Chapter 7

Exercise 7.1
Input:

$$x(t) \in \{a, b, c\}$$

Output:

$$z(t) \in \{p, q\}$$

Function:

$$z(t) = \begin{cases} q & \text{if number of a's in } x(0, t-1) \text{ is even and number of b's is odd.} \\ p & \text{otherwise} \end{cases}$$

State:

$$\begin{aligned} \underline{s}(t) &= (s_a(t), s_b(t)) \\ s_a(t) &= (\text{number of a's}) \bmod 2 \\ s_b(t) &= (\text{number of b's}) \bmod 2 \end{aligned}$$

Initial state:

$$\begin{aligned} s_a(0) &= 0 \\ s_b(0) &= 0 \end{aligned}$$

Transition function:

$$\begin{aligned} s_a(t+1) &= \begin{cases} s_a(t)' & \text{if } x(t) = a \\ s_a(t) & \text{otherwise} \end{cases} \\ s_b(t+1) &= \begin{cases} s_b(t)' & \text{if } x(t) = b \\ s_b(t) & \text{otherwise} \end{cases} \end{aligned}$$

Output function:

$$z(t) = \begin{cases} q & \text{if } \underline{s}(t)=(0,1) \\ p & \text{otherwise} \end{cases}$$

The state transition and output table is shown below. The state diagram for the system is presented in Figure **??** on page **??**.

	Input			
PS	$x = a$	$x = b$	$x = c$	
(0,0)	(1,0)	(0,1)	(0,0)	p
(0,1)	(1,1)	(0,0)	(0,1)	q
(1,0)	(0,0)	(1,1)	(1,0)	p
(1,1)	(0,1)	(1,0)	(1,1)	p
	NS			Output (z)

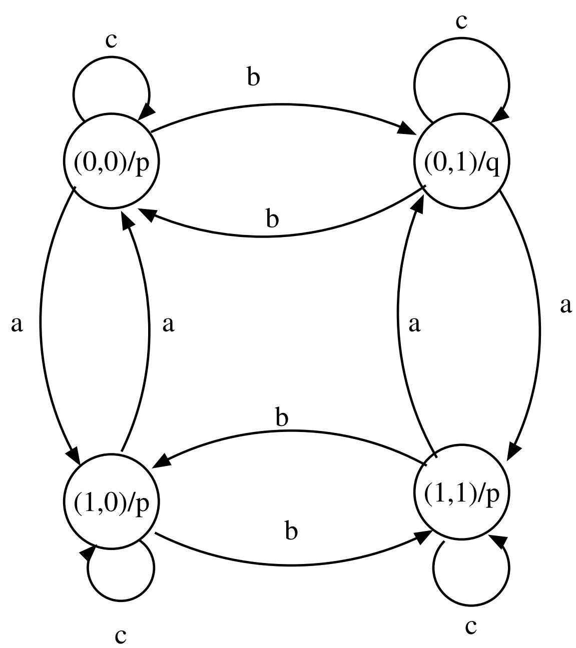

Figure 7.1: State Transition Diagram of Exercise 6.1

Exercise 7.3

State table:

	$x(t)$	
PS	$x = a$	$x = b$
S_0	$S_0, 01$	$S_1, 11$
S_1	$S_2, 11$	$S_3, 00$
S_2	$S_3, 11$	$S_3, 11$
S_3	$S_3, 00$	$S_4, 01$
S_4	$S_4, 01$	$S_0, 00$
	NS, z	

Exercise 7.5 The state diagram for this exercise is shown in Figure **??** on page **??**

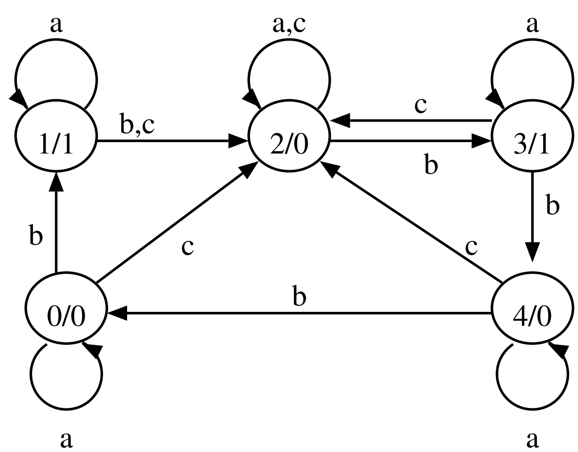

Figure 7.2: State diagram of Exercise 7.5

Exercise 7.7

There are sixteen possible states. To obtain the state diagram, we first obtain the state table, by an evaluation of the expressions. To simplify the notation, we label the states with an integer $0 \leq j \leq 15$ whose binary representation is the bit-vector (s_3, s_2, s_1, s_0). The state table is

