonredundant representation of its combined *A*- and *Z*-structures; thus it consists of (1) a set of atomic components which form a minimal decomposition for the given relation, and (2) a minimal cover set of its elementary FDs. A formal procedure for schema design given the FDs and MDs of the relations was then presented. Its design objective is to ensure completeness of representation (complete relatability) with minimum redundancy. The basic decomposition steps of the design procedure were taken from [33], where they were shown to be useful for the design of third normal form schemata.

Graphical representations of our schemata in the form of *CAZ*-graphs and E-R diagrams were then defined and illustrated by a number of examples. Finally, we discussed the application of these diagrams as a design aid and as a conceptual schema.

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