Towards Querying Probabilistic Knowledge Bases

Guy Van den Broeck

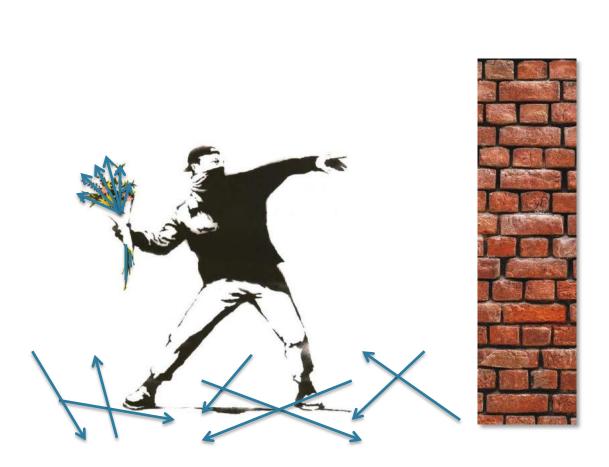
First Conference on Automated Knowledge Base Construction





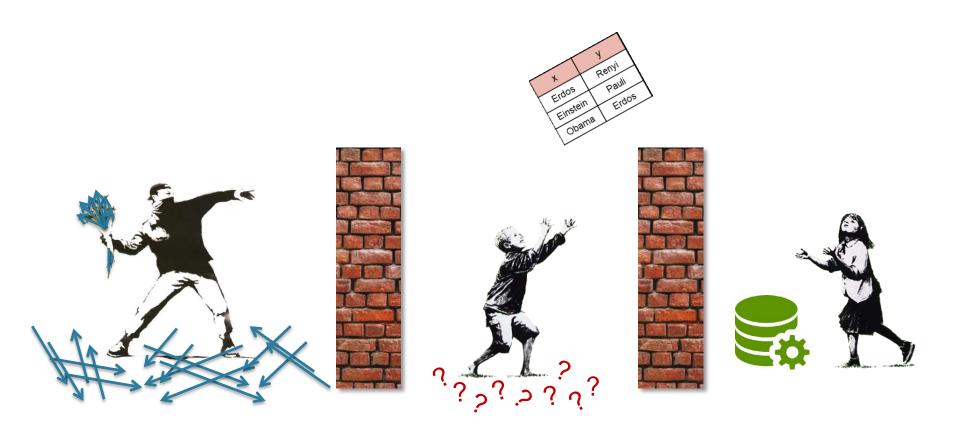
Cartoon Motivation

Cartoon Motivation: Relational Embedding Models

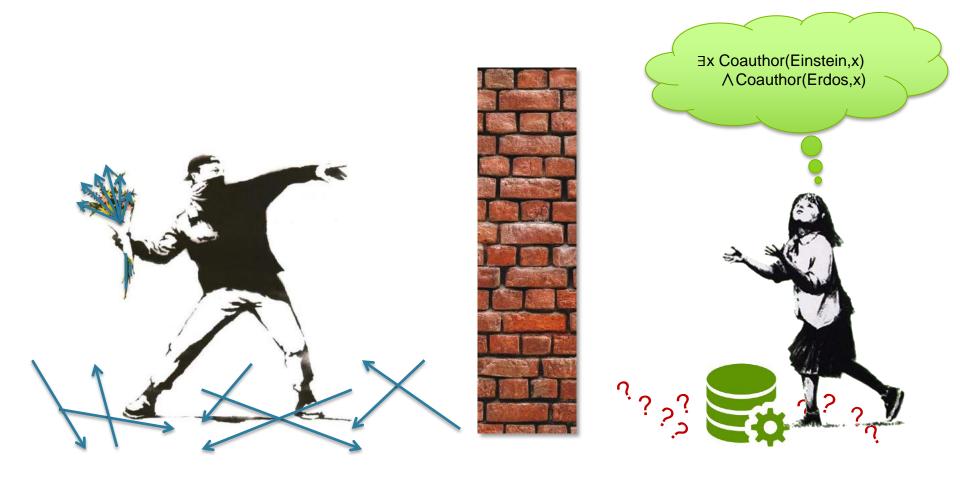




Cartoon Motivation 2: Relational Embedding Models

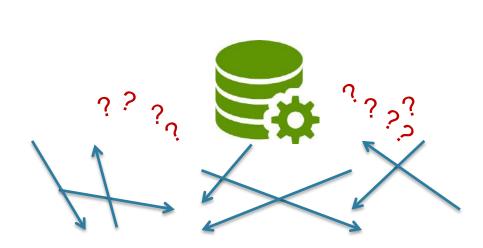


Goal 1: Probabilistic Query Evaluation



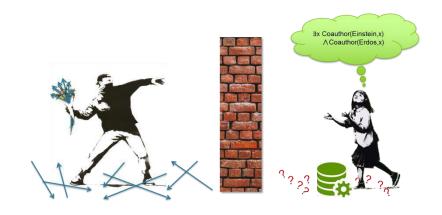
Goal 2: Querying Relational Embedding Models

∃x Coauthor(Einstein,x) ∧Coauthor(Erdos,x)





Probabilistic Query Evaluation



What we'd like to do...

