

# SIMPSON'S PARADOX, ACTIONS, DECISIONS, AND FREE WILL

Judea Pearl  
UCLA  
www.cs.ucla.edu/~judea

## SIMPSON'S PARADOX

(Pearson et al. 1899; Yule 1903; Simpson 1951)

- Any statistical relationship between two variables may be reversed by including additional factors in the analysis.

Application: The adjustment problem

- Which factors should be included in the analysis.

## SIMPSON'S PARADOX (1951 – 1994)

$\frac{M}{T}$	$R$	$\bar{R}$		+	$\frac{\bar{M}}{T}$	$R$	$\bar{R}$		=	$\frac{T}{T}$	$R$	$\bar{R}$	
18	12	30			2	8	10			20	20	40	
7	3	10			9	21	30			16	24	40	
25 15 40					11 29 40					36 44 80			
0.6 < 0.7					0.2 < 0.3					0.5 > 0.4			

$T$  – Treated                       $\bar{T}$  – Not treated  
 $R$  – Recovered                     $\bar{R}$  – Dead  
 $M$  – Males                          $\bar{M}$  – Females

Easy question (1950-1994)

- When / why the reversal?

Harder questions (1994)

- Is the treatment useful? Which table to consult?
- Why is Simpson's reversal a paradox?

## SIMPSON'S REVERSAL

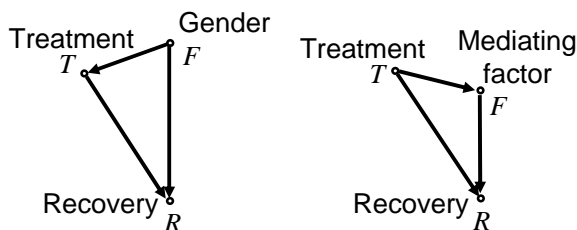
Group behavior:

$\Pr(\text{recovery} \mid \text{drug, male}) < \Pr(\text{recovery} \mid \text{no-drug, male})$   
 $\Pr(\text{recovery} \mid \text{drug, female}) < \Pr(\text{recovery} \mid \text{no-drug, female})$

Overall behavior:

$\Pr(\text{recovery} \mid \text{drug}) > \Pr(\text{recovery} \mid \text{no-drug})$

## TO ADJUST OR NOT TO ADJUST?



Solution:

Adjust iff  $F$  blocks all back-door paths

## THE INEVITABLE CONCLUSION: THE PARADOX STEMS FROM CAUSAL INTERPRETATION

**TWO PROOFS:**

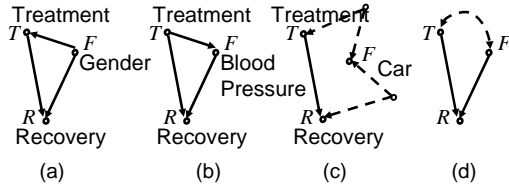
- Surprise surfaces only when we speak about "efficacy," not about evidence for recovery.  $P(R \mid do(T)) \neq P(R \mid T)$
- When two causal models generate the same statistical data and in one we decide to use the drug yet in the other not to use it, our decision must be driven by causal and not by statistical considerations.

Thus, there is no statistical criterion to warn us against consulting the wrong table.

Q. Can Temporal information help?

A. No!, see Figure (c).  $F = \text{Car Type}$

## WHY TEMPORAL INFORMATION DOES NOT HELP



- In (c),  $F$  may occur before or after  $T$ , and the correct answer is to consult the combined table.
- In (d),  $F$  may occur before or after  $T$ , and the correct answer is to consult the  $F$ -specific tables.

## WHY SIMPSON'S PARADOX EVOKES SURPRISE

1. People think causes, not proportions.
2. "Reversal" is possible in the calculus of proportions but impossible in the calculus of causes.

## CAUSAL CALCULUS PROHIBITS REVERSAL

Group behavior:

$$\Pr(\text{recovery} \mid \text{do}\{\text{drug}\}, \text{male}) < \Pr(\text{recovery} \mid \text{do}\{\text{no-drug}\}, \text{male})$$

$$\Pr(\text{recovery} \mid \text{do}\{\text{drug}\}, \text{female}) < \Pr(\text{recovery} \mid \text{do}\{\text{no-drug}\}, \text{female})$$

Assumption:

$$\Pr(\text{male} \mid \text{do}\{\text{drug}\}) = \Pr(\text{male} \mid \text{do}\{\text{no-drug}\})$$

Entailed overall behavior:

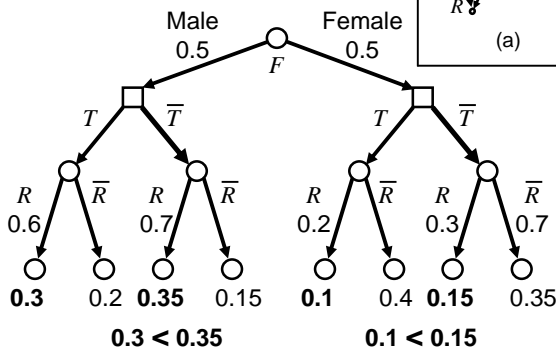
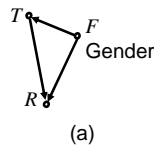
$$\Pr(\text{recovery} \mid \text{do}\{\text{drug}\}) < \Pr(\text{recovery} \mid \text{do}\{\text{no-drug}\})$$

## THE SURE THING PRINCIPLE

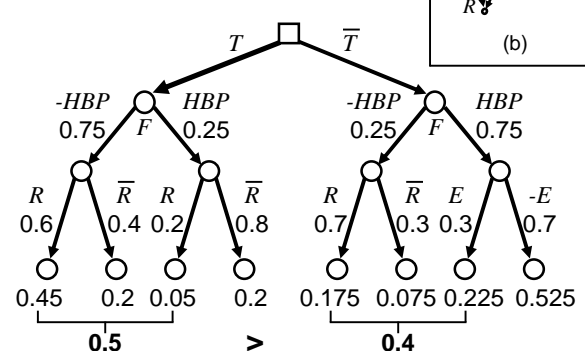
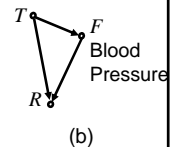
Theorem 6.1.1

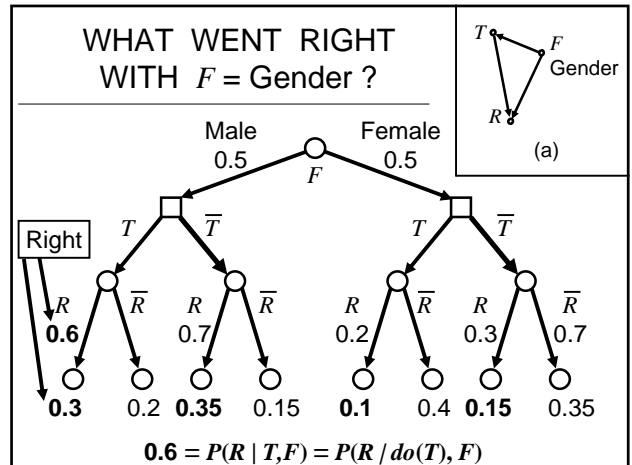
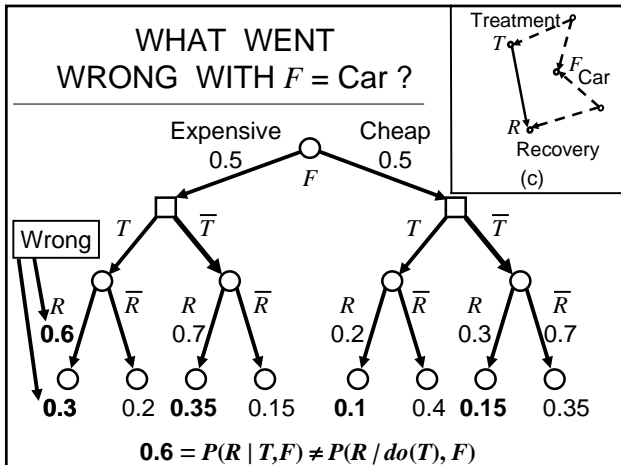
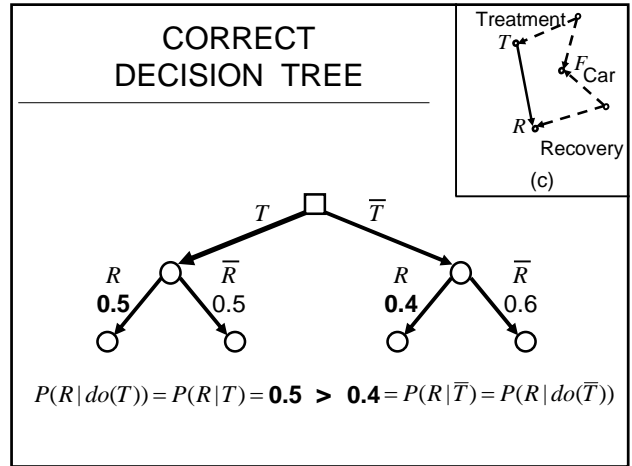
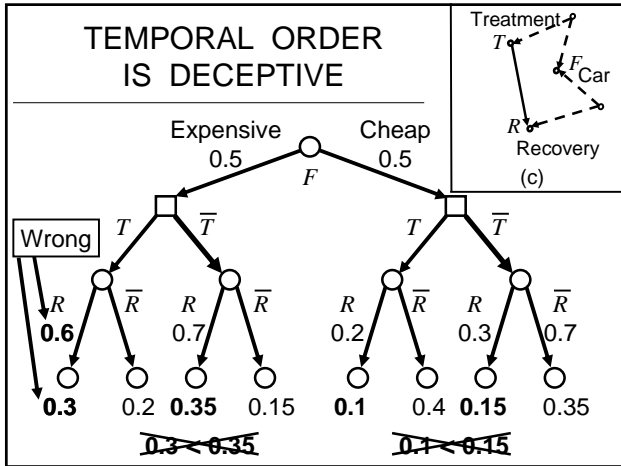
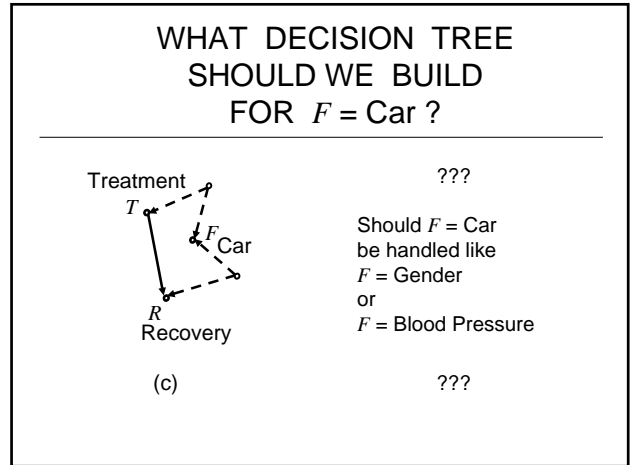
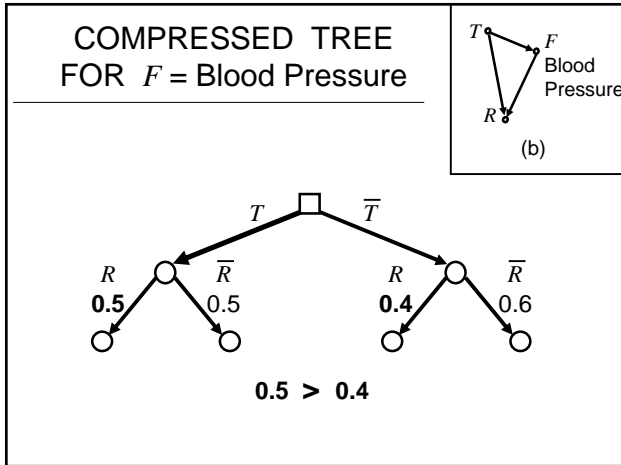
An action  $A$  that increases the probability of an event  $E$  in each subpopulation must also increase the probability of  $E$  in the population as a whole, provided that the action does not change the distribution of the subpopulations.

