

Relational Algebra

Module 3, Lecture 1



Relational Query Languages

- Query languages: Allow manipulation and retrieval of data from a database.
- Relational model supports simple, powerful QLs:
 - Strong formal foundation based on logic.
 - Allows for much optimization.
- Query Languages != programming languages!
 - QLs not expected to be “Turing complete”.
 - QLs not intended to be used for complex calculations.
 - QLs support easy, efficient access to large data sets.



Formal Relational Query Languages

Two mathematical Query Languages form the basis for “real” languages (e.g. SQL), and for implementation:

- ¶ *Relational Algebra*: More operational, very useful for representing execution plans.
- *Relational Calculus*: Lets users describe what they want, rather than how to compute it. (Non-operational, *declarative*.)
- * *Understanding Algebra & Calculus is key to*
- * *understanding SQL, query processing!*



Preliminaries

- A query is applied to *relation instances*, and the result of a query is also a relation instance.
 - *Schemas* of input relations for a query are fixed (but query will run regardless of instance!)
 - The schema for the *result* of a given query is also fixed! Determined by definition of query language constructs.
- Positional vs. named-field notation:
 - Positional notation easier for formal definitions, named-field notation more readable.
 - Both used in SQL



Example Instances

- “Sailors” and “Reserves” relations for our examples.
- We’ll use positional or named field notation, assume that names of fields in query results are ‘inherited’ from names of fields in query input relations.

R1

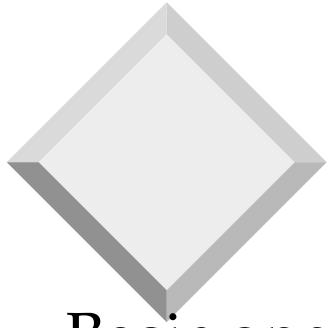
<u>sid</u>	<u>bid</u>	<u>day</u>
22	101	10/10/96
58	103	11/12/96

S1

<u>sid</u>	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S2

<u>sid</u>	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0



Relational Algebra

- Basic operations:
 - Selection (σ) Selects a subset of rows from relation.
 - Projection (π) Deletes unwanted columns from relation.
 - Cross-product (\times) Allows us to combine two relations.
 - Set-difference ($-$) Tuples in reln. 1, but not in reln. 2.
 - Union (\cup) Tuples in reln. 1 and in reln. 2.
- Additional operations:
 - Intersection, join, division, renaming: Not essential, but (very!) useful.
- Since each operation returns a relation, operations can be *composed!* (Algebra is “closed”.)

Projection

- Deletes attributes that are not in *projection list*.
- *Schema* of result contains exactly the fields in the projection list, with the same names that they had in the (only) input relation.
- Projection operator has to eliminate *duplicates!* (Why??)
 - Note: real systems typically don't do duplicate elimination unless the user explicitly asks for it. (Why not?)

sname	rating
yuppy	9
lubber	8
guppy	5
rusty	10

$\pi_{sname, rating}(S2)$

age
35.0
55.5

$\pi_{age}(S2)$



Selection


- Selects rows that satisfy *selection condition*.
- No duplicates in result! (Why?)
- *Schema* of result identical to schema of (only) input relation.
- *Result* relation can be the *input* for another relational algebra operation! (*Operator composition*.)

sid	sname	rating	age
28	yuppy	9	35.0
58	rusty	10	35.0

$$\sigma_{rating > 8}(S2)$$

sname	rating
yuppy	9
rusty	10

$$\pi_{sname, rating}(\sigma_{rating > 8}(S2))$$



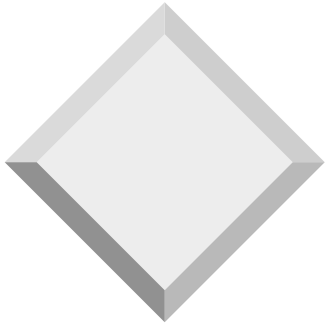
Union, Intersection, Set-Difference

- All of these operations take two input relations, which must be union-compatible:
 - Same number of fields.
 - ‘Corresponding’ fields have the same type.
- What is the *schema* of result?

$$S1 \cup S2$$

$$S1 \cap S2$$

$$S1 - S2$$



Union of Relations

$S1$

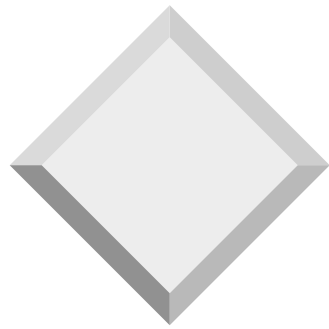
<u>sid</u>	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

$S2$

<u>sid</u>	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

$S1 \cup S2$

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0
44	guppy	5	35.0
28	yuppy	9	35.0



Intersection and Set-Difference

$S1$

<u>sid</u>	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

$S2$

<u>sid</u>	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

$S1 \cap S2$

sid	sname	rating	age
31	lubber	8	55.5
58	rusty	10	35.0

$S1 - S2$

sid	sname	rating	age
22	dustin	7	45.0

Cross-Product

- Each row of S1 is paired with each row of R1.
- *Result schema* has one field per field of S1 and R1, with field names `inherited' if possible.
 - *Conflict*: Both S1 and R1 have a field called *sid*.