	Input		
PS	$x = 0$	$x = 1$	
0	0	1	0
1	2	3	0
2	4	5	0
3	6	7	0
4	8	9	0
5	10	11	0
6	12	13	0
7	14	15	0
8	1	0	1
9	3	2	1
10	5	4	1
11	7	6	1
12	9	8	1
13	11	10	1
14	13	12	1
15	15	14	1
	NS		z

Observe that the state transition function is:

$$s(t+1) = (2s(t) + (\left\lfloor \frac{s(t)}{8} \right\rfloor + x(t)) \bmod 2) \bmod 16$$

The corresponding state diagram is shown in Figure **??**.

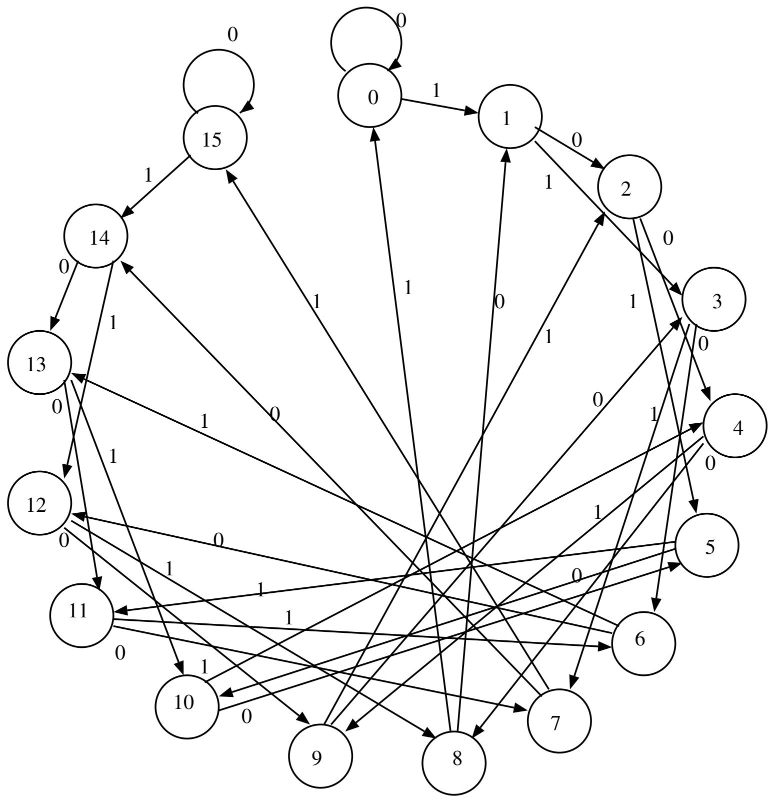

Figure 7.3: State diagram for Exercise 7.7

Exercise 7.9 The sequential systems described by each table are:
(a) Mealy machine. The output function is a function of the Present State and of the Input.
(b) Moore machine. The output function does not depend on the input.
(c) Moore machine.

Exercise 7.11

The time behavior of the module-p counter is given by the arithmetic expressions

$$z(t) = (\sum_{i=0}^{t} s(0) + x(t)) \bmod p$$

$$s(0) = S_{IN}$$

where $S_{IN} \in \{0, 1, \ldots, p-1\}$ and $x(t) \in \{0, 1\}$

Exercise 7.13

The state diagram of Figure 7.11(a) of the textbook results in the following state table:

PS	Input 0	Input 1
S_{init}	$S_0, 0$	$S_1, 0$
S_0	$S_{00}, 0$	$S_{01}, 0$
S_1	$S_{10}, 0$	$S_{11}, 0$
S_{00}	$S_{00}, 0$	$S_{01}, 0$
S_{01}	$S_{10}, 0$	$S_{11}, 0$
S_{10}	$S_{00}, 0$	$S_{01}, 1$
S_{11}	$S_{10}, 0$	$S_{11}, 0$
	$NS, Output$	

From the table we get

$$P_1 = (S_{init}, S_0, S_1, S_{00}, S_{01}, S_{11})(S_{10})$$

To obtain P_2, we determine the group of states P_1 to which the successors of each state belong.