Has anyone published a paper with both Erdos and Einstein





All

News

Images

Videos

Shopping

More ▼

Search tools

About 82,400 results (0.73 seconds)

Erdős number - Wikipedia, the free encyclopedia

https://en.wikipedia.org/wiki/Erdős_number ▼ Wikipedia ▼

He published more papers during his lifetime (at least 1,525) than any other ...

Anybody else's Erdős number is k + 1 where k is the lowest Erdős number of any coauthor. .. Albert Einstein and Sheldon Lee Glashow have an Erdős number of 2.

and mathematician Ruth Williams, both of whom have an Erdös number of 2.

Erdős-Bacon number - Wikipedia, the free encyclopedia

https://en.wikipedia.org/wiki/Erdős-Bacon_number ▼ Wikipedia ▼ This article possibly contains previously unpublished synthesis of published ... Her paper gives her an Erdős number of 4, and a Bacon number of 2, both of ...

What we'd like to do...

 $\exists x \; Coauthor(Einstein,x) \; \land \; Coauthor(Erdos,x)$





All News

Images

Videos

Shopping

More ▼

Search tools

About 82,400 results (0.73 seconds)

Erdős number - Wikipedia, the free encyclopedia

https://en.wikipedia.org/wiki/Erdős_number ▼ Wikipedia ▼ He published more papers during his lifetime (at least 1,525) than any other ... Anybody else's Erdős number is k + 1 where k is the lowest Erdős number of any coauthor. ... Albert Einstein and Sheldon Lee Glashow have an Erdős number of 2. ... and mathematician Ruth Williams, both of whom have an Erdős number of 2.

Erdős-Bacon number - Wikipedia, the free encyclopedia

https://en.wikipedia.org/wiki/Erdős-Bacon_number ▼ Wikipedia ▼ This article possibly contains previously unpublished synthesis of published ... Her paper gives her an Erdős number of 4, and a Bacon number of 2, both of ...

Einstein is in the Knowledge Graph

Albert Einstein



Q

All News

Images

Books

Videos

More ▼

Search tools

About 82,800,000 results (0.45 seconds)

The Official Licensing Site of Albert Einstein

Walcome to the Offici

Welcome to the Official Licensing Site of Albert Einstein. Learn more about Albert Einstein and contact us today for any commercial licensing inquiries.

Albert Einstein - Wikipedia, the free encyclopedia

https://en.wikipedia.org/wiki/Albert_Einstein ▼ Wikipedia ▼

Albert Einstein (/ˈaɪnstaɪn/; German: [ˈalbɛɐ̯t ˈaɪnʃtaɪn] (listen); 14 March 1879 – 18 April 1955) was a German-born theoretical physicist.

Hans Albert Einstein - Mass-energy equivalence - Eduard Einstein - Elsa Einstein

Albert Einstein (@AlbertEinstein) | Twitter

https://twitter.com/AlbertEinstein

16 hours ago - View on Twitter

ICYMI, Albert Einstein knew a thing or two about being romantic. Learn about the love letters he wrote. guff.com/didnt-know-einst...

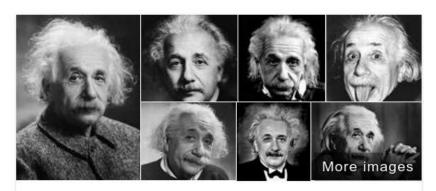
20 hours ago - View on Twitter

An interesting read on Einstein's superstar status. What are your thoughts? twitter.com/aeonmag/statu...



Albert Einstein - Biographical - Nobelprize.org

www.nobelprize.org/nobel_prizes/physics/.../einstein-bio.htm... ▼ Nobel Prize ▼ Albert Einstein was born at Ulm, in Württemberg, Germany, on March 14, 1879. ...



