Learning Probabilistic Sentential Decision Diagrams Under Logic Constraints by Sampling and Averaging

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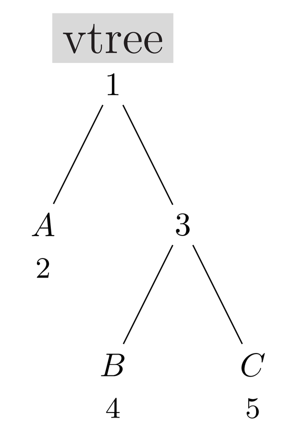
1. Motivation

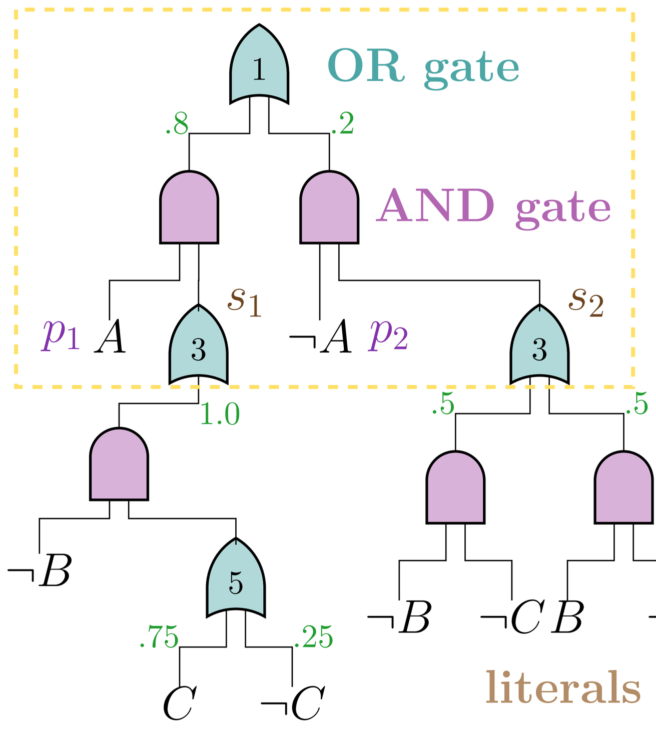
Probabilistic Sentential Decision Diagrams (PSDDs):

- Structured Decomposable probabilistic circuits
- Encode certain knowledge as logic constraints
- Encode uncertain knowledge as probabilities
- Interpretable syntax
- Many inferences are exact and tractable:
  - ☑ Evidence
  - ☑ Marginals
  - ☑ MLE Parameter Learning
  - ☑ Most Probable Explanation
  - ☑ Expectations
  - ☑ KL-divergence

A	B	C	Pr
0	0	0	**0.1**
0	1	0	**0.1**
1	0	0	**0.2**
1	0	1	**0.6**

s.t. $(A \to \neg B) \wedge (C \to A)$





- PSDD circuit represents recursive decomposition of formula:

$$\bigvee_{i=1}^{k} (p_i \wedge s_i),$$ where each *prime* p_i and *sub* s_i are logical formulae

Existing PSDD learners:

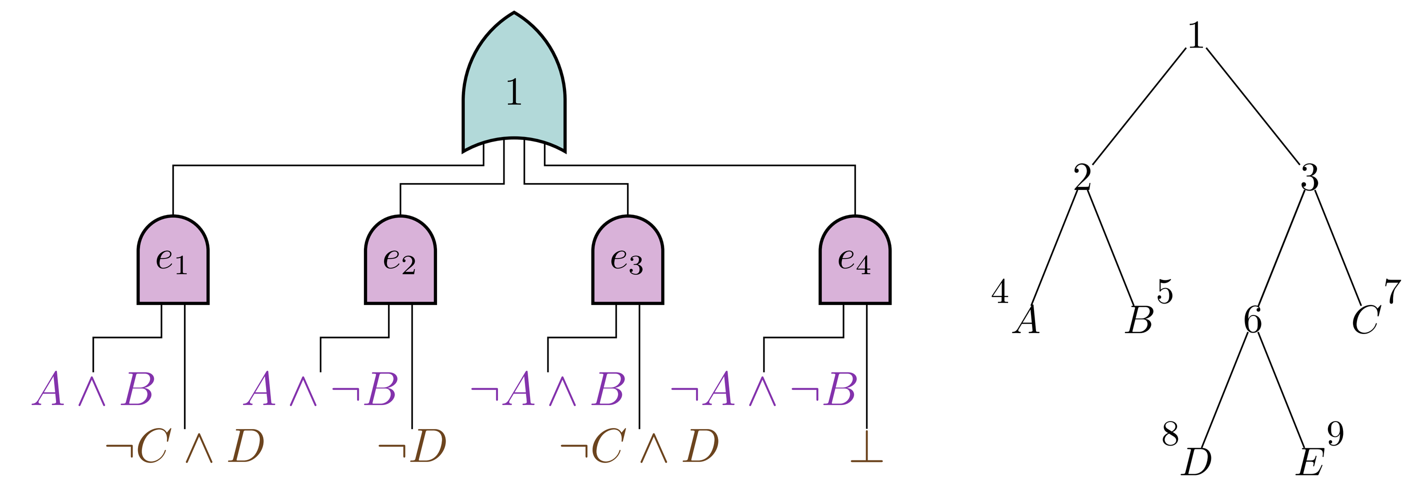
- require an initial PSDD encoding the support;
- scale poorly to complex formulae and/or high dimension.