	group 1 $(S_{init}, S_0, S_1, S_{00}, S_{01}, S_{11})$	group 2 (S_{10})
0	1 1 2 1 2 2	1
1	1 1 1 1 1 1	1

Consequently,

$$P_2 = (S_{init}, S_0, S_{00})(S_1, S_{01}, S_{11})(S_{10})$$

To obtain P_3, we determine the group of states of P_2 to which the successors of each state belong.

	group 1 (S_{init}, S_0, S_{00})	group 2 (S_1, S_{01}, S_{11})	group 3 (S_{10})
0	1 1 1	3 3 3	1
1	2 2 2	2 2 2	2

Thus, $P_3 = P_2 = P = (S_{init}, S_0, S_{00})(S_1, S_{01}, S_{11})(S_{10}) = (S_{init}, A, B)$

Using this partition we construct the state table (from the original table):

PS	Input 0	Input 1
S_{init}	$S_{init}, 0$	$A, 0$
A	$B, 0$	$A, 0$
B	$S_{init}, 0$	$A, 1$
	$NS, Output$	

The corresponding state diagram is equivalent to the one presented in Figure 7.11(b) of the textbook, thus both diagrams represent the same system.

Exercise 7.15

From the state table we get

$$P_1 = (a, b, c, e)(d, h)(f)(g)$$

To obtain P_2, we determine the class of P_1 to which the successors of the states belong.

	1 (a, b, c, e)	2 (d, h)	3 (f)	4 (g)
0	3 2 3 2	4 4	3	4
1	1 1 1 1	1 1	1	2

Thus,

$$P_2 = (a, c)(b, e)(d, h)(f)(g)$$

To obtain P_3, we determine the group of states of P_2 to which the successors of the state belong.

	1 (a, c)	2 (b, e)	3 (d, h)	4 (f)	5 (g)
0	4 4	3 3	5 5	4	5
1	2 2	1 1	1 1	2	3

Therefore, $P = P_3 = P_2 = (a, c)(b, e)(d, h)(f)(g)$ and the reduced table is

	Input	
PS	$x = 0$	$x = 1$
a	$f, 0$	$b, 0$
b	$d, 0$	$a, 0$
d	$g, 1$	$a, 0$
f	$f, 1$	$b, 1$
g	$g, 0$	$d, 1$
	$NS, Output$	

Exercise 7.17

Based on the outputs for each state we get the first partition

$$P_1 = (A, C, G, H)(B, D, E)(F)$$

To obtain P_2, we determine the class of P_1 to which the successors of the states belong.

	group 1				group 2			group 3
	A	C	G	H	B	D	E	F
a	2	2	2	2	1	1	1	
b	1	1	1	1	3	3	3	
c	2	2	2	2	2	2	2	
d	2	3	3	3	2	2	2	

Partition P_2 is

	group 1	group 2			group 3			group 4
	A	C	G	H	B	D	E	F
a		3	3	3	2	2	2	
b		1	1	2	4	4	4	
c		3	3	3	3	3	3	
d		4	4	4	3	3	3	

Partition P_3 is

	group 1	group 2		group 3	group 4			group 5
	A	C	G	H	B	D	E	F
a		4	4		2	2	2	
b		1	1		5	5	5	
c		4	4		4	4	4	
d		5	5		4	4	4	

STOP.

The equivalent states are: {A}, {B,D,E}, {C,G}, {F}, {H}

Minimal state transition table:

PS	$x = a$	$x = b$	$x = c$	$x = d$
A	$B/1$	$C/0$	$B/1$	$B/1$
B	$C/0$	$F/1$	$B/1$	$B/0$
C	$B/1$	$A/0$	$B/1$	$F/1$
F	$C/1$	$F/1$	$B/0$	$H/0$
H	$B/1$	$C/0$	$B/1$	$F/1$
	$NS/output$			

Exercise 7.19

Input: $x(t) \in \{a, b, c, d\}$

Output: $z(t) \in \{0, 1\}$

State: Since the pattern to be recognized has four symbols, using the vector-state approach, we represent the state as the vector

$$\underline{s} = (s_2, s_1, s_0)$$

and the states are $s(t) \in \{0, 1, 2, ..., 7\}$.