Albert Einstein

Theoretical Physicist

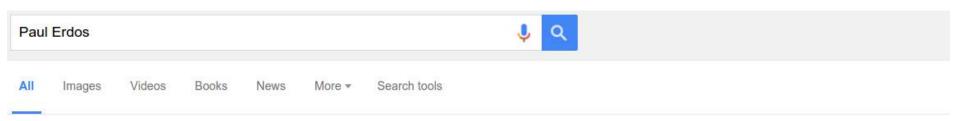
Albert Einstein was a German-born theoretical physicist. He developed the general theory of relativity, one of the two pillars of modern physics. Einstein's work is also known for its influence on the philosophy of science. Wikipedia

Born: March 14, 1879, Ulm, Germany Died: April 18, 1955, Princeton, NJ

Influenced by: Isaac Newton, Mahatma Gandhi, More

Children: Eduard Einstein, Lieserl Einstein, Hans Albert Einstein Spouse: Elsa Einstein (m. 1919–1936), Mileva Marić (m. 1903–1919)

Erdős is in the Knowledge Graph



About 333,000 results (0.35 seconds)

Paul Erdős - Wikipedia, the free encyclopedia

https://en.wikipedia.org/wiki/Paul Erdős ▼ Wikipedia ▼ Paul Erdős was a Hungarian Jewish mathematician. He was one of the most prolific mathematicians of the 20th century. He was known both for his social ... Fan Chung - Ronald Graham - Béla Bollobás - Category: Paul Erdős

The Man Who Loved Only Numbers - The New York Times

https://www.nvtimes.com/books/.../hoffman-man.ht... ▼ The New York Times ▼ Paul Erdös was one of those very special geniuses, the kind who comes along only once in a very long while yet he chose, quite consciously I am sure, to share ...

Paul Erdos | Hungarian mathematician | Britannica.com

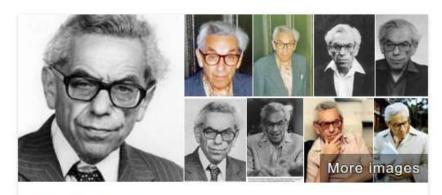
www.britannica.com/biography/Paul-Erdos ▼ Encyclopaedia Britannica ▼ Paul Erdős, (born March 26, 1913, Budapest, Hungary-died September 20, 1996, Warsaw, Poland), Hungarian "freelance" mathematician (known for his work ...

Paul Erdős - University of St Andrews

www-groups.dcs.st-and.ac.uk/~history/Biographies/Erdos.html > Paul Erdős came from a Jewish family (the original family name being Engländer) although neither of his parents observed the Jewish religion. Paul's father ...

[PDF] Paul Erdős Mathematical Genius, Human - UnTruth.org www.untruth.org/~iosh/math/Paul%20Erdös%20bio-rev2.pdf ▼

by J Hill - 2004 - Related articles



Paul Erdős

Mathematician

Paul Erdős was a Hungarian Jewish mathematician. He was one of the most prolific mathematicians of the 20th century. He was known both for his social practice of mathematics and for his eccentric lifestyle. Wikipedia

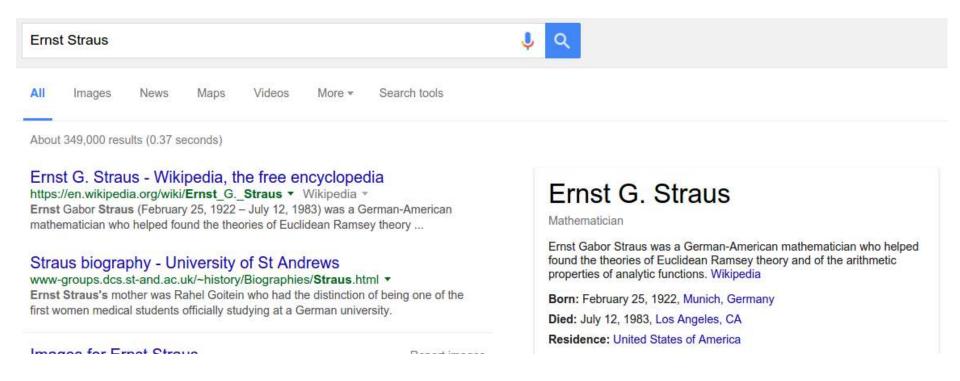
Born: March 26, 1913, Budapest, Hungary Died: September 20, 1996, Warsaw, Poland Education: Eötvös Loránd University (1934)

Books: Probabilistic Methods in Combinatorics, More

Notable students: Béla Bollobás, Alexander Soifer, George B. Purdy,

Incanh Kruckal

This guy is in the Knowledge Graph



... and he published with both Einstein and Erdos!

Desired Query Answer

Has anyone published a paper with both Erdos and Einstein







Ernst Straus



Barack Obama, ...



Justin Bieber, ...

- Fuse uncertain information from web
 - **⇒** Embrace probability!
- 2. Cannot come from labeled data
 - **⇒** Embrace query eval!