This Work: How to effectively learn PSDDs s.t. complex formula?

2. SamplePSDD

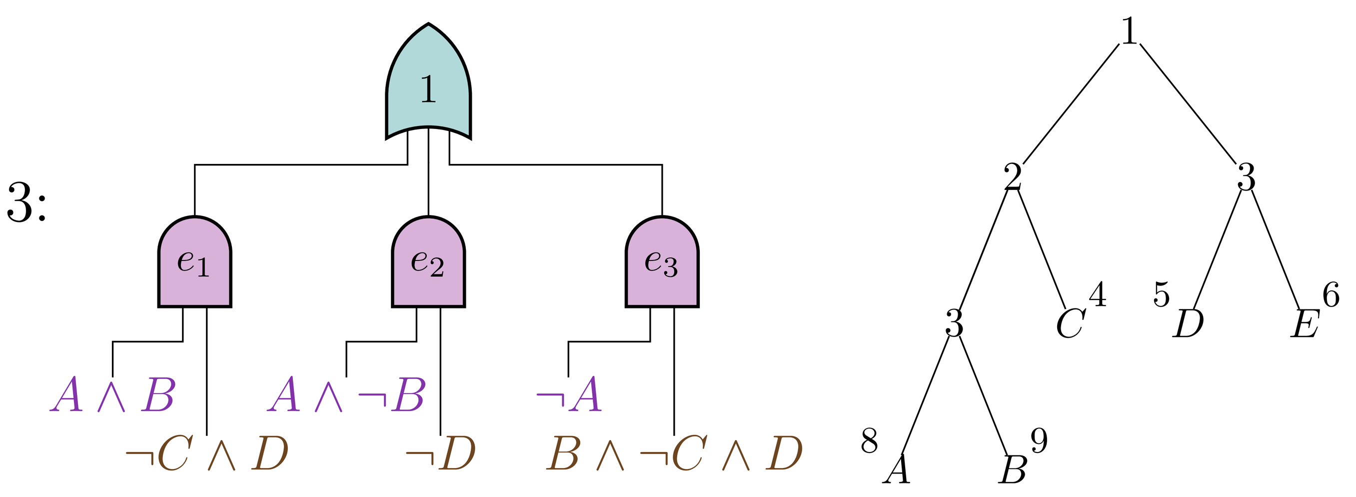
- Common assumption: primes p_i are conjunctions of literals.

$$\phi(A,B,C,D) = (A \wedge \neg B \wedge \neg D) \vee (B \wedge \neg C \wedge D)$$