Function: The corresponding state transition and output functions are:

$$s_i(t+1) = s_{i-1}(t) \quad 1 \leq i \leq 2$$
$$s_0(t+1) = x(t)$$

$$z(t) = \begin{cases} 1 & \textbf{if } (\underline{s}(t), x(t)) = abca \\ 0 & \textbf{otherwise} \end{cases}$$

For the minimum-number-of-states approach, we define a state with the following four values:

$$\begin{aligned} s(t) &= A \textbf{ if } x(t-1) = a \\ s(t) &= B \textbf{ if } x(t-2, t-1) = ab \\ s(t) &= C \textbf{ if } x(t-3, t-1) = abc \\ s(t) &= D \textbf{ if } \text{none of the above (Initial)} \end{aligned}$$

The state description is given by the state diagram shown in Figure **??**and the following table:

	Input			
PS	*a*	*b*	*c*	*d*
A	*A*, 0	*B*, 0	*D*, 0	*D*, 0
B	*A*, 0	*D*.0	*C*, 0	*D*, 0
C	*A*, 1	*D*, 0	*D*, 0	*D*, 0
D	*A*, 0	*D*, 0	*D*, 0	*D*, 0
	NS, *z*			

The output sequence corresponding to the given input sequence is

$x(t)$	a	a	b	c	a	b	c	a	d	a	a	a	b	c	a	
$s(t)$	D	A	A	B	C	A	B	C	A	D	A	A	A	B	C	A
$z(t)$	0	0	0	0	1	0	0	1	0	0	0	0	0	0	1	

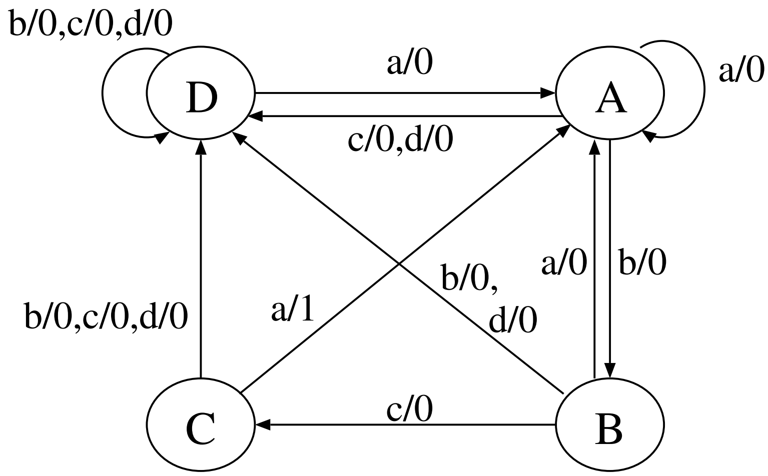

Figure 7.4: State diagram for Exercise 7.19

Exercise 7.21

The state diagram that describes the system is presented in Figure **??** page **??**, using the minimum state approach. The vector state approach is not possible to be used since the system is not a finite memory system.