### DECISION TREE FOR $F = \text{Gender}$



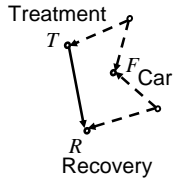
### DECISION TREE FOR $F = \text{Blood Pressure}$





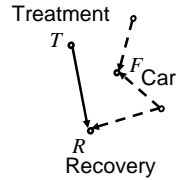
## THE PERCEPTION OF FREE WILL

Compelled Act  
(Explained reaction)



(c)

Free Act  
(Decision)



(c)

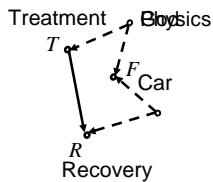
## DISAMBIGUATING THE TWO ROLES OF ACT

*Whatever evidence an act might provide  
On motives that cause such acts,  
Should never be used to help one decide  
On whether to choose that same act.*

(p. 109)

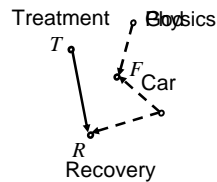
## THE PARADOX OF FREE WILL

Compelled Act  
(Explained reaction)



(c)

Free Act  
(Decision)



(c)

## COMPUTATIONAL RESOLUTION OF THE PARADOX OF FREE WILL

Free will is an illusion that demands a computational model, to explain:

1. What mental activity generates the illusion of acting freely?
2. Why/how is such illusion generated from the activity above?
3. How can we survive the distortion generated by the illusion? Is the distortion harmless? Is the illusion dispensable?
4. What is the computational usefulness of the illusion?

## CONCLUSIONS

1. Simpson's Paradox is resolved through the calculus of causation.
2. People think causally, not probabilistically
3. Decision trees cannot replace causal models
4. Free-will is a software illusion to be explained in computational terms.