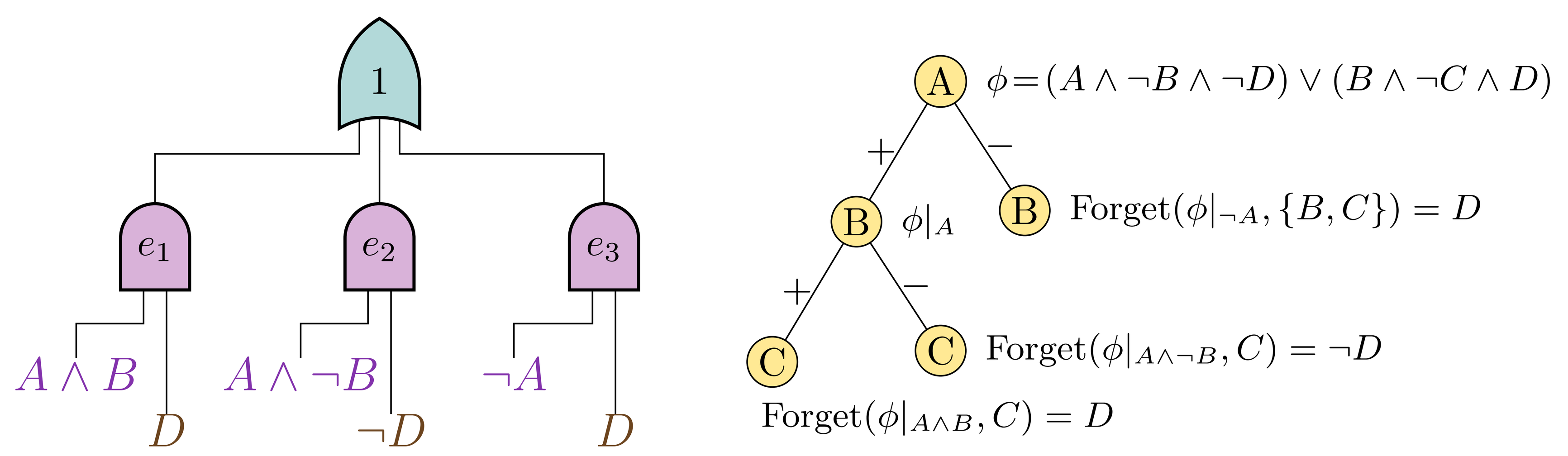


- **Problem:** Size of circuit is exponential in the size of p_i
- **Solution:** randomly sample a bounded number (k) of p_i
- **But:** this violates structure decomposability

Example, $k = 3$:



- **New solution:** Relax logical constraints ϕ



3. Experiments

Evaluation: we sample 30 PSDDs and use 5 ensemble strategies:

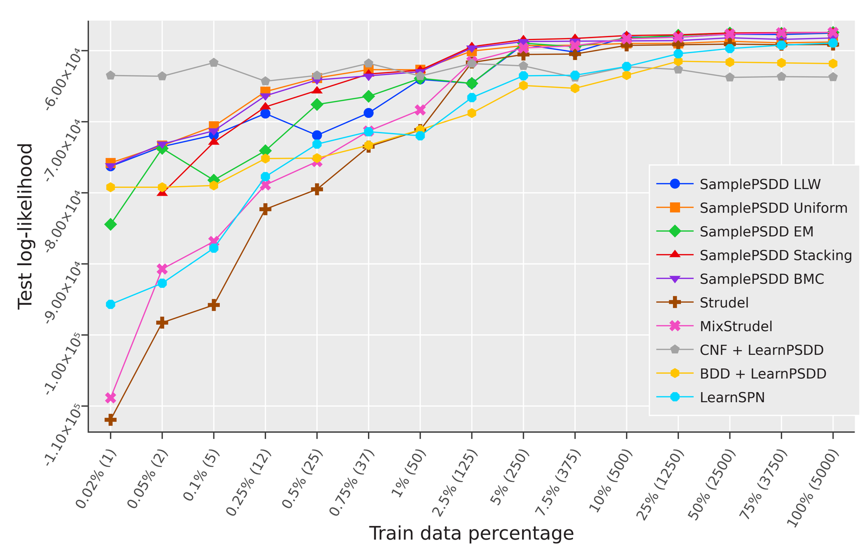
1. Likelihood weighting (LLW),
2. Uniform weights,
3. Expectation Maximization (EM),
4. Stacking,
5. Bayesian Model Combination;

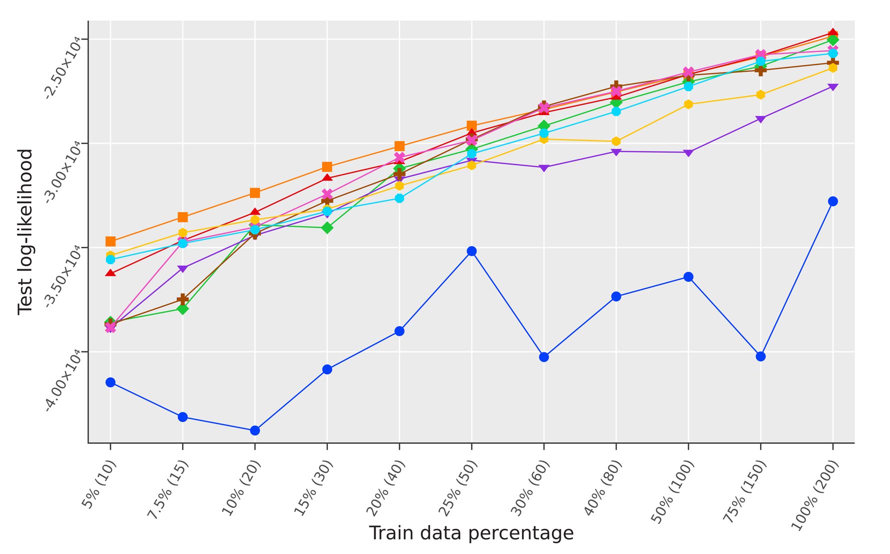
comparing with Strudel, LearnPSDD and LearnSPN.