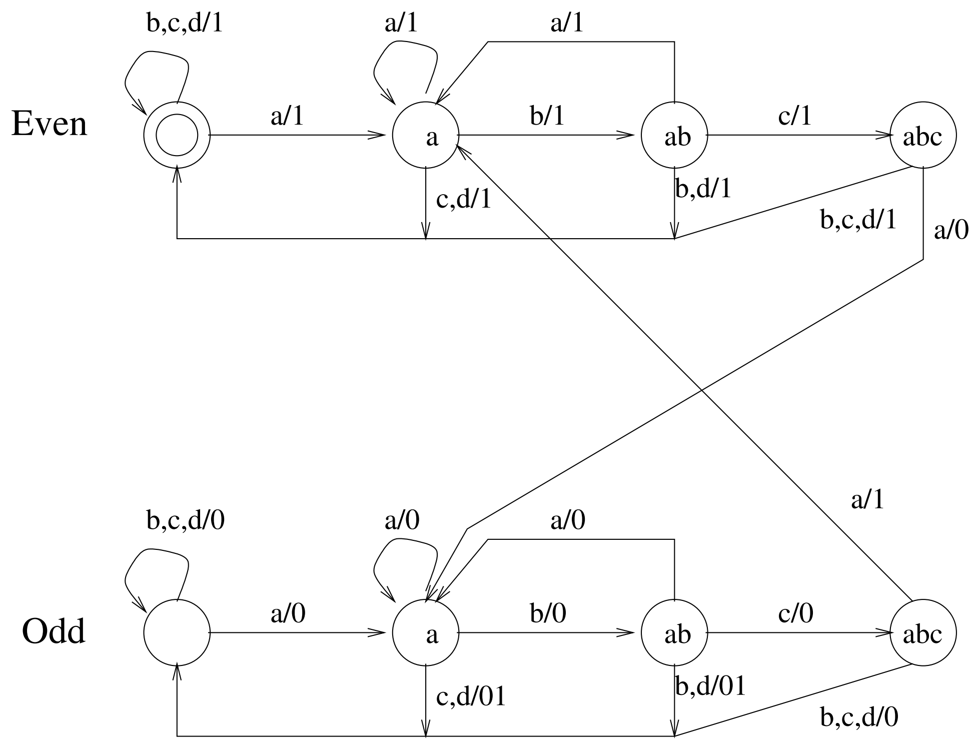

Figure 7.5: State Diagram for Exercise 7.21

Exercise 7.23

Following the hint given in the exercise, the sequential system is decomposed into two. System A recognizes the pattern 0110, and system B recognizes the pattern 1001, as shown in Figure **??**. The output $z(t)$ is defined as a function of the system A states ($S_A(t)$) and system B states ($S_B(t)$) as follows:

$S_A(t)$	$S_B(t)$	$z(t)$
$\{0,1,2,3\}$	–	0
4	$\{0,1,2,3\}$	1
4	4	2

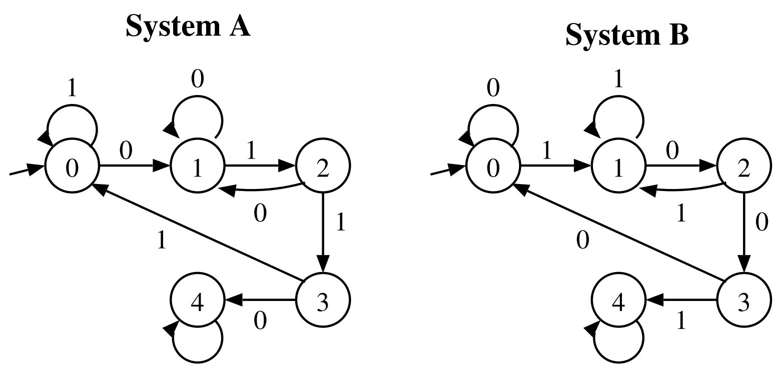

Figure 7.6: State Transition Diagram of Exercise 7.23

The timing behavior for the given input sequence, considering $s(t) = (S_A(t), S_B(t))$ is:

t	0	1	2	3	4	5	6	7
$x(t)$	0	0	1	0	1	1	1	0
$s(t)$	(0,0)	(1,0)	(2,1)	(1,2)	(2,1)	(3,1)	(0,1)	(1,2)
$z(t)$	0	0	0	0	0	0	0	0
t	8	9	10	11	12	13	14	15
$x(t)$	1	1	0	0	1	0	1	0
$s(t)$	(2,1)	(3,1)	(4,2)	(4,3)	(4,4)	(4,4)	(4,4)	(4,4)
$z(t)$	0	0	1	1	2	2	2	2

Exercise 7.25
To make the design simple and tractable, we make the following assumptions:

- Selection of stamps to buy is the first step
- If amount of accumulated coins is at least 15c greater than the face value of the selected stamp, return all coins

Then the inputs and outputs are as follows (ref. Figure **??**):
Inputs:

$$\begin{aligned} Reset: &\in \{T, F\} \\ CoinType: &\in \{N, D, Q\} \\ StampType: &\in \{S1(20c), S2(40c), S3(50c)\} \\ ReturnCoinRequest: &\in \{T, F\} \end{aligned}$$

Outputs:

$RS1$	–	Release stamp 1 (20c)
$RS2$	–	Release stamp 2 (40c)
$RS3$	–	Release stamp 3 (50c)
RC	–	Return Coin
RN	–	Return Nickel
RD	–	Return Dime

All outputs take values from the set {T, F}. The state diagram is shown in Figure **??**.