Probabilistic Databases

Has anyone published a paper with both Erdos and Einstein





Tuple-independent probabilistic database

st	X	Р
Scientist	Erdos	0.9
Scie	Einstein	0.8
0,	Pauli	0.6

o	Х	у	Р
oauthor	Erdos	Renyi	0.6
oal	Einstein	Pauli	0.7
S	Obama	Erdos	0.1

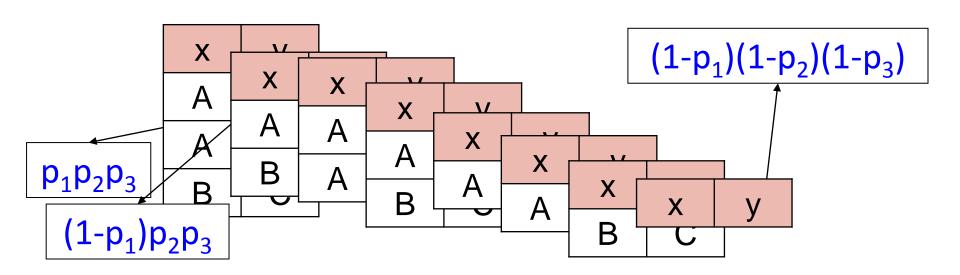
 Learned from the web, large text corpora, ontologies, etc., using statistical machine learning.

Tuple-Independent Probabilistic DB

Probabilistic database D:

Jor	Х	у	Р
Coauthor	Α	В	p ₁
Ö	Α	С	p ₂
	В	С	p_3

Possible worlds semantics:

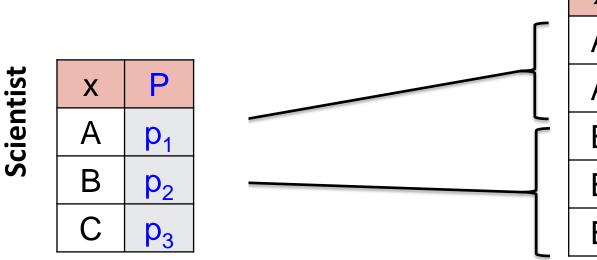


Probabilistic Query Evaluation

 $Q = \exists x \exists y \ Scientist(x) \land Coauthor(x,y)$

$$P(Q) = 1 - \{1 - p_1^*[1 - (1 - q_1)^*(1 - q_2)] \}^*$$

$$\{1 - p_2^*[1 - (1 - q_3)^*(1 - q_4)^*(1 - q_5)] \}$$



X	У	Ω.
Α	D	q_1
Α	Ш	q_2
В	F	q_3
В	G	q_4
В	Ι	q_5

Coauthor

Lifted Inference Rules

Preprocess Q (omitted),
Then apply rules (some have preconditions)

$$P(\neg Q) = 1 - P(Q)$$

Negation

$$P(Q1 \land Q2) = P(Q1) P(Q2)$$

 $P(Q1 \lor Q2) = 1 - (1 - P(Q1)) (1 - P(Q2))$

Decomposable Λ,V

$$P(\forall z \ Q) = \prod_{A \in Domain} P(Q[A/z])$$

$$P(\exists z \ Q) = 1 - \prod_{A \in Domain} (1 - P(Q[A/z]))$$

Decomposable ∃,∀

$$P(Q1 \land Q2) = P(Q1) + P(Q2) - P(Q1 \lor Q2)$$

 $P(Q1 \lor Q2) = P(Q1) + P(Q2) - P(Q1 \land Q2)$

Inclusion/ exclusion

Example Query Evaluation

 $Q = \exists x \exists y \ Scientist(x) \land Coauthor(x,y)$

```
Decomposable ∃-Rule
```

```
P(Q) = 1 - \Pi_{A \in Domain} (1 - P(Scientist(A) \land \exists y Coauthor(A,y))
```

Check independence:

```
Scientist(A) \land \exists y \text{ Coauthor}(A,y)
Scientist(B) \land \exists y \text{ Coauthor}(B,y)
```

```
= 1 - (1 - P(Scientist(A) ∧ ∃y Coauthor(A,y))
x (1 - P(Scientist(B) ∧ ∃y Coauthor(B,y))
x (1 - P(Scientist(C) ∧ ∃y Coauthor(C,y))
x (1 - P(Scientist(D) ∧ ∃y Coauthor(D,y))
x (1 - P(Scientist(E) ∧ ∃y Coauthor(E,y))
x (1 - P(Scientist(F) ∧ ∃y Coauthor(F,y))
```

Complexity PTIME

Limitations

 $H_0 = \forall x \forall y \text{ Smoker}(x) \lor \text{ Friend}(x,y) \lor \text{ Jogger}(y)$

The decomposable ∀-rule:

$$P(\forall z \ Q) = \Pi_{A \in Domain} P(Q[A/z])$$

... does not apply:

 $H_0[Alice/x]$ and $H_0[Bob/x]$ are dependent:

Dependent

∀y (Smoker(Alice) V Friend(Alice,y) V Jogger(y))

∀y (Smoker(Bob) V Friend(Bob,y) V Jogger(y))

Lifted inference sometimes fails.

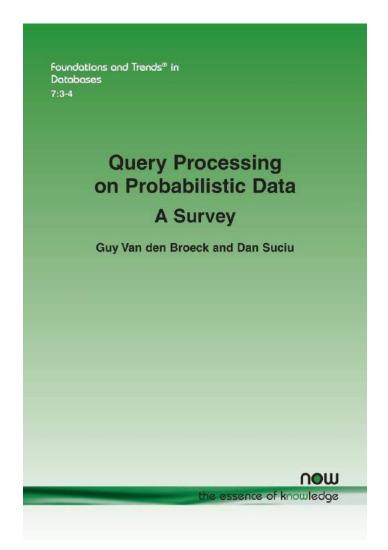
Are the Lifted Rules Complete?

Dichotomy Theorem for Unions of Conjunction Queries / Monotone CNF

- If lifted rules succeed, then PTIME query
- If lifted rules fail, then query is #P-hard

Lifted rules are complete for UCQ!

Commercial Break

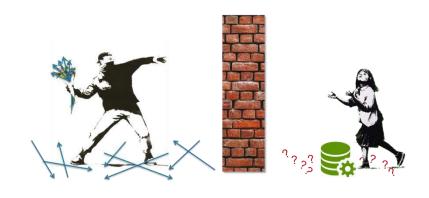


• Survey book

http://www.nowpublishers.com/article/Details/DBS-052

• IJCAI 2016 tutorial http://web.cs.ucla.edu/~guyvdb/talks/IJCAI16-tutorial/

Throwing Relational Embedding Models Over the Wall



- Notion of distance d(h, r, t) in vector space (Euclidian, 1-cosine, ...)
- Probabilistic semantics:
 - Distance d(h, r, t) = 0 is certainty
 - Distance d(h, r, t) > 0 is uncertainty

$$P(r(h,t)) \approx e^{-\alpha \cdot d(h,r,t)}$$

What About Tuple-Independence?

- Deterministic databases
 - = tuple-independent
- Relational embedding models
 - = tuple-independent

At no point do we model joint uncertainty between tuples

- We can capture correlations, but query evaluation becomes much harder!
 - See probabilistic database literature
 - See statistical relational learning literature

So everything is solved?

What we'd like to do...

 $\exists x \ Coauthor(Einstein,x) \ \land \ Coauthor(Erdos,x)$







Ernst Straus



Kristian Kersting, ...



Justin Bieber, ...

Open World DB

What if fact missing?

Probability 0 for:

Coauthor

X	Υ	Р
Einstein	Straus	0.7
Erdos	Straus	0.6
Einstein	Pauli	0.9
Erdos	Renyi	0.7
Kersting	Natarajan	8.0
Luc	Paol	0.1

 $Q1 = \exists x \ Coauthor(Einstein, x) \land Coauthor(Erdos, x)$

 $Q2 = \exists x \text{ Coauthor}(\text{Bieber}, \mathbf{x}) \land \text{ Coauthor}(\text{Erdos}, \mathbf{x})$

Q3 = Coauthor(Einstein, Straus) ∧ Coauthor(Erdos, Straus)

 $Q4 = Coauthor(Einstein, Bieber) \land Coauthor(Erdos, Bieber)$

Q5 = Coauthor(Einstein, Bieber) ∧ ¬Coauthor(Einstein, Bieber)

Intuition

Х	Υ	Р
Einstein	Straus	0.7
Erdos	Straus	0.6
Einstein	Pauli	0.9
Erdos	Renyi	0.7
Kersting	Natarajan	0.8
Luc	Paol	0.1

$Q1 = \exists x Coauthor(Einstein, x)$	∧ Coauthor(Erdos,x)
--	---------------------

$$Q2 = \exists x \text{ Coauthor}(\text{Bieber}, \mathbf{x}) \land \text{ Coauthor}(\text{Erdos}, \mathbf{x})$$

Q3 = Coauthor(Einstein, Straus) ∧ Coauthor(Erdos, Straus)

Q4 = Coauthor(Einstein, Bieber) \(\Lambda\) Coauthor(Erdos, Bieber)

 $Q5 = Coauthor(Einstein, Bieber) \land \neg Coauthor(Einstein, Bieber)$

We know for sure that $P(Q1) \ge P(Q3)$, $P(Q1) \ge P(Q4)$ and $P(Q3) \ge P(Q5)$, $P(Q4) \ge P(Q5)$ because P(Q5) = 0.

We have strong evidence that $P(Q1) \ge P(Q2)$.

Problem: Curse of Superlinearity

Reality is worse: tuples intentionally missing!
Sibling

X	у	Р

0.8

0.6

0.4

0.2

0.0

NELL(Movie) | NELL | [61939 tuples] | [2467442 t | 3.22 G |]

S. of data

Facebook scale ⇒ 200 Exabytes of data All Google storage is 2 exabytes...

Randall Munroe. Google's datacenters on punch cards, 2015.

ICE SHEET

Bayesian Learning Loop

Bayesian view on learning:

1. Prior belief:

```
P(Coauthor(Straus, Pauli)) = 0.01
```

2. Observe page

```
P(Coauthor(Straus, Pauli| | ) = 0.2
```

3. Observe page



Principled and sound reasoning!

Problem: Broken Learning Loop

Bayesian view on learning:

- 1. Prior belief:
 - P(Coauthor(Straus, Pauli)) = 0
- 2. Observe page
 P(Coauthor(Straus, Pauli)
 - Sheek Lin (b. Nat)

 | Control of the Control of the

- 3. Observe page
 - P(Coauthor(Straus, Pauli) | [] ,



Not desirate to the New Control of the New Control

This is mathematical nonsense!

Problem: Model Evaluation

Given:

Coauthor

Х	у	Р
Einstein	Straus	0.7
Erdos	Straus	0.6
Einstein	Pauli	0.9

Learn:

0.8::Coauthor(x,y) :- Coauthor(z,x) \land Coauthor(z,y).

OR

0.6::Coauthor(x,y) :- Affiliation(x,z) \land Affiliation(y,z).

What is the likelihood, precision, accuracy, ...?

Open-World Prob. Databases

Intuition: tuples can be added with $P < \lambda$

 $Q2 = Coauthor(Einstein, Straus) \land Coauthor(Erdos, Straus)$

$$0.7 * \lambda \geq P(Q2) \geq 0$$

Coauthor

X	Y	P
Einstein	Straus	0.7
Einstein	Pauli	0.9
Erdos	Renyi	0.7
Kersting	Natarajan	8.0
Luc	Paol	0.1

Coauthor

X	Y	Р
Einstein	Straus	0.7
Einstein	Pauli	0.9
Erdos	Renyi	0.7
Kersting	Natarajan	0.8
Luc	Paol	0.1
Erdos	Straus	λ

Open-world query evaluation

UCQ / Monotone CNF

- Lower bound = closed-world probability
- Upper bound = probability after adding all tuples with probability λ

- Polynomial time ©
- Quadratic blow-up
- 200 exabytes ... again ☺

Closed-World Lifted Query Eval

 $Q = \exists x \exists y \ Scientist(x) \land Coauthor(x,y)$

```
Decomposable ∃-Rule

o

o

o
```

```
P(Q) = 1 - \Pi_{A \in Domain} (1 - P(Scientist(A) \land \exists y Coauthor(A,y))
```

Check independence:

```
Scientist(A) \land \exists y \text{ Coauthor}(A,y)
Scientist(B) \land \exists y \text{ Coauthor}(B,y)
```

```
= 1 - (1 - P(Scientist(A) ∧ ∃y Coauthor(A,y))
x (1 - P(Scientist(B) ∧ ∃y Coauthor(B,y))
x (1 - P(Scientist(C) ∧ ∃y Coauthor(C,y))
x (1 - P(Scientist(D) ∧ ∃y Coauthor(D,y))
x (1 - P(Scientist(E) ∧ ∃y Coauthor(E,y))
x (1 - P(Scientist(F) ∧ ∃y Coauthor(F,y))
```

. . .

Complexity PTIME

Closed-World Lifted Query Eval

 $Q = \exists x \exists y \ Scientist(x) \land Coauthor(x,y)$

```
P(Q) = 1 - \Pi_{A \in Domain} (1 - P(Scientist(A) \land \exists y Coauthor(A,y))
```

```
= 1 - (1 - P(Scientist(A) ∧ ∃y Coauthor(A,y))
x (1 - P(Scientist(B) ∧ ∃y Coauthor(B,y))
x (1 - P(Scientist(C) ∧ ∃y Coauthor(C,y))

x (1 - P(Scientist(D) ∧ ∃y Coauthor(D,y))
x (1 - P(Scientist(E) ∧ ∃y Coauthor(E,y))
x (1 - P(Scientist(F) ∧ ∃y Coauthor(F,y))

...

No supporting facts in database!

Probability 0 in closed world

Ignore these sub-queries!
```

Complexity linear time!

Open-World Lifted Query Eval

 $Q = \exists x \exists y \ Scientist(x) \land Coauthor(x,y)$

```
P(Q) = 1 - \Pi_{A \in Domain} (1 - P(Scientist(A) \land \exists y Coauthor(A,y))
```

```
= 1 - (1 - P(Scientist(A) Λ ∃y Coauthor(A,y))
x (1 - P(Scientist(B) Λ ∃y Coauthor(B,y))
x (1 - P(Scientist(C) Λ ∃y Coauthor(C,y))

x (1 - P(Scientist(D) Λ ∃y Coauthor(D,y))
x (1 - P(Scientist(E) Λ ∃y Coauthor(E,y))
x (1 - P(Scientist(F) Λ ∃y Coauthor(F,y))

...

Probability λ in open world
```

Complexity PTIME!

Open-World Lifted Query Eval

 $Q = \exists x \exists y \ Scientist(x) \land Coauthor(x,y)$

```
P(Q) = 1 - \Pi_{A \in Domain} (1 - P(Scientist(A) \land \exists y Coauthor(A,y))
```

```
= 1 - (1 - P(Scientist(A) ∧ ∃y Coauthor(A,y))
x (1 - P(Scientist(B) ∧ ∃y Coauthor(B,y))
x (1 - P(Scientist(C) ∧ ∃y Coauthor(C,y))

x (1 - P(Scientist(D) ∧ ∃y Coauthor(D,y))
x (1 - P(Scientist(E) ∧ ∃y Coauthor(E,y))
x (1 - P(Scientist(F) ∧ ∃y Coauthor(F,y))

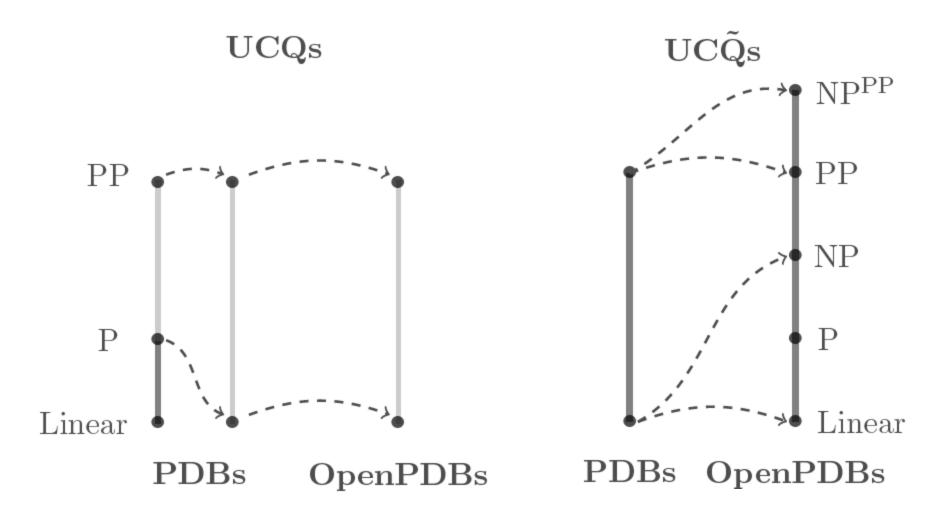
...

Probability p in closed world
```

Complexity linear time!

All together, probability (1-p)^k Exploit symmetry Lifted inference

Complexity Results



 $Linear \subseteq P \subseteq NP \subseteq PP \subseteq P^{PP} \subseteq NP^{PP} \subseteq PSpace \subseteq ExpTime$

Implement PDB Query in SQL

Convert to nested SQL recursively

 $Q = \exists x P(x) \land Q(x)$

Open-world existential quantification

0.0001 = open-world probability; 4 = # open-world query instances ior = Independent OR aggregate function

Conjunction

```
SELECT q9.c5,

COALESCE(q9.pUse, λ) *COALESCE(q10.pUse, λ) AS pUse

FROM

SQL(Q(X)) OUTER JOIN SQL(P(X))

SELECT Q.v0 AS c5,

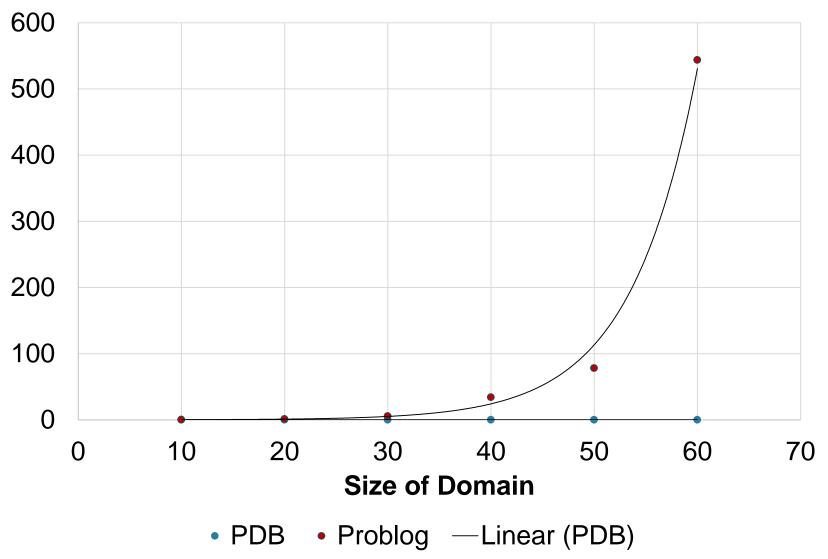
p AS pUse

FROM Q
```

Run as single \(\square

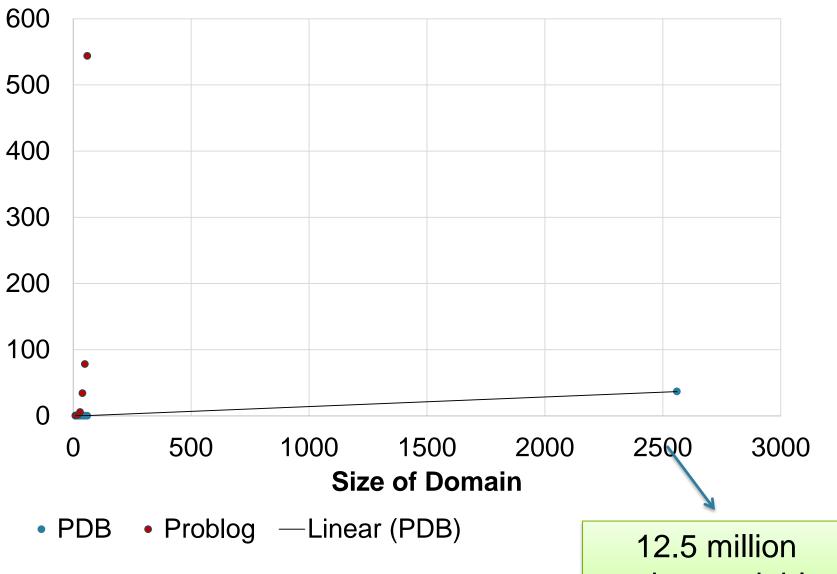


OpenPDB vs Problog Running Times (s)



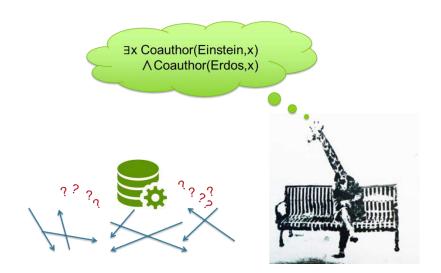
Out of memory trying to run the ProbLog query with 70 constants in domain

OpenPDB vs Problog Running Times (s)



random variables!

Querying Relational Embedding Models



Ongoing Work

- Run query evaluation in vector space?
 - DistMult model on Wordnet18RR
 - Query most likely answers to:

$$Q(h) = R(h, c')$$

$$Q(t) = R(c', t)$$

Solver: gradient descent on query probability

Prediction Method	MRR	hits@5	hits@1
Ranking Entities	0.305	0.383	0.234
Vector Search	0.301	0.377	0.232

Fast approximate query answering

Ongoing Work

- Join algorithms in vector space?
 - DistMult model on Wordnet18RR
 - Query most likely answers to:

```
Q(x) = Hypernym ('OliveTree', x) \land Hypernym(x, 'FloweringTree')
```

- Answer: x = 'SpiceTree'
- Projection (∃x)?
 - Skolemization in vector space
- More to come!

Conclusions

- You can do much more with knowledge bases/graphs than just completing missing tuples
- Let's tear down the wall(s) between
 - statistical models for knowledge base completion and
 - query evaluation systems
- Relational probabilistic reasoning is frontier and integration of AI, KR, ML, DB, TH, etc.
- Forward pointers @AKBC:
 - Arcchit Jain, Tal Friedman, Ondrej Kuzelka, Guy Van den Broeck and Luc De Raedt. <u>Scalable Rule Learning in Probabilistic Knowledge Bases</u>
 - Tal Friedman and Guy Van den Broeck.
 On Constrained Open-World Probabilistic Databases

QUESTIONS?