Datasets: we evaluate with 5 data + knowledge as logic constraints:

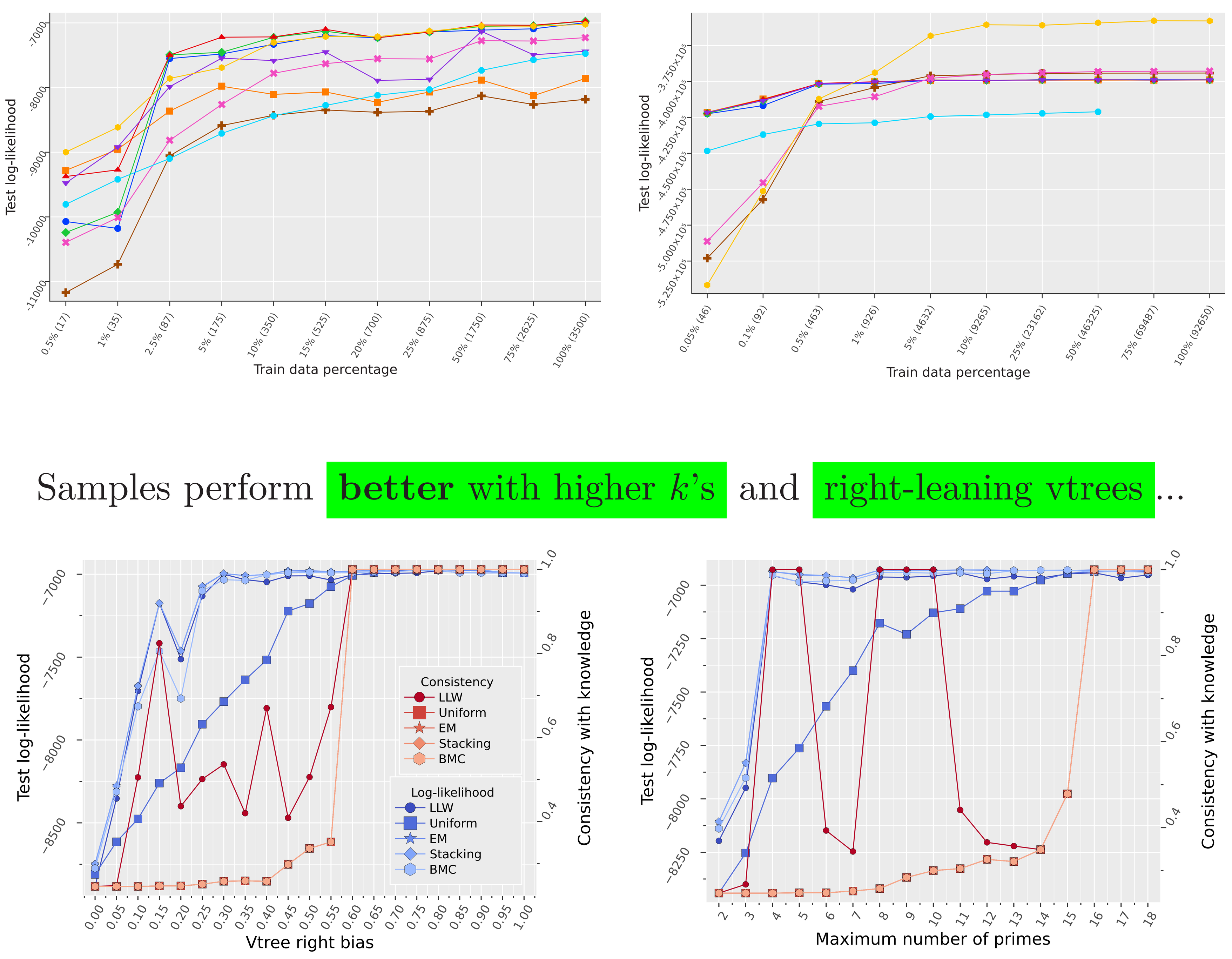
Dataset	#vars	#train	ϕ's size
LED	14	5000	23
LED + Images	157	700	39899
Sushi Ranking	100	3500	17413
Sushi Top 10	10	3500	37
Dota 2 Games	227	92650	1308

Our approach fares **better with fewer** data, yet remains **competitive** under **lots of data**.





Samples perform **better with higher k's** and **right-leaning vtrees**...



...but at a **cost** to complexity.