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	22	101	10/10/96
22	dustin	7	45.0	58	103	11/12/96
31	lubber	8	55.5	22	101	10/10/96
31	lubber	8	55.5	58	103	11/12/96
58	rusty	10	35.0	22	101	10/10/96
58	rusty	10	35.0	58	103	11/12/96

* Renaming operator: $\rho (C(1 \rightarrow sid1, 5 \rightarrow sid2), S1 \times R1)$

Joins

- Condition Join: $R \bowtie_c S = \sigma_c (R \times S)$

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	58	103	11/12/96
31	lubber	8	55.5	58	103	11/12/96

$$S1 \bowtie_{S1.sid < R1.sid} R1$$

- *Result schema* same as that of cross-product.
- Fewer tuples than cross-product, might be able to compute more efficiently
- Sometimes called a *theta-join*.

Joins

- Equi-Join: A special case of condition join where the condition c contains only *equalities*

sid	sname	rating	age	bid	day
22	dustin	7	45.0	101	10/10/96
58	rusty	10	35.0	103	11/12/96

$$S1 \bowtie_{sid} R1$$

- *Result schema* similar to cross-product, but only one copy of fields for which equality is specified.
- Natural Join: Equijoin on *all* common fields.

Division

- Not supported as a primitive operator, but useful for expressing queries like:
Find sailors who have reserved all boats.
- Let A have 2 fields, x and y ; B have only field y :
 - $A/B = \{ \langle x \rangle \mid \forall y \in B \langle x, y \rangle \in A \}$
 - i.e., A/B contains all x tuples (sailors) such that for every y tuple (boat) in B , there is an xy tuple in A .
 - Or: If the set of y values (boats) associated with an x value (sailor) in A contains all y values in B , the x value is in A/B .
- In general, x and y can be any lists of fields; y is the list of fields in B , and $x \cup y$ is the list of fields of A .

Examples of Division A/B

sno	pno
s1	p1
s1	p2
s1	p3
s1	p4
s2	p1
s2	p2
s3	p2
s4	p2
s4	p4

A

pno
p2

B1

sno
s1
s2
s3
s4

A/B1

pno
p2
p4

B2

sno
s1
s4

A/B2

pno
p1
p2
p4

B3

sno
s1


A/B3

Expressing A/B Using Basic Operators

- Division is not essential op; just a useful shorthand.
 - (Also true of joins, but joins are so common that systems implement joins specially.)
- *Idea*: For A/B , compute all x values that are not 'disqualified' by some y value in B .
 - x value is *disqualified* if by attaching y value from B , we obtain an xy tuple that is not in A .


Disqualified x values: $\pi_x((\pi_x(A) \times B) - A)$

A/B : $\pi_x(A) -$ all disqualified tuples



Find names of sailors who've reserved boat #103

- Solution 1: $\pi_{sname}((\sigma_{bid=103} Reserves) \bowtie Sailors)$
- Solution 2: $\rho(Temp1, \sigma_{bid=103} Reserves)$
 $\rho(Temp2, Temp1 \bowtie Sailors)$
 $\pi_{sname}(Temp2)$
- Solution 3 $\pi_{sname}(\sigma_{bid=103}(Reserves \bowtie Sailors))$



Find names of sailors who've reserved a red boat


- Information about boat color only available in Boats; so need an extra join:

$$\pi_{sname}((\sigma_{color='red'} Boats) \bowtie Reserves \bowtie Sailors)$$

* A more efficient solution:

$$\pi_{sname}(\pi_{sid}((\pi_{bid} \sigma_{color='red'} Boats) \bowtie Res) \bowtie Sailors)$$

* *A query optimizer can find this given the first solution!*



Find sailors who've reserved a red or a green boat

- Can identify all red or green boats, then find sailors who've reserved one of these boats:

$$\rho (\text{Tempboats}, (\sigma_{color='red' \vee color='green'} \text{Boats}))$$
$$\pi_{sname}(\text{Tempboats} \bowtie \text{Reserves} \bowtie \text{Sailors})$$

- * Can also define Tempboats using union! (How?)
- * What happens if \vee is replaced by \wedge in this query?

Find sailors who've reserved a red and a green boat

- Previous approach won't work! Must identify sailors who've reserved red boats, sailors who've reserved green boats, then find the intersection (note that *sid* is a key for Sailors):

$$\rho (Tempred, \pi_{sid}((\sigma_{color='red'} Boats) \bowtie Reserves))$$
$$\rho (Tempgreen, \pi_{sid}((\sigma_{color='green'} Boats) \bowtie Reserves))$$
$$\pi_{sname}((Tempred \cap Tempgreen) \bowtie Sailors)$$



Find the names of sailors who've reserved all boats

- Uses division; schemas of the input relations to / must be carefully chosen:

$$\rho (Tempkids, (\pi_{sid, bid} Reserves) / (\pi_{bid} Boats))$$

$$\pi_{sname} (Tempkids \bowtie Sailors)$$

- * To find sailors who've reserved all 'Interlake' boats:

$$\dots / \pi_{bid} (\sigma_{bname='Interlake'} Boats)$$