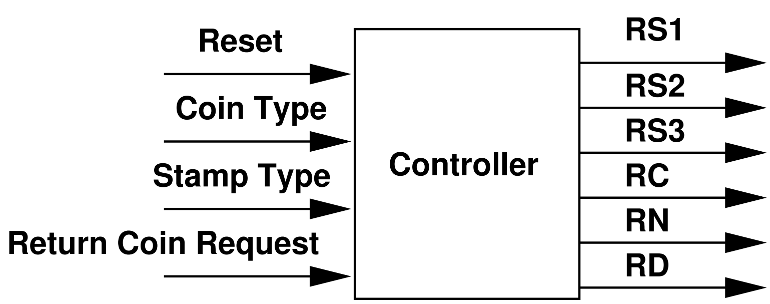

Figure 7.7: Inputs & Outputs of Vending Machine Controller

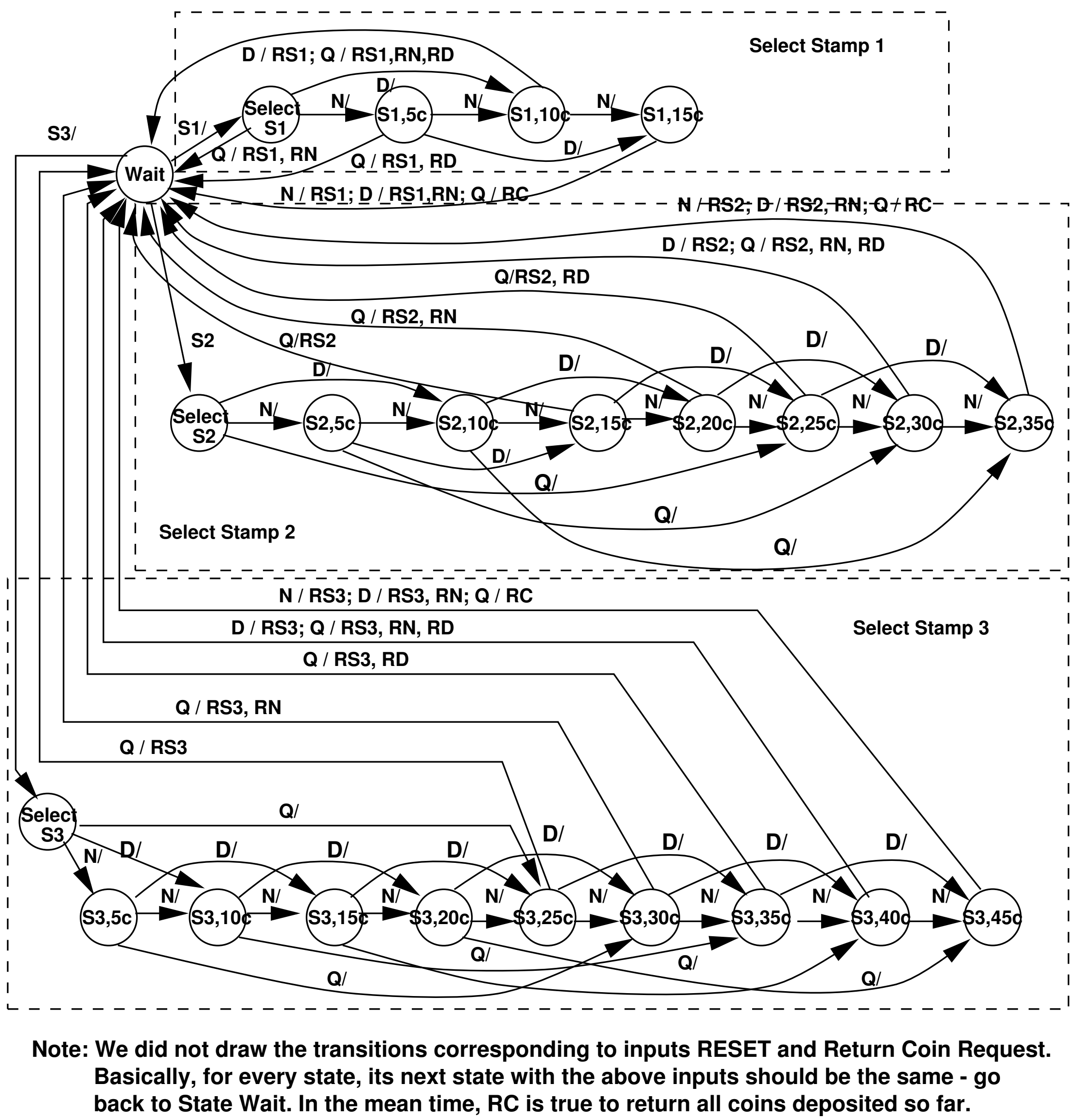

Figure 7.8: State Diagram of Exercise 